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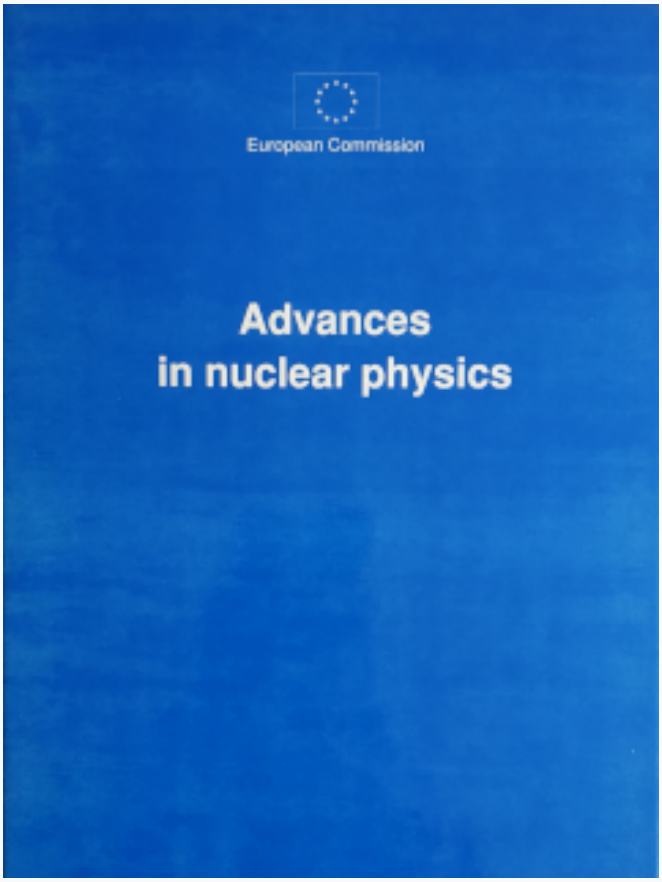

Study of the Generalized Momentum Distribution of Model Nuclear Matter

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STUDY OF THE GENERALIZED MOMENTUM DISTRIBUTION OF MODEL NUCLEAR MATTER *

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Abstract

Valuable information on the correlation structure of the nuclear medium is stored in the generalized momentum distribution $n(\boldsymbol{p},\boldsymbol{Q})$, the Fourier transform of the half-diagonal two-body density matrix $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$. In this paper, we present a numerical calculation of $n(\boldsymbol{p},\boldsymbol{Q})$ for two Jastrow-correlated models of symmetrical nuclear matter based on the structural decomposition of $n(\boldsymbol{p},\boldsymbol{Q})$ derived by Ristig and Clark and on a Fermi-hypernetted-chain procedure. Results exhibit significant departures from the ideal Fermi gas case in certain kinematic domains; this behaviour indicates the strong short-range correlations present in these models. Nevertheless, such deviations are less prominent than in earlier low-cluster-order calculations. The results are also used to judge the quality of Silver's approximation for $n(\boldsymbol{p},\boldsymbol{Q})$.

1.Introduction

There is currently increasing interest in the development of a detailed and quantitative description of the generalized momentum distribution $n(\boldsymbol{p},\boldsymbol{Q})$ and the associated two-body density matrix of finite nuclei. This interest is mainly stimulated by the fact that accurate interpretation of a range of recent and planned experiments on inclusive quasi-elastic (e,e') scattering [1] as well as exclusive $(e,e'N)$ [2] scattering, etc. hinges on a quantitative study of the propagation of ejected nucleons and their final-state interactions (FSI) due to their collisions

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mainly with the repulsive cores of the potentials on neighbouring nucleons. Reliable extraction of quantities such as momentum distributions, spectral functions and transparency from the experimental data requires an accurate accounting of final-state effects. As we progress beyond mean-field, optical-model descriptions, theoretical treatments of FSI are found to involve, as input, the diagonal and half-diagonal portions of the two-body density matrix [3-11]. Finally, $n(\boldsymbol{p},\boldsymbol{Q})$ is involved in fundamental sum rules that provide insight into the nature of elementary excitations of quantum many-body systems [12].

