Influence of model assumptions on radial flow investigations in heavy-ion collisions

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Abstract

An effective way for the investigation of nuclear matter under the extreme conditions of high density and temperature is the study of nuclear fragmentation, realized in intermediate-energy heavy-ion collisions. The potential energy accumulated in the compression zone of the collision is converted into collective flow of matter in the later stages of the reaction. During the expansion of the system, fragments are formed in a clusterization process. In the final stage of the reaction, the fragments are moving along Coulomb trajectories and may deexcite by particle emission or secondary break-up. In order to verify the influence of the collective energy on the experimental energy spectra and extract quantitative information on the thermal and collective components, a model simulation was developed. In
this procedure, the evolution of the disintegrating system in a multi-
fragmentation process was described after the freeze-out stage with a
Monte-Carlo approach. A reconstruction procedure for the fragment
kinetic energies has shown the possibility to obtain the thermal and
collective components from measured fragment spectra. In the present
work, we scrutinize the model simulation for the evolution of the disin-
tegrating system by testing the sensitivity of the collective expansion
ergy extraction on the model assumptions concerning the freeze-out
characteristics, such as the size and shape of the freeze-out volume.
Furthermore, thermal aspects of the early stage of the expansion are
tested with an examination of the relationship between the assump-
tion of the degree of thermalization of the emitting source and the
extracted value of the collective energy.

1 Introduction

A great deal of attention in nuclear physics research is focused on the in-
vestigation of nuclear matter under the extreme conditions of high density
and temperature. An effective way for such an investigation is the study of
nuclear fragmentation. In energetic violent heavy-ion collisions not only the
heating up, but also significant compression of nuclear matter is possible.
The first predictions of this phenomenon resulted from hydrodynamic calcu-
lations of the nuclear collision [1], [2] and the first experimental evidence of
the occurrence of the radial flow was obtained ten years latter [3], [4]. The
further investigations allowed to establish systematic trends realated to the
collective radial energy [5], [6], [7]. The potential energy accumulated in the
compression zone is converted into collective flow of matter in the later stages
of the reaction, after a initial partition of fragments is formed. In the final
stage of the reaction, the fragments are moving along Coulomb trajectories
and may deexcite by particle emission or secondary break-up.

The study of collective flow in heavy-ion collisions can provide informa-
tion about the stiffness of nuclear matter and the parameters of the equation
of state (EOS) involved in the disassembly mechanisms of excited nuclear
matter. Due to a weak coupling between the intrinsic and collective degrees
do freedom, the fragment kinetic energy can be decomposed into two terms,
a thermal one and the radial one. A signature of the collective motion can
be found in the kinetic energy spectra of the emitted fragments. It is usu-
ally assumed that the average collective energy of the species is proportional to their masses, whereas the average thermal energy is distributed independently of the fragment mass. Consequently, the thermal component plays a predominant role in the emitted light charged particles (LCP), whereas the collective phenomena prevail in the kinetic energies of heavy species, i.e. the intermediate mass fragments (IMF).

In the following, we study the influence of the collective energy on the fragment energy spectra using a model simulation. The evolution of the disintegrating system in a multifragmentation [8] process is described after the freeze-out stage with a Monte-Carlo approach. Furthermore, a reconstruction procedure for the fragment kinetic energies has shown the possibility to obtain the thermal and collective components from experimentally measured fragment energy spectra [9]. The purpose of the present paper is to scrutinize the model simulation for the evolution of the disintegrating system, by testing the sensitivity of the collective expansion energy on model assumptions concerning the freeze-out stage characteristics.

