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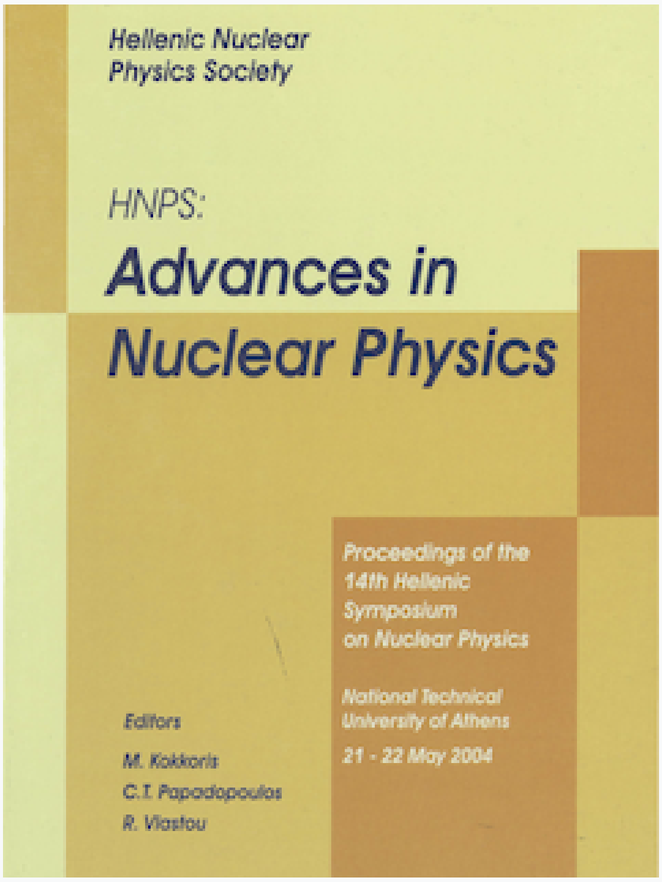

The Nuclear Equation of State in Heavy Ion Collisions

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The Nuclear Equation of State in Heavy Ion Collisions

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Abstract

We present several possibilities offered by the dynamics of intermediate energy heavy ion collisions to investigate the nuclear matter equation of state (EoS) beyond the ground state. In particular the relation between the reaction dynamics and the high density nuclear EoS is discussed by comparing theoretical results with experiments.

Key words: Nuclear matter, equation of state, symmetry energy, heavy ion collisions

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1. Introduction

Heavy ion collisions (HIC) open the unique possibility to study nuclear matter under extreme conditions of baryon density, temperature and isospin asymmetry. Such investigations, which are still an object of current interest [1], are important for the understanding of astrophysics like the physical mechanism of supernovae explosions and neutron starts [2]. The traditional method to study the high density dependence of the nuclear equation of state (EoS) in HIC at relativistic bombarding energies (0.1-2 AGeV) consists of comparisons between theoretical predictions and experimental data. Theoretically a HIC is modeled by the Boltzmann equation which describes the phase space evolution of a semi-classical phase space distribution function f(x,p) under the influence of a baryonic mean field and binary collisions, see Refs. [3,4]. The physical input are therefore the nuclear EoS (mean field) and the nucleon-nucleon (NN) cross sections for 2-body collisions.

In these proceedings we give an overview on recent studies on HIC at relativistic energies from 0.1 up to 2 AGeV bombarding energy per nucleon.