Following the microscopic evaluation of the momentum distribution $n(p)$ and its Fourier inverse, the one-body density matrix $\rho_1(\boldsymbol{r}_1,\boldsymbol{r}_1')$, variational theory has been extended to the investigation of the half-diagonal two-body density matrix $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$ of the ground states of infinite symmetrical nuclear matter and other uniform strongly interacting Fermi systems [13]. Initial calculations of $n(\boldsymbol{p},\boldsymbol{Q})$ have recently been performed [14] for simple models of nuclear matter defined by Jastrow-correlated wave functions, applying low-order cluster approximations within the theory of Ref. [13]. In this paper as well as in Ref.[26] we present the results of our calculations obtained within the framework of the Fermi-hypernetted-chain procedure.

We consider uniform, isospin-symmetrical, spin saturated nuclear matter at density ρ, with corresponding Fermi wave number $k_F=(6\pi^2\rho/\nu)^{1/3}$, where $\nu=4$ is the level degeneracy of plane-wave single-particle states. For a given state vector $|\Psi\rangle$, the generalized momentum distribution $n(\boldsymbol{p},\boldsymbol{Q})$ is defined by:

$$n(\boldsymbol{p},\boldsymbol{Q})=\sum_{\hat{\boldsymbol{k}}}<\Psi|\alpha^{+}_{\hat{\boldsymbol{k}}+\boldsymbol{Q}}\alpha^{+}_{\hat{\boldsymbol{p}}-\boldsymbol{Q}}\alpha_{\hat{\boldsymbol{p}}}\alpha_{\hat{\boldsymbol{k}}}|\Psi> \tag{1}$$

Here, $\hat{\boldsymbol{k}}$ labels the single-particle orbital with wave vector $\boldsymbol{k}$ and spin/isospin projections σ,τ while $\hat{\boldsymbol{k}}+\boldsymbol{Q}=(\boldsymbol{k}+\boldsymbol{Q},\sigma,\tau)$. The role played by the generalized momentum distribution in final-state interactions emerges clearly by introducing the density fluctuation operator $\rho_{\boldsymbol{Q}}=\sum_{\hat{\boldsymbol{k}}}\alpha^{+}_{\hat{\boldsymbol{k}}+\boldsymbol{Q}}\alpha_{\hat{\boldsymbol{k}}}$ $(\boldsymbol{Q}\neq 0)$ and writing definition (1) in the form

$$n(\boldsymbol{p},\boldsymbol{Q})=\langle\Psi|\rho_{\boldsymbol{Q}}\alpha^{+}_{\hat{\boldsymbol{p}}-\boldsymbol{Q}}\alpha_{\hat{\boldsymbol{p}}}|\Psi\rangle-n(p) \tag{2}$$

The first term on the right may be interpreted as a transition matrix element for scattering a particle out of orbital $\hat{\boldsymbol{p}}=(\boldsymbol{p},\sigma',\tau')$ to another orbital

$\hat{\boldsymbol{p}} - \boldsymbol{Q} = (\boldsymbol{p} - \boldsymbol{Q}, \sigma', \tau')$, the process being mediated by a density fluctuation of wave vector $\boldsymbol{Q}$. The function $n(\boldsymbol{p},\boldsymbol{Q})$ is connected to the half-diagonal two-body density matrix

$$\rho_{2h}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1') = A(A-1)\int \Psi^*(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)\Psi(\boldsymbol{r}_1', \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)d\boldsymbol{r}_3 \ldots d\boldsymbol{r}_A \quad (3)$$

by the Fourier transformation

$$n(\boldsymbol{p},\boldsymbol{Q}) = \frac{1}{\nu}\frac{\rho}{A}\int \rho_{2h}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1')e^{-i\boldsymbol{p}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_1')}e^{-i\boldsymbol{Q}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_2)}d\boldsymbol{r}_1 d\boldsymbol{r}_2 d\boldsymbol{r}_1' \quad (4)$$

In writing (3), the spin/isospin variables have been suppressed for the sake of economy. For the noninteracting system, the Pauli exclusion principle generates kinematic particle-particle correlations and $n(\boldsymbol{p},\boldsymbol{Q})$ takes the form

$$n_F(\boldsymbol{p},\boldsymbol{Q}) = \delta_{Q0}(A-1)\theta(k_F - p) - (1-\delta_{Q0})\theta(k_F - p)\theta(k_F - |\boldsymbol{p} - \boldsymbol{Q}|) \quad (5)$$