2 Monte-Carlo simulation of the system evolution after the freeze-out stage

The Monte-Carlo simulation of the system disassembly starts with an initial configuration of the freeze-out stage characterized by a given shape and size of the freeze-out volume, a fragment partition, the position distributions of fragments within this volume, the average collective energy per nucleon and average fragment thermal energy. In the following simulations we employ initial parameters pertaining to experimental data of $^{64}$Zn + $^{64}$Zn collisions at 47 AMeV [10]. The choice of values of average collective energy per nucleon and average fragment thermal energy follows the values estimated from different experimental data analyses [11], [12]. In the next step of the procedure, the freeze-out conditions are defined for the excited nuclear system. For a given fragment the collective energy is assumed to be proportional to the fragment mass and to the initial distance of the fragment from the center of mass of the freeze-out volume. The evolution of the system is followed, taking into account the Coulomb interaction in the fragment trajectories. As a result, the total energy spectra for fragments consisting of protons up to
12C are obtained. A detailed account of the Monte-Carlo procedure is given in [13].

3 Consequences of model assumptions

We test the influence of model assumptions concerning the freeze-out characteristics on the collective expansion energy derivation. Simulations were performed for a variety of sets of assumed sizes and shapes for the freeze-out volume. Furthermore, thermal aspects of the early stage of the expansion were tested with an examination of the relationship between the degree of thermalization of the emitting source and the collective energy. The results presented in the following Sections are based exclusively on a comparative analysis of model simulations. The experience gathered from an analysis with fully controllable initial conditions is necessary for a successful application of the method to the experimental energy spectra.

The influence of model assumptions on collective energy investigations was studied with comparisons made with respect to a reference simulation for the 64Zn + 64Zn reaction with the following initial conditions: average collective energy per nucleon \( \langle E_{\text{coll}} \rangle = 2 \, \text{MeV} \), average thermal energy \( \langle E_{\text{th}} \rangle = 7 \, \text{MeV} \), spherical freeze-out volume, minimum distance between fragment surfaces \( d = 2 \, \text{fm} \), and global thermalization in the freeze-out volume.

3.1 Deformation of the shape of the freeze-out volume

In central collisions, the assumption of a spherical freeze-out volume may could be questioned, since an oblate configuration may be more appropriate. Furthermore, in peripheral collisions, a prolate configuration is certainly more reasonable. In order to take into account such deformations, we introduce an axially symmetric ellipsoidal shape parametrization (with principal axes \( a \) and \( b \)) for the freeze-out volume. We consider a broad range of prolate and oblate deformations with an axis ratio \( a/b = 0.3, 0.5 \) and \( a/b = 2, 3 \), respectively. In a collision, the deformed ellipsoid attains random orientations with respect to the beam direction. For a given deformation, the size of the system was adjusted in order to comply with volume conservation.

In the case of deformed freeze-out volumes, an averaging of the fragment
position distribution occurs, due to the randomization of the orientation in space of the emission zone. The collective velocity is proportional to the fragment distance from the center of the emission zone. Thus, the energy spectra of fragments arising from a deformed source are expected to reach a higher energy limit than in the spherical case. However, this effect may be compensated by the Coulomb energy decrease induced by the increased average fragment distance in the deformed freeze-out volume. From the simulation, the energy spectra originate from an emission zone with different orientations of the ellipsoidal shape with respect to the beam direction. Therefore, substantial differences between the energy spectra of fragments emitted from spherical and deformed sources are expected. In order to examine the possibility of compensation of deformation effects in the energy spectra through a variation of the energy variables, simulations were performed for a combination of average collective energies $\langle E_{\text{coll}} \rangle = 0, 2, 4 \text{ AMeV}$ and average thermal energies $\langle E_{\text{th}} \rangle = 1, 3, 7, 10 \text{ MeV}$, assuming oblate and prolate ellipsoidal deformations ($a/b = 2$ and $a/b = 0.5$). Figure 1 shows the calculated spectra for alpha particles assuming an oblate deformation with $a/b = 2$. In each panel, solid lines show the energy spectra with the indicated values of $\langle E_{\text{coll}} \rangle$ and $\langle E_{\text{th}} \rangle$. Superimposed is the energy spectrum with $\langle E_{\text{coll}} \rangle = 2 \text{ AMeV}$ and $\langle E_{\text{th}} \rangle = 7 \text{ MeV}$, represented with a dotted line, in all panels. Figure 2 shows the results of this simulation for $^{12}$C fragments, with the same meaning for the lines. From Figure 1, the strong differences between the two sets of spectra imply that deformation cannot be compensated by some combination of $\langle E_{\text{coll}} \rangle$ and $\langle E_{\text{th}} \rangle$. From Figure 2, we see that compensation can be achieved for $^{12}$C fragments with $\langle E_{\text{coll}} \rangle = 2 \text{ AMeV}$ and $\langle E_{\text{th}} \rangle = 10 \text{ MeV}$. However, this combination of $\langle E_{\text{coll}} \rangle$ and $\langle E_{\text{th}} \rangle$ does not compensate the alpha particle spectrum. Similar conclusions can be drawn from an inspection of a simulation assuming an oblate deformation with $a/b = 0.5$, shown in Figs. 5 and 6, for alpha particles and $^{12}$C fragments, respectively.