2. The equation of state (EoS)

The traditional approach to investigate the properties of nuclear matter at and beyond the ground state consists of the so-called mean field (MF) approximation of the Quantumhadrodynamics [7]. In the MF theory the baryons (protons and neutrons), described relativistically by the Dirac equation, interact trough a classical mean field that is given by mesons with distinct Lorentz properties. In the iso-scalar sector there are the Lorentz-scalar σ and Lorentz-vector ω classical fields with the first one being responsible for the attractive and the second one for the repulsive part of the NN-interaction, respectively. Finite nuclei studies show that these two fields are very strong and comparable with the nucleon mass of 939 MeV [8]. Their different Lorentz structure (cancellation effects between attractive and repulsive character) leads to a very small value of the binding energy per nucleon of ca. −16 MeV. On the other hand, they can predict the empirically known strong spin-orbit potential, which is proportional to the sum between them. This is one (among many others which cannot be listed here) of the reasons why a relativistic description is meaningful. The iso-vector part of the nuclear matter EoS is characterized by the symmetry energy E_{sym} defined as the second derivative of the energy density with respect to the asymmetry parameter α = oρ_nt$\rho_p$pvρg(g$\rho_n$sg$\rho_p$sgρgeing the neutron-, proton and total density, respectivelypughe asymmetry term at saturation density is well known from the mass formula. It can be described by an iso-vector, Lorentz-vector ρ-field, or by the presence of an additional iso-vector, Lorentz-scalar δ-field. As in the iso-scalar case, the ρ-field is responsible for the repulsive and the δ-field for the attractive character of the iso-vector potential. However, for the description of the bulk asymmetry parameter a_4 there are several possibilities, in contrast to the iso-scalar case: the inclusion of the ρ-meson, or of both, the consideration of the ρ- and δ-fields. In the later case one has to increase the coupling of the ρ-field to describe the same value of a_4. The different Lorentz-structure of the two mesons will influence the high density behaviour of E_{sym}.

The description of ground state nuclear matter and finite nuclei has been widely studied within non-relativistic and covariant models in the spirit of phenomenological Hartree-Fock and more sophisticated Brueckner-Hartree-Fock approaches. It is not the aim of this contribution to go into further details. The situation is summarized in Fig.1 in terms of the nuclear matter EoS (the binding energy per nucleon on the left and the symmetry energy on the right). In summary, all models can describe the saturation properties, i.e. the binding energy of ca. −16 MeV, the compression modulus of ca. 200-250 MeV and the asymmetry parameter of about 30 MeV. However, they differ significantly at supra-normal densities (ρ¿0.3 fm^{-3}). They can be classified into two groups with a moderate (strong) compressional energy at high densities respectively a "soft" ("hard") nuclear EoS. Therefore it turns out that in HIC the high density behaviour is tested, and not necessarily the compression modulus.

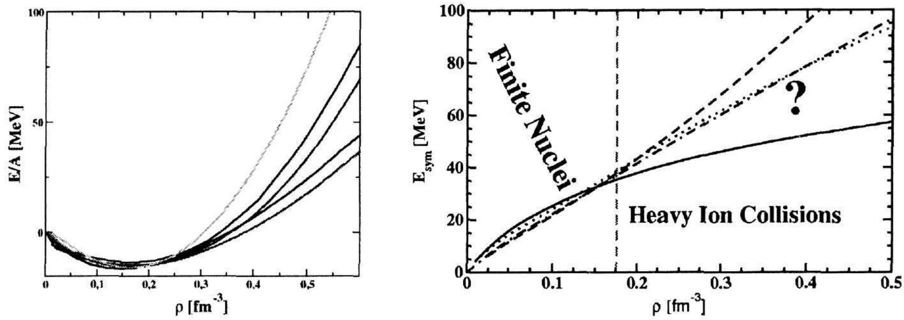

Fig. 1. Density depenendence of the EoS in terms of the binding energy per nucleon (left) and the symmetry energy (right) for different models of nuclear structure (see [6,9] for details).

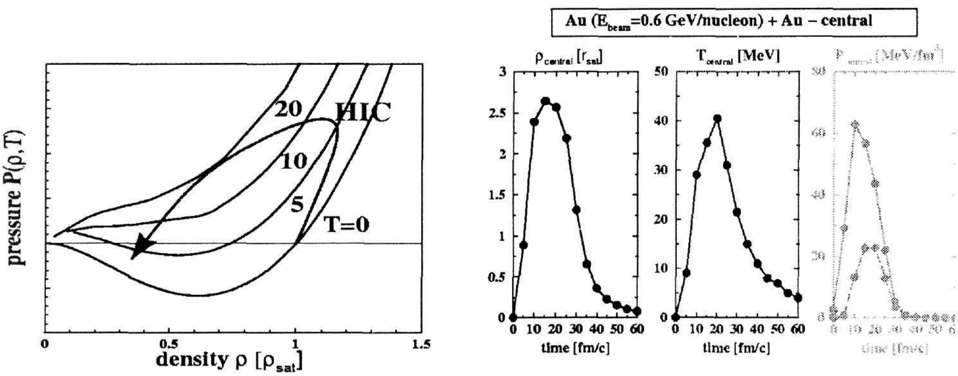

Fig. 2. Phase diagram of nuclear matter at different temperatures (as indicated) and trajectory of nuclear matter through the phase diagram in a HIC. (Right) Time evolution of the local density (left), the local temperature (middle) and the longitudinal (solid) and transversal (dashed) pressures (right) in a central Au on Au collision at 0.6 AGeV bombarding energy per nucleon.