In addition to time-reversal invariance, the generalized momentum distribution $n(\boldsymbol{p},\boldsymbol{Q})$ has the following formal properties that arise from the corresponding properties of $\rho_{2h}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1')$ [13]. The sequential relation in configuration space

$$\int \rho_{2h}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1')d\boldsymbol{r}_2 = (A-1)\rho_1(\boldsymbol{r}_1, \boldsymbol{r}_1') \quad (6)$$

relating $\rho_{2h}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1')$ and the one-body density matrix $\rho_1(\boldsymbol{r}_1, \boldsymbol{r}_1')$, may be transformed to momentum space to yield a relation between $n(\boldsymbol{p},\boldsymbol{Q})$ and the momentum distribution $n(p)$

$$n(\boldsymbol{p}, \boldsymbol{Q} = 0) = (A-1)n(p) \quad (7)$$

For the full-diagonal case ($\boldsymbol{r}_1' = \boldsymbol{r}_1$), Eq. (3) reduces to $\rho_{2h}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1) = \rho^2 g(r_{12})$ and, summing over $\boldsymbol{p}$ in Eq. (4), we arrive at the so-called $\boldsymbol{p}$ sum rule

$$A^{-1}\sum_{\hat{\boldsymbol{p}}} n(\boldsymbol{p},\boldsymbol{Q}) = A\delta_{Q0} + S(Q) - 1 \quad (8)$$

where $S(Q)$ is the static structure function. In the case of strong short-range repulsions, $n(\boldsymbol{p},\boldsymbol{Q})$ also obeys the $\boldsymbol{Q}$ sum rule

$$\sum_{Q} n(\boldsymbol{p},\boldsymbol{Q}) = 0 \tag{9}$$

Three approximations [16-18] have been proposed for estimating $n(\boldsymbol{p},\boldsymbol{Q})$. In formulating his hard-core perturbation theory of FSI, Silver [17] has proposed the simple approximation

$$n(\boldsymbol{p},\boldsymbol{Q}) \approx \frac{n(p)}{A} \sum_{p} n(\boldsymbol{p},\boldsymbol{Q}) \tag{10}$$

which, combined with Eq. (8), leads to the following equation for $n(\boldsymbol{p},\boldsymbol{Q})$ for $Q \neq 0$:

$$n(\boldsymbol{p},\boldsymbol{Q}) \approx n(p)[S(Q)-1] \tag{11}$$

This form obeys the $\boldsymbol{p}$ and $\boldsymbol{Q}$ sum rules and meets the sequential relation, but violates time-reversal invariance.

A microscopic analysis for the evaluation of $n(\boldsymbol{p},\boldsymbol{Q})$ and $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$ at the variational level of correlated-basis-function (CBF) theory, for both Bose and Fermi systems, has been developed by Ristig and Clark [15,13]. For the Fermi case, the ground state wave function is approximated by the Jastrow-Slater Ansatz

$$\Psi(1,\ldots,A) = N^{-1} \prod_{i<j}^{A} f(r_{ij}) \Phi(1,\ldots,A) \tag{12}$$

where Φ is a Slater determinant of A plane-wave orbitals filling the Fermi sea up to k_F, $f(r_{ij})$ is the Jastrow two-body correlation function and N is a normalisation constant. Cluster-diagrammatic decomposition was followed by graphical resummations to yield structural formulas for $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$ and for $n(\boldsymbol{p},\boldsymbol{Q})$. The quantities building up these functions can be computed either by cluster expansion to some (low) order or by Fermi hypernetted-chain (FHNC) techniques [19-21]. In Ref. [14], numerical calculations of $n(\boldsymbol{p},\boldsymbol{Q})$ within this framework were begun for nuclear matter described by a Jastrow-Slater wave function. Two approximation schemes were investigated:

(i) *LO approximation.* Evaluation to lowest (two-body) cluster order in the factorised-Iwamoto-Yamada cluster expansion generated for $n(\boldsymbol{p},\boldsymbol{Q})$ directly from the definition (1).