From Figs. 1 and 2 we can draw the following conclusions. The deformation signatures in simulated spectra are not sufficiently well pronounced to be used for the shape investigations, even with comparison to a high quality experimental data. This means that the thermal and collective energy extraction from the energy spectra is insensitive to the moderate deviation of the shape from the spherical. Although the compensation of deformation effects, as manifested by the smooth and monotonic change of the energy spectra, is possible for individual fragments, a single combination of $\langle E_{\text{th}} \rangle$


and \( \langle E_{\text{coll}} \rangle \) is not able to account for a simultaneous compensation for all fragments.

3.2 The size of the freeze-out volume

The size of the freeze-out volume is a model assumption not fully justified from the ground of theoretical predictions or experimental evidence. Usually, chemical equilibrium in the freeze-out stage is assumed, in order to be concerned with a well defined system. This assumption implies a fixed partition of fragments in the disintegration zone, due to the cessation of the interaction between the fragments. The condition for no-interaction is achieved by considering a sufficiently large freeze-out volume or by setting a requirement on a minimum distance between the fragments inside the freeze-out volume. The last approach follows the concept of the proximity potential [REF], which suggests that the nuclear interaction between two fragments vanishes at a distance between their surfaces larger than 2 fm. This value has been adopted by default in many investigations dealing with the concept of the freeze-out system. There is no convincing evidence of the influence of this assumption on the stability of the radial flow extraction. This topic was investigated with additional simulations.

In Fig. 3 we show the average position \( \langle r \rangle \) of protons, \(^4\text{He}\) and \(^{12}\text{C}\) as a function of the minimum distance between their surfaces in the spherical freeze-out volume. For all fragments, the non-linear trend of \( \langle r \rangle \) is understood from the geometry of the system. For a given distance between the fragment surfaces, the relative values of \( \langle r \rangle \) are also understood in terms of the relative numbers of fragments within the freeze-out volume.

One may expect some influence of freeze-out volume size on the fragment energy spectra as an interplay of two opposing factors. On one hand, a compact freeze-out configuration implies an increase in the Coulomb potential energy, which is subsequently converted into kinetic energy. This results in an increase of the average kinetic energy of the emitted fragments. On the other hand, the kinetic energy of the fragments is predefined by their positions in the freeze-out volume. This is inherent to the assumption that the fragment velocities are proportional to their distances from the center of mass of the expanding system.

In order to study this effect, the following model simulations were performed. Two values of minimum fragment distance were chosen, namely,
\(d = 1\, fm\) and \(d = 3\, fm\), deviating from the commonly used value of \(d = 2\, fm\) by 1\, fm. Then, an attempt was made to obtain consistency between the calculated spectra by adjusting the collective energy, in each case. It was found that, at a qualitative level, consistency may be achieved with a variation of the collective energy within ±25\% its nominal value. Results of these simulations are given in Fig. 4. In this figure, the dotted lines show the simulated energy spectra of fragments ranging from protons up to \(^{12}\text{C}\), with \(\langle E_{\text{coll}}\rangle = 1\, AMeV\) and \(d = 1\, fm\). The solid lines correspond to \(\langle E_{\text{coll}}\rangle = 1.5\, AMeV\) and \(d = 3\, fm\). In both cases, \(\langle E_{\text{th}}\rangle = 7\, MeV\) was assumed. Apart from small differences, the two sets of spectra seem close to one another. The effect of using the greater value of \(d = 3\, fm\) was almost compensated by assuming an average collective energy of 1.5\, MeV. We realize that the spectra of \(p\), \(d\) and \(^{3}\text{He}\) fragments remain almost unaffected, whereas stronger differences appear in the rest of the spectra, especially of the heaviest fragments.