3. Probing the high density behavior of the EoS in heavy ion collisions

The theoretical description of HIC is based on transport equations of a Boltzmann-type [3,4]. They describe the space-time evolution of a classical phase-space distribution function f(x,p) in the presence of a nuclear mean field, i.e. the nuclear EoS, and 2-body collisions including possible inelastic processes, see [3,4,9].

Fig. 2 on the right shows the dynamical situation in a HIC at an intermediate energy of 0.4 AGeV in terms of the temporal evolution of the local density (left), local temperature (middle) and longitudinal and transversal pressures (right) obtained in the central shell in the c.m.s. reference frame of the two colliding nuclei. One sees the existence of a highly compressed and hot nuclear matter, however, with a very short life time (note that 1 fm/c$\approx 10^{-24}$sec). This so-called fireball expands then rapidly due to the high pressure gradients and reaches finally densities below saturation, local temperatures near the critical one and "negative" pressures (not visible in the figure)

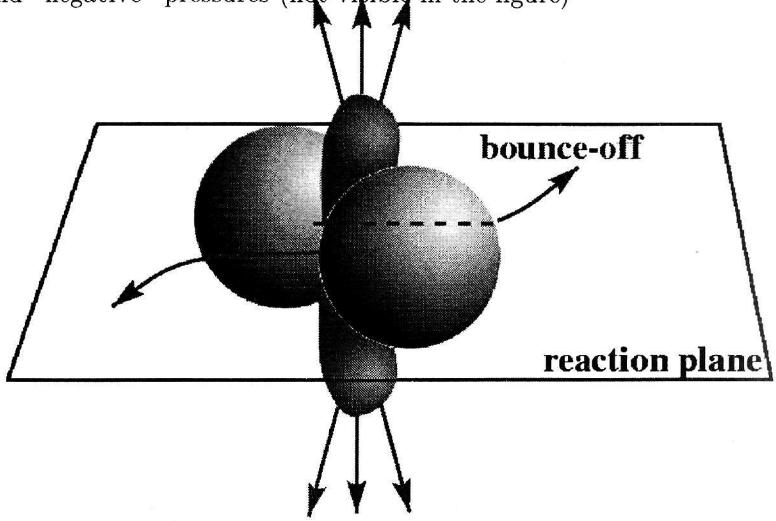

Fig. 3. Characterization of the reaction dynamics in terms of collective flow effects (see text).

Physically the compression/expansion scenario is as follows: In the initial stage the two nuclei approaches each other and starts to get influenced by the common nuclear mean-field and to undergo 2-body collisions. The mean-field has a repulsive character at these high energies which bounce-off the particles from each other. The collisions stop the matter in the central shell by filing up the central region in the momentum space. This scenario continues forming a highly compressed (and hot) nuclear matter. However, the strong pressure gradients start then in a second phase to influence the fireball: it begins to expand rapidly with a velocity of sound given by cs=$\partial\varepsilon/\partial$p [1] ($\varepsilon$, p being the energy and pressure density, respectively). Thus, the nuclear EoS in terms of the energy density and the pressure gradients affects the reaction dynamics. A stiff (soft) EoS leads to less (more) compression and higher (smaller) collectivity, i.e. stronger (less) repulsion.

This dynamical situation is also displayed in the nuclear phase diagram (HIC trajectory in the left panel of Fig. 2). Starting from ground state nuclear matter at zero temperature and pressure the HIC trajectory passes through regions of the phase diagram with high densities and temperatures. Finally it