(ii) *LOIC1 approximation.* Lowest-cluster-order evaluation of the ingredients of the renormalised structural expression for $n(\boldsymbol{p},\boldsymbol{Q})$ (See Eq. (13) below).

The results of the pilot studies of Ref. [14] differ significantly from the noninteracting Fermi gas case. However, a detailed investigation of individual contributions to this behaviour, for models of nuclear matter with different correlation strengths, indicates that the contribution of neglected higher order cluster terms may be quite important. The occurrence of large violations of the sequential relation (7) within the approximation schemes of Ref. [14] supports also this conclusion. The above considerations indicate that it would be advisable to proceed to a Fermi hypernetted-chain (FHNC) treatment of $n(\boldsymbol{p},\boldsymbol{Q})$. The goal of the present paper is to perform such an evaluation at the FHNC/0 level and then to compare it, specifically, with Silver's approximation (Eq.(11)). Section 2 sketches the framework of our calculation. The numerical results for two simple nuclear-matter models are reported and discussed in Section 3. Some perspectives for future work are presented in Section 4.

2.Fermi Hypernetted Chain Analysis

Our calculation is based on the microscopic analysis of $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$ developed for Fermi fluids by Ristig and Clark [13] within the variational CBF theory. For a uniform Fermi system described by a Jastrow-Slater wave function, application of the factorised Iwamoto-Yamada (FIY) cluster expansion scheme [22] leads to an infinite cluster series whose addends are generally reducible, i.e. they can be factorised into products of cluster diagrams. Resummation of graphical subseries with the aid of hypernetted chain techniques results in a closed form expression for $n(\boldsymbol{p},\boldsymbol{Q})$ in terms of a small number of irreducible quantities [13],

$$\begin{aligned} n(\boldsymbol{p},\boldsymbol{Q}) = {} & (A-1)\delta_{Q0}n(p)+(1-\delta_{Q0})F_{Qdd}(Q)[n(p)-n(|\boldsymbol{p}-\boldsymbol{Q}|)] \\ & +(1-\delta_{Q0})F_{Qde}(Q)[n_{DI}(p)+n_{DI}(|\boldsymbol{p}-\boldsymbol{Q}|)] \\ & -n_0(1-\delta_{Q0})[\theta(k_F-p)-F_{Qcc}(p)][\theta(k_F-|p-Q|)-F_{Qcc}(|\boldsymbol{p}-\boldsymbol{Q}|)] \\ & +(1-\delta_{Q0})n^{(2')}(\boldsymbol{p},\boldsymbol{Q})+(1-\delta_{Q0})n^{(3')}(\boldsymbol{p},\boldsymbol{Q}) \end{aligned} \tag{13}$$

(The Q index appearing in Eq.(13) is introduced to make the necessary connection with Ristig's notation [21]: it should not be confused with the momentum variable

Q.) In the above expression, n_0 is the strength factor that arises in the structural formula for the momentum distribution $n(k)$ [19-21]. The modified momentum distribution $n_{DI}(k)$ is defined by

$$n_{DI}(k) = \frac{1}{\nu}\int \rho_{1D}(r)\ell(r)e^{-ik\cdot r}dr \tag{14}$$

where $\rho_{1D}(r)$ is the direct-direct (dd) component of the full Fermi one-body density matrix $\rho_1(r_1, r_1')$. In addition, we have the "two-point" quantities $F_{Qxy}(k)$ (with xy=dd, de or cc -- for the meaning of the subscripts dd, de, cc see Appendix) which serve as form factors, and the "three-point" quantities $n^{(2')}(\boldsymbol{p},\boldsymbol{Q})$ and $n^{(3')}(\boldsymbol{p},\boldsymbol{Q})$. The designations "two-point" and "three-point" refer to the graphical topology of the corresponding configuration space functions. The "three-point" quantity $n^{(2')}(\boldsymbol{p},\boldsymbol{Q})$ is given by a three-dimensional integral over a sum of products of two-point functions

$$n^{(2')}(\boldsymbol{p},\boldsymbol{Q}) = \frac{1}{\nu}\frac{\rho}{A}\int K(r_1, r_2, r_1')e^{-ip\cdot(r_1-r_1')}e^{-iQ\cdot(r_1-r_2)}dr_1 dr_2 dr_1' \tag{15}$$