From Fig. 4, we realize that considering a freeze-out volume with \(d = 3\, fm\), it is possible to compensate the calculated energy spectra with an increase in the average collective energy. This may happen in a reaction system with a small average collective energy. In both cases, the effect of increasing the freeze-out volume size in the calculated energy spectra is moderate. This is consistent with our previous discussion, suggesting a partial cancellation of the two opposing factors which influence the calculated fragment energy spectra.
3.3 The thermal equilibrium assumption

The assumption of thermal and chemical equilibrium at the freeze-out stage is commonly used in multifragmentation investigations due to the convenience of the thermodynamic approach. However, the justification of this assumption is not straightforward and is difficult to verify experimentally. Problems arise for the following reasons:

- The disintegrating nuclear system is characterized by a limited number of degrees of freedom and does not conform with the requirements of the thermodynamic limit \( N \to \infty \)
- The finite size of the system implies fluctuations of the thermodynamic parameters and smears out the signatures of phenomena like phase transitions, and
- Nuclear reactions inevitably involve coupling of the internal degrees of freedom with the dynamics of the reaction.

In experimental data analyses, the selection of events with a well defined temperature is very difficult. Methods of selecting temperature limits are usually based on estimates of the centrality of the collision. The collision centrality is considered to be related to the multiplicity of emitted light particles and intermediate mass fragments. Even if a rigorous filter is imposed on the data, the selection does not guarantee the strict definition of the emitting source temperature.

For the purpose of our simulations, we employ an experimental fragment partition distribution for central collisions of \(^{64}Zn + ^{64}Zn\) at 47 AMeV [REF]. The partition distribution shown in Fig. 5 corresponds to selected events by requiring the detected total LCP and IMF charge number, at backward angles, to be greater than 25. This is an appropriate selection filter for mid-central and central collisions.

For the investigation of the energy spectra of fragments emitted from sources of partial thermal equilibrium, we introduce a global ratio coefficient

\[
p_{\text{glob}} = \frac{E_{\text{COLL}}}{E_{\text{TOT}}} = 1 - \frac{E_{\text{TH}}}{E_{\text{TOT}}} \tag{1}
\]
where $E_{\text{COLL}}$ is the total collective energy and $E_{\text{TOT}}$ is the sum of total thermal, collective and Coulomb energy available in a collision event. The total energy is written as

$$E_{\text{TOT}} = E_N \cdot N_{\text{nuc}}$$  \hspace{1cm} (2)$$

where $E_N$ is the total energy per nucleon and $N_{\text{nuc}}$ is the number of nucleons of the emitting source in the $k$–th partition.

For reference, we simulated fragment energy spectra corresponding to a spherical freeze-out volume and $\langle E_{\text{th}} \rangle = 7$ MeV and $\langle E_{\text{coll}} \rangle = 2$ AMeV. These energy spectra were compared with the results of simulations using $E_N$ and $p_{\text{glob}}$ as input parameters. A two dimensional search in the $(E_N, p_{\text{glob}})$ space allowed us to find that the combination $(E_N = 4$ MeV, $p_{\text{glob}} = 0.5)$ closely reproduces the LCP and IMF energy spectra obtained from simulations assuming a global thermal equilibrium.