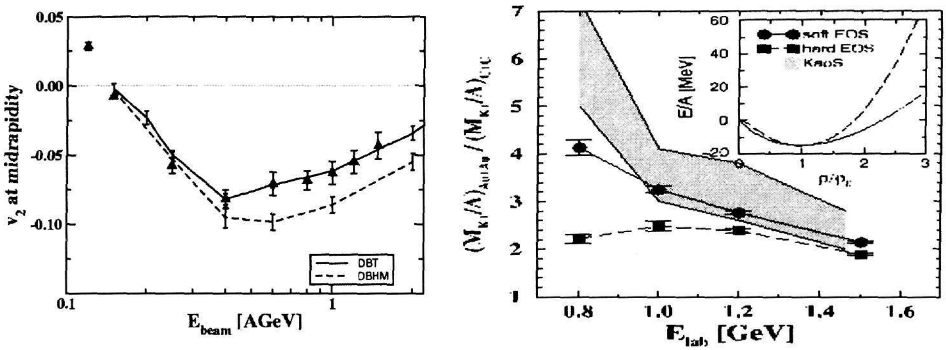

Fig. 4. Energy dependence of the elliptic flow v2 at mid rapidity. Theoretical calculations using the microscopic Dirac-Brueckner (DB) model in two different versions (from Tuebingen-group (DBT, solid curve) and from B. ter Haar&R. Malfliet (DBHM, dashed curve)) are compared to data (filled diamonds), see for details [9]. (Right) energy dependence of the Kaon (K^+) multiplicities for Au+Au collisions (normalized to C+C collisions) as obtained from theoretical calculations (solid-circles and dashed-squares) and from experimental analyses (taken from [11]).

ends up to the so-called instability region determined by negative values of the compression modulus indicating the existence of a phase transition from a liquid to a gas phase. This mechanism gives rise to the fragmentation of the nuclear "liquid" [10]. In the following, however, we will discuss only the physics of the intermediate highly excited fireball and the question of determining from its analysis the high density behaviour of the nuclear EoS. More information on the topic of phase transitions can be found in [10].

4. Comparison with experiments

The determination of the high density dependence of the nuclear EoS, see e.g. Fig. 1, consists of a comparison between theoretical and experimental studies of the reaction dynamics. To do so, one has first to select observables, which, on the one hand, depend on the fireball dynamics, i.e. on the high density behaviour of the nuclear matter, and, on the other hand, are accessible from experiment and theory. The signals can be divided into two groups: (a) collective flow (momentum space distributions) and (b) particle production.

Collective (momentum) effects have been extensively investigated theoretically and experimentally during the last 20 years (see for an overview [1]). The situation is schematically depicted in Fig. 3. In non-central collisions a part of the nucleus collides with that of the second one. The particles participating in the collision process are called the "participants" and they form the highly excited fireball. The rest of the two nuclei, which is not participating in the process and just passes near the reaction zone, is called "spectator". However, due to the common nuclear mean field spectator and participant matter is not

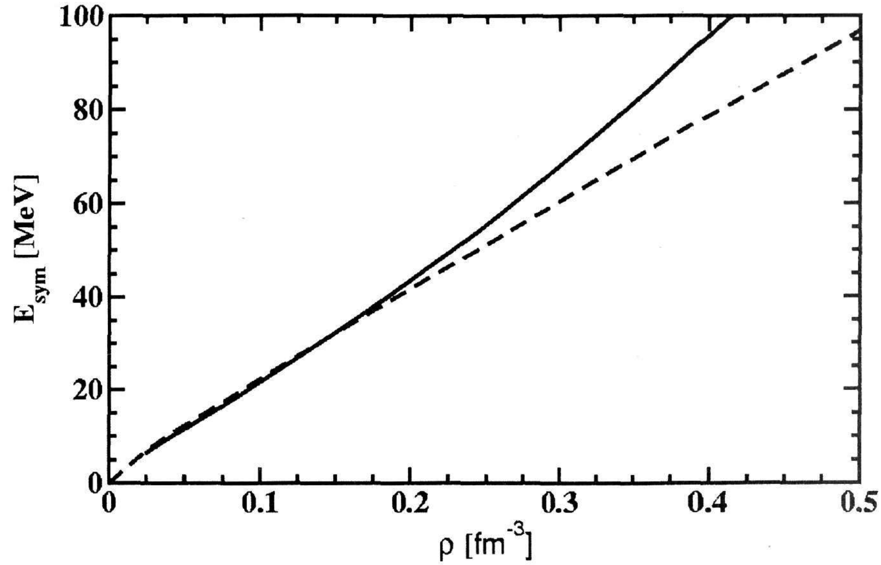

Fig. 5. Density dependence of the symmetry energy E_{sym} using only the ρ-meson field (dashed curve) and both, the ρ- and δ-meson fields (solid curve), taken from [5,12].

exactly separated. The entire situation can be characterized by momentum distributions projected into or out of the reaction plane (RP). The RP is characterized by the beam (z-direction) and transversal (x-direction) axes. The transversal x-direction is given be the finite impact parameter, which shifts the two nuclei in this direction.