where (with $r \equiv |r_1 - r_2|$ and $r' \equiv |r_1' - r_2|$)

$$\begin{aligned} K(r_1, r_2, r_1') = {} & \rho\,\rho_1(r_1, r_1')F_{Qdd}(r)F_{Qdd}(r') \\ & + \rho\,\rho_{1D}(r_1, r_1')\ell(r_1, r_1')[F_{Qdd}(r)F_{Qde}(r') + F_{Qdd}(r')F_{Qde}(r)] \\ & - \nu\,\rho[\rho_{1D}(r_1, r_1') - \rho n_0][v^{-1}\ell(r) - F_{Qcc}(r)][v^{-1}\ell(r') - F_{Qcc}(r')] \end{aligned} \tag{16}$$

The remaining "three-point" quantity $n^{(3')}(\boldsymbol{p},\boldsymbol{Q})$ is an integral over a sum of terms, each of which involves at least one irreducible three-point function.

The inputs of Eq.(13), namely n_0, $n(k)$, $n_{DI}(k)$, the $F_{Qxy}(k)$ and $n^{(2')}(\boldsymbol{p},\boldsymbol{Q})$ were calculated by implementing the FHNC algorithm at the level in which elementary diagrams are omitted (FHNC/0). (The contribution of elementary diagrams is generally expected to be significant only at higher densities; thus, elementary diagrams are commonly ignored in calculations on nuclear systems [22,23].) The corresponding sets of FHNC equations are too extensive to appear in this presentation; the interested reader may find the detailed expressions in Refs. [13,20,21]. The results obtained were further used to calculate the corresponding FHNC results for $n(\boldsymbol{p},\boldsymbol{Q})$. In our calculation, the term $n^{(3')}(\boldsymbol{p},\boldsymbol{Q})$

was omitted; the reasoning is similar to that applied in neglecting elementary diagrams.

Expression (13) for $n(\boldsymbol{p},\boldsymbol{Q})$ assumes that the sequential relation (7) is satisfied. In terms of the ingredients of Eq.(13) , this condition is equivalent to

$$\begin{aligned} &2F_{Qdd}(0)n(p)+2F_{Qde}(0)n_{DI}(p)-n_0[\theta(k_F-p)-F_{Qcc}(p)]^2 \\ &+n^{(2')}(p,0)+n^{(3')}(p,0)=-n(p) \end{aligned} \tag{17}$$

The FHNC/0 evaluation necessarily compromises the sequential relation (7) (and hence (17)) to some extent, due to the absence of elementary diagrams and the term $n^{(3')}(\boldsymbol{p},\boldsymbol{Q})$. For the same reason, it also fails to meet the $\boldsymbol{p}$ sum rule (8) (although the violation may be small). On the other hand, the FHNC/0 approximation does conserve time-reversal invariance and obeys the $\boldsymbol{Q}$ sum rule (9).

3.Numerical results

Numerical calculations of the generalized momentum distribution are performed for two models of nuclear matter near its saturation density. These models provide a representative picture of the short-range repulsive correlations of nuclear systems while the intermediate and long-range correlations are described in an average way [24]. Both models refer to the density value $\rho = 0.182\,\mathrm{fm}^{-3}$, corresponding to $k_F = 1.392\,\mathrm{fm}^{-1}$.

The "Monte Carlo" (MC) model is drawn from a variational Monte Carlo study of Ceperley et al [25]; it entails the following form of the Jastrow correlation function

$$f(r)=\exp[-C_1 e^{-C_2 r}\frac{(1-e^{r/C_3})}{r}] \tag{18}$$

The parameters $C_1 = 1.7\,\mathrm{fm}$, $C_2 = 1.6\,\mathrm{fm}^{-1}$, $C_3 = 0.1\,\mathrm{fm}$ were determined by minimisation of the Jastrow-Slater energy expectation value corresponding to the ground state of symmetrical nuclear matter. The assumed interaction is the υ_2 potential; this state-independent potential consists of the central part of the Reid soft-core interaction in the ${}^3S_1-{}^3D_1$ channel, acting in all partial waves.