In the next step of analysis, we investigated the temperature distributions corresponding to the assumption of non-global thermal equilibrium of the emitting source. Denoting by $T_k$ the temperature parameter associated with the $k$–th partition of fragments in the freeze-out stage, involving $N_{\text{frag}}$ fragments, we have

$$T_k = \frac{2}{3} \frac{E_{\text{TH}}}{N_{\text{frag}}}$$  \hspace{1cm} (3)$$

Combining Eqs. 1, 2 and 3, we obtain

$$T_k = \frac{2}{3} E_N (1 - p_{\text{glob}}) \frac{N_{\text{nuc}}}{N_{\text{frag}}}$$  \hspace{1cm} (4)$$

From this expression and the experimental partition distribution of Fig. 5, temperature distributions corresponding to different values of the parameters $E_N$ and $p_{\text{glob}}$ were produced. Temperature distributions corresponding to $p_{\text{glob}} = 0.2$ and 0.5, are shown in the two panels of Fig. 6, for different values of the parameter $E_N$. From this figure, it is seen that the average temperature of the distribution with $E_N = 4$ MeV and $p_{\text{glob}} = 0.5$ is close to 6 MeV, whereas the parameters used in the global thermal equilibrium simulation correspond to a temperature of $T = \frac{2}{3} \langle E_{\text{th}} \rangle = 4.6$ MeV. Therefore, using the global or non-global thermalization assumptions leads to temperature parameters differing by about 1.5 MeV. Furthermore, substantial differences in the mean values of fragment collective energies are noticed in
each case. In the global thermalization scenario \( \langle E_{\text{coll}} \rangle = 2 \text{AMeV} \) was introduced, whereas the average collective energy calculated using \( E_N = 4 \text{MeV} \) and \( p_{\text{glob}} = 0.5 \) was found to be close to 1 MeV, for all fragments under consideration.

A comparison was made of the results of simulations performed for global and non-global equilibrium hypothesis using in both cases the similar values for the ratio coefficient \( p \) per nucleon. In the equilibrium simulation, the fixed values of the mean thermal and collective energies, \( \langle E_{\text{th}} \rangle = 7 \text{MeV} \) and \( \langle E_{\text{coll}} \rangle = 2 \text{AMeV} \), yield a value for the ratio coefficient \( p = \frac{\langle E_{\text{coll}} \rangle}{\langle E_{\text{coll}} \rangle + \langle E_{\text{th}} \rangle} = 0.22 \). For this set of parameters, \( \langle T \rangle = 4.6 \text{MeV} \). From Fig. 6, it is seen that for \( p_{\text{glob}} = 0.2 \), the average temperature of about 4.6 MeV corresponds to the temperature distribution for \( E_N = 2 \text{MeV} \).

Figure 7 shows the comparison of LCP and IMF energy spectra obtained in these simulations. Dotted lines show the energy spectra of the indicated fragments under the assumption of global thermal equilibrium. Thin lines correspond to the assumption of non-global thermal equilibrium with \( E_N = 2 \text{MeV} \), \( p_{\text{glob}} = 0.2 \), and the thick ones to \( E_N = 4 \text{MeV} \), \( p_{\text{glob}} = 0.5 \). As seen in this figure, the application of different hypotheses concerning the global or not degree of thermalization of the emitting source, assuming however a similar value for the ratio coefficient and a similar value for the mean temperature, results in substantial differences in the fragment energy distributions and strong differences in the collective energies.
4 Conclusions

The analysis presented in this paper concerns the influence of simulation assumptions on the results of investigations of the collective expansion energy in intermediate energy heavy-ion collisions. For this purpose, we examined the sensitivity of the fragment energy spectra on the freeze-out volume deformation, freeze-out volume size, as well as implications arising from non-global thermal equilibrium.

Our analysis shows that the freeze-out volume deformation does not affect substantially the shape of energy spectra and does not have a significant influence on the collective energy extraction. Therefore, the investigation of initial shape of the expanding nuclear system is not feasible. The randomization of orientation of deformed freeze-out configurations about the beam direction, in inclusive experiments, smears out the deformation signatures in the fragment energy spectra. In this respect, experiments performed with polarized beams, or experiments with a well defined reaction plane could be more helpful.