One important observable which depends on the "stiffness" of the nuclear EoS is the so-called "bounce-off" effect: due to the repulsion of the mean field the particle momenta are shifted in transverse direction (with respect to the beam axis) as shown in Fig. 3. Here one defines the mean transverse in-plane flow ¡$p_x(y)$¿ as the average transverse momentum $p_x$at each rapidity interval (the rapidity y presents another characterization of the beam momentum p_z[1]).

Another important flow signal is the so-called "squeeze-out" effect, which strongly depends on the nuclear EoS. Its formation takes place very earlier during the pre-equilibrium stage. Thus, it contains directly information on the physical properties of the fireball. The relation to the nuclear EoS is as follows [1]: In non-central collisions the expansion phase of the fireball occurs with a velocity of sound cs=$\partial\varepsilon/\partial$p. The stiffness of the EoS determines therefore the dynamics of the highly excited nuclear matter. On the other hand, the fireball expansion is "blocked" from the spectators, which at the same time pass near the reaction zone. Thus, the emitted particles from the fireball can escape preferentially perpendicular to the RP since the beam direction is blocked from spectator matter (see again Fig. 3). This preferential emission out of the reaction plane is the stronger the stiffer the nuclear EoS is (at high densities): A stiffer nuclear EoS is related with stronger pressure gradients and, therefore, with a faster fireball expansion. Due to the "shadowing" of spectator matter a stiffer EoS will enhance the squeeze-out effect. It is experimentally determined from a Fourier analysis of azimuthal distributions and its given by the so-called elliptic flow coefficient $v_2=(p_x^2-p_y^2)/(p_x^2+p_y^2)$. Negative (positive) v_2–values indicates a preferential out-of-plane (in-plane) emission with respect to the RP.

A third possibility to explore high density effects of the nuclear EoS is the abundances of produced particles. At intermediate energies up to 1-2 AGeV these are pions ($\pi^{\pm,0}$) and kaons ($K^{\pm,0}$). Although their masses are small (0.138 and 0.494 GeV, respectively) respecting the nucleon mass (0.939 GeV), they are produced indirectly via higher excited nucleonic states or together with strange particles: Pions are produced via the decay of the $\Delta(1232)$-resonance. The resonances are created early in the preequilibrium phase in NN collisions. The threshold energy is 2.014 GeV (in terms of the invariant c.m.s. energy $\sqrt{s}$). The early-created resonances decay then according exponential laws into pions and nucleons. Furthermore, the production mechanism for kaons is as follows: they are created via BB$\diamondsuit$BYK and B$\pi\diamondsuit$YKgollisions (B stands for a nucleon or a resonance and Y for hyperons (ΣsΛ)). Since kaons are particle with strangeness they has to be created together with particles with anti-strangeness (hyperons ΣsΛ) due to strangeness number conservation. The threshold energy for kaon production is very high (in the 3-body channel) due to the high masses of the hyperons.

However, an important feature of high energetic nucleus-nucleus collisions is the so-called "sub-threshold production" which means that even if the initial bombarding energy is smaller than the production threshold, one is able to produce pions and kaons. This is because of the high energy densities achieved in the fireball region the required energy becomes available through many sequential 2-body collisions. The threshold energy strongly depends on the compressional energy of the fireball, i.e. on the high density dependence (soft or stiff) of the nuclear EoS. On the other hand, the experimental accessible particle multiplicities directly depend on the threshold energy available. Therefore, particle production also helps to explore compressional effects of the fireball.

As discussed above, with a given neutron to proton asymmetry one can explore the iso-vector sector of the nuclear EoS. The procedure is the same as described here, i.e. by using the same or similar observables, which are related to the high density behaviour of the symmetry energy. In the following we will discuss in more detail how one can extract the high density nuclear EoS of symmetric and asymmetric matter in HIC.

To study compressional effects in HIC one performs theoretical simulations by applying different models of the nuclear EoS. Fig. 4 shows two typical examples: the energy dependence of the squeeze-out effect (given in terms of the elliptic flow v_2 calculated at mid rapidity) and of the relative ratio of K^+-multiplicities. The theoretical calculations were performed with a soft and stiff nuclear EoS. One can be seen that these observables are very sensitive to the high density behaviour of the nuclear EoS.

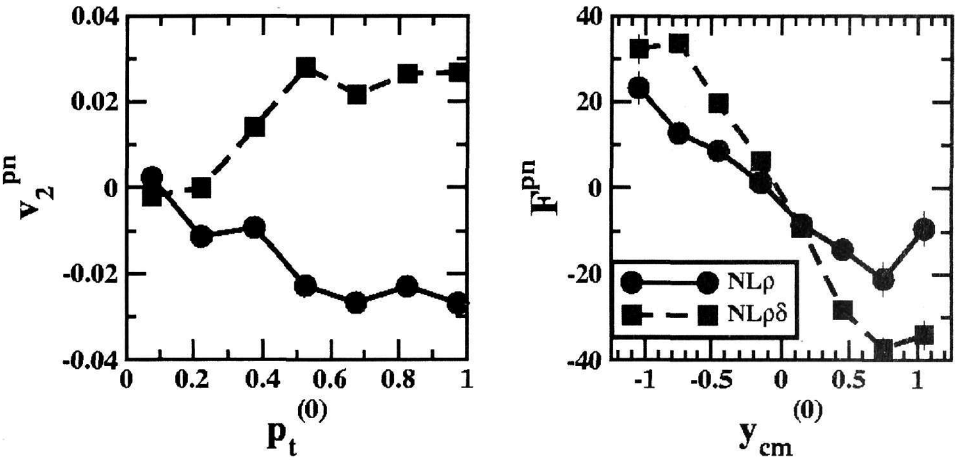

Fig. 6. (Left) Isospin in-plane (right) and out-of-plane (left) flows as function of the normalized rapidity and transverse momentum, respectively. Calculations with the same iso-scalar EoS, but different choices in the iso-vector case are shown: by taking into account only the ρ-meson field (NLρ) and both the ρ- and δ-fields (NL$\rho\delta$). The considered reaction is a $^{132}Sn+^{132}Sn$ peripheral (6fm of impact parameter) collision at 1.5 AGeV bombarding laboratory energy (taken from [12]).

The energy dependence of the elliptic flow can be reproduced very satisfactory by using a soft EoS (at high densities). The repulsive character in the second theoretical calculation (dashed) generates a stronger out-of-plane emission as indicated be the larger negative values of the elliptic flow at all the energies considered. However, the comparison with the data does not support such a stiff behaviour of the nuclear EoS at supra-normal densities [9]. From an analyis on kaon multiplicities one has arrived to similar conclusions. In Fig. 4 (on the right) one sees the energy dependence of the kaon (K^+) multiplicities in Au+Au collisions (relative to that in C+C collisions). Again the experimental data support a soft nuclear EoS at high compressions. A stiffer EoS leads to less compression in the fireball region and thus too less 2-body collisions and finally too less binary processes with strangeness production. This effect becomes stronger as the beam energy decreases (see the discussion on sub-threshold production above).

The iso-vector part of the nuclear EoS has not been so far studied in detail. Recently one has started to explore the properties of highly excited asymmetric nuclear matter in terms of the symmetry energy E_{sym} as function of the baryon density. The saturation point is well known from the mass formula with a value of about 30 MeV for the bulk asymmetry parameter a_4 of the Bethe-Weizaecker formula, but not its behaviour beyond the saturation point.

However, the mechanism leading to the empirical value of the bulk asymmetry parameter is not trivial and even more complicated than in the iso-scalar case: there one needs at least two strong fields with opposite Lorentz properties (scalar attractive and vector repulsive fields) to simultaneously describe the small value of the binding energy per nucleon and the large value of the spin-orbit potential. In the iso-vector case one has different possibilities to describe the same bulk asymmetry parameter: by introducing only one iso-vector, Lorentz-vector ρ-field or by the considering an additional field, the iso-vector, Lorentz-scalar δ-field. The symmetry energy can be relativistically expressed [5] by $E_{sym} \propto (f_\rho-(m^*/\rho_s)f_\delta)\rho_B$. $f_{\rho s\delta}$sgre the ρ- and δ- coupling constants, which describes the interaction between the nucleons and the corresponding mesons. $\rho_{s,B}$ are the scalar and baryon densities, respectively, and m^* the nucleon effective mass. Last quantity characterizes the scalar interaction between the nucleons and mesonic fields being responsible for the modification of the free nucleon mass of the baryons inside the mesonic classical background field [5].