Our investigation on the correlation between the calculated radial expansion energy and the freeze-out volume size was based on adjusting the minimum relative fragment distance. It was found that a variation of the standard minimum distance of $d = 2 \text{ fm}$ by 1 fm may influence the energy spectra due to the change in the Coulomb potential energy. A compensation of the Coulomb energy change is possible with a modification in the radial expansion energy. We conclude that there is an ambiguity in the two-dimensional space of freeze-out size and radial energy parameters. This ambiguity is consistent with a 25% modification in the radial energy, for a variation of the minimum distance between fragments from $d = 1 \text{ fm}$ to $d = 3 \text{ fm}$. However, it is anticipated that using the more realistic value of $d = 2 \text{ fm}$ results in an inappreciable ambiguity in the expansion energy derived from experimental data analyses.

The influence of the thermal equilibrium assumption on the calculated fragment energy spectra was studied with simulations performed by using a distribution of freeze-out stage temperatures about a mean value. Thermo-
dynamic aspects resulting in such a temperature distribution and a numerical
method of realization were discussed. The results of this analysis seem to moderate the power of global thermal equilibrium assumption on the reconstruction of the radial energy from experimental fragment energy spectra. The assumption of global thermal equilibrium presents a convenient simplification rather than an experimentally confirmed fact. From the experimental point of view, the influence of partial thermalization could be effectively reduced by imposing multidimensional rules selecting the centrality of the collision. A rigorous selection confines the region of system excitation, resulting in diminishing temperature fluctuations in the population of the analyzed collision events. In conclusion, the obtained results allow us to state that despite the ambiguities of procedures in the determination of the expansion energy, the experimental procedures may offer us opportunities for their limitation in cognitive processes.
References


of the IWM2001 International Workshop on Multifragmentation and Related Topics, Catania, Italy, November 28 - December 1, 2001, p. 49.


FIGURE CAPTIONS

Fig. 1 Energy spectra of alpha particles from the freeze-out volume with an oblate deformation \((a/b = 2)\). Calculated spectra with the indicated values of \(\langle E_{\text{coll}} \rangle = 0, 2, 4\) \(\text{AMeV}\) and \(\langle E_{\text{th}} \rangle = 1, 3, 7, 10\) \(\text{MeV}\) are shown with solid lines, in each panel. They are compared with the calculated energy spectrum with \(\langle E_{\text{coll}} \rangle = 2\) \(\text{AMeV}\) and \(\langle E_{\text{th}} \rangle = 7\) \(\text{MeV}\), spherical shape, represented with dotted lines, in all panels.

Fig. 2 Same as Fig. 3, for \(^{12}\text{C}\) fragments.

Fig. 3 Average position of protons, \(^{4}\text{He}\) and \(^{12}\text{C}\) fragments versus their minimum distance in the spherical freeze-out volume.

Fig. 4 Simulated energy spectra of the indicated fragments, with \(\langle E_{\text{coll}} \rangle = 1\) \(\text{AMeV}\) , \(d = 1\) \(\text{fm}\) (dotted lines), and \(\langle E_{\text{coll}} \rangle = 1.5\) \(\text{AMeV}\) , \(d = 3\) \(\text{fm}\) (solid lines). In both cases, \(\langle E_{\text{th}} \rangle = 7\) \(\text{MeV}\) was assumed.

Fig. 5 Distribution of the number of partitions versus nucleon and fragment numbers for a \(^{64}\text{Zn} + ^{64}\text{Zn}\) central collision.

Fig. 6 Temperature distributions for the indicated values of \(E_N\), assuming \(p_{\text{glob}} = 0.2\) (left panel) and \(p_{\text{glob}} = 0.5\) (right panel).

Fig. 7 Dotted lines show the simulated energy spectra of the indicated fragments under the assumption of thermal equilibrium with \(\langle E_{\text{th}} \rangle = 7\) \(\text{MeV}\) and \(\langle E_{\text{coll}} \rangle = 2\) \(\text{AMeV}\) . Thin lines correspond to the assumption of non-global thermal equilibrium with \(E_N = 2\) \(\text{MeV}\) , \(p_{\text{glob}} = 0.2\), and the thick ones to \(E_N = 4\) \(\text{MeV}\) , \(p_{\text{glob}} = 0.5\).