By fixed bulk asymmetry parameter a_4 one has therefore two possibilities: (a) using only the ρ-meson field to determine the density dependence of the symmetry energy E_{sym} or (b) both the ρ- and δ-meson fields. In the last case, however, one has to increase the ρ-meson coupling in order to reproduce the same fixed empirical value of a_4. The importance of the introduction of the new Lorentz-scalar δ-field can be seen in the high density behaviour of the symmetry energy in Fig. 5. At densities near saturation E_{sym} shows similar density dependence in the two cases. At supra-normal densities, however, crucial differences are visible which arises from the explicit consideration of the δ-meson in the description of the iso-vector part of the nuclear EoS. This difference has a relativistic origin: at high densities the Lorentz-scalar δ-field is suppressed by the factor (m^*/ρ_s), whereas the ρ-meson is just proportional to the baryon density. On the other hand, one has to increase the ρ-meson coupling (in the presence of the δ-field) which finally leads to the observed density behaviour of the symmetry energy.

In order to explore the high density behaviour of the symmetry energy one has to study HIC of asymmetric systems at relativistic energies where central baryon densities of more that $2\times\rho_{sat}$ are achieved. Such dynamical processes are Sn+Sn or Au+Au reactions at bombarding energies of around 1 AGeV. The collective flow effects and particle production are again important as they were for the iso-scalar case (see previous section).

Fig. 6 shows the influence of the different treatment of the high density behaviour of the symmetry energy on different components of the collective flow.

To study isospin effects on collective flow one defines for all the components of the collective flow (in-plane and out-of-plane) the so-called isospin flow that is determined as the difference between the proton and neutron flow components (F^{pn} for the in-plane flow and v_2^{pn} for the out-of-plane flow, respectively). One sees from the Fig. 6 that the consideration of the δ-field in the iso-vector EoS generally enhances the isospin flows (the effect is more pronounced for the out-of-plane flow as function of transverse momentum). This can be understood from the high density dependence of the symmetry energy (see Fig. 5): the δ-field generates a stiffer E_{sym} with a more repulsive iso-vector field for neutrons (than for protons). More repulsion, on the other hand, means stronger pressure gradients being responsible for a faster expansion of the fireball. In non-central collisions (which is here the case) a faster fireball expansion generates more in-plane and out-of-plane flows due to the spectator "shadowing"-effect described in the previous section. Thus, one observes higher collective phenomena when the δ-field is taken into account in the dynamical descriptions.

Therefore, relativistic effects are very important in order to characterize the high density dependence of the iso-vector EoS (in terms of the symmetry energy discussed here). Similar effects have been also found for particle production such as pions [6] and kaons [13] (in the last case below threshold). The isospin equilibration (or stopping degree of the colliding system) is another very interesting feature of intermediate energy HIC to investigate the still unknown structure of the symmetry energy at supra-normal densities [14]. In summary, one preliminarily concludes that the iso-vector EoS might exhibit a stiff behaviour at high densities, which can be characterized (in the spirit of a relativistic framework) only by the presence of the iso-vector, Lorentz-scalar δ-meson field.

5. Final remarks

We have discussed the possibility to explore the EoS under extreme conditions of density and isospin degree of freedom by studying HIC at relativistic energies up to 1-2 AGeV. Such studies are of crucial importance for astrophysical processes like super-novae explosions and neutron stars where hadronic matter under similar extreme conditions exists. A relativistic description of nuclear matter systems appeared to be essential in understanding the properties of highly excited nuclear matter in terms of the EoS in the iso-scalar and iso-vector cases.

The study of nuclear matter under extreme conditions in high energetic nucleus-nucleus collisions, which has been an object of debate over more than 20 years, has provided the possibility to set constraints on the still unknown behaviour of hadronic matter at supra-normal densities. Concerning the properties of symmetric nuclear matter it seems that the nuclear EoS should exhibit a "soft" character at high densities as a conclusion from comparisons between theory and experiment. The situation in the iso-vector case, i.e. the properties of asymmetric nuclear matter, is still an object of current investigations, since there is not yet enough experimental information. Preliminary comparisons conclude a "stiff" behaviour of the high density symmetry energy, which can be only described by the presence of the iso-vector, Lorentz-scalar δ-mesonic field.

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