The "Gaussian" model (designated G2) is specified by the Jastrow correlation function

$$f(r) = 1 - \exp(-\beta^2 r^2) \tag{19}$$

with $\beta = 1.478\,\mathrm{fm}^{-1}$. This model has no direct connection with any familiar two-nucleon interaction. However, it could be associated with a potential containing a soft repulsive core -- softer than the Yukawa core present in the v_2 interaction.

The correlation functions $f(r)$ of the two models are plotted in Fig.1. Qualitatively similar, they nevertheless show significant differences in behaviour both in the core region and at medium distances. The corresponding wound parameters, $\kappa_{dir} = \rho\int (f(r)-1)^2 dr$ for the two models are $\kappa_{dir} = 0.297$ (MC) and 0.111 (G2).

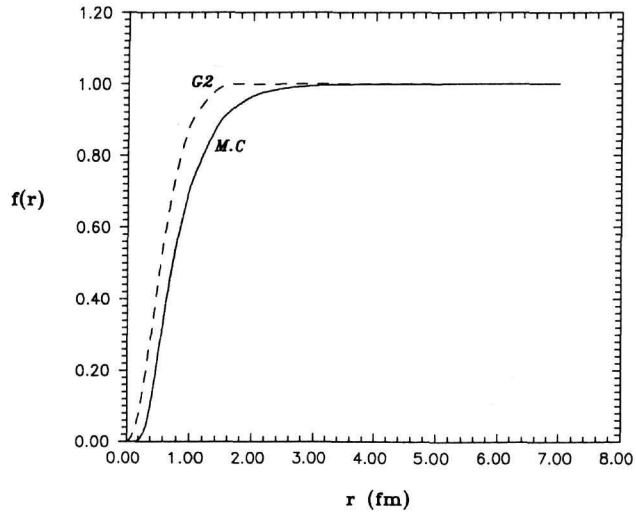

Fig. 1. Pair correlation functions $f(r)$ defining the Monte Carlo (MC) and the Gaussian (G2) models of the correlation structure of nuclear matter, plotted against radial distance r.

The FHNC/0 procedure was employed to construct the dimensionless quantity $n(\boldsymbol{p},\boldsymbol{Q})$ from Eq. (13) (disregarding the term $n^{(3')}(\boldsymbol{p},\boldsymbol{Q})$) at selected points in the ranges $[0,3k_F]$ and $(0,4k_F]$ of the momentum variables p and Q respectively. Attention is restricted to $Q \neq 0$, since as it can be seen from Eq. (13), at $Q=0$, the generalized momentum distribution just reproduces the single particle momentum distribution $n(p)$, with the large factor $A-1$. We focus on the case in which $\boldsymbol{p}$ and $\boldsymbol{Q}$ are parallel. In Ref. [14], the dependence of $n(\boldsymbol{p},\boldsymbol{Q})$ on the angle

$\theta_{p,Q}$ between $\boldsymbol{p}$ and $\boldsymbol{Q}$ was studied for the MC model in the LO approximation. Similar behaviour is to be expected in the FHNC/0 treatment.

Fig. 2 displays $n(\boldsymbol{p},Q\,\boldsymbol{p}/p)$ for the MC model as given by the FHNC/0 approximation. Figs. 3 and 4 present $n(\boldsymbol{p},Q\,\boldsymbol{p}/p)$ as a function of Q at $p = k_F$ and $p = 2k_F$ respectively, the results for the MC and G2 models being compared with that for noninteracting fermions. It should be noted that the function $n(\boldsymbol{p},Q\,\boldsymbol{p}/p)$ is discontinuous. For $p < k_F$ it is discontinuous at $Q = p + k_F$ whereas for $p > k_F$ it has discontinuities at $Q = p - k_F$ and $Q = p + k_F$. Also, as can be seen from Eq. (5), for $p < k_F$, deviations of $-n(\boldsymbol{p},Q\,\boldsymbol{p}/p)$ from unity for $Q < p + k_F$ and from zero for $Q > p + k_F$ measure the effects of dynamical correlations. For $p > k_F$, dynamical correlations are also responsible for any nonvanishing values of $n(\boldsymbol{p},\boldsymbol{Q})$. As expected from the corresponding sizes of the wound parameters, the calculated deviations from the Fermi gas limit are generally somewhat larger for the MC model than for G2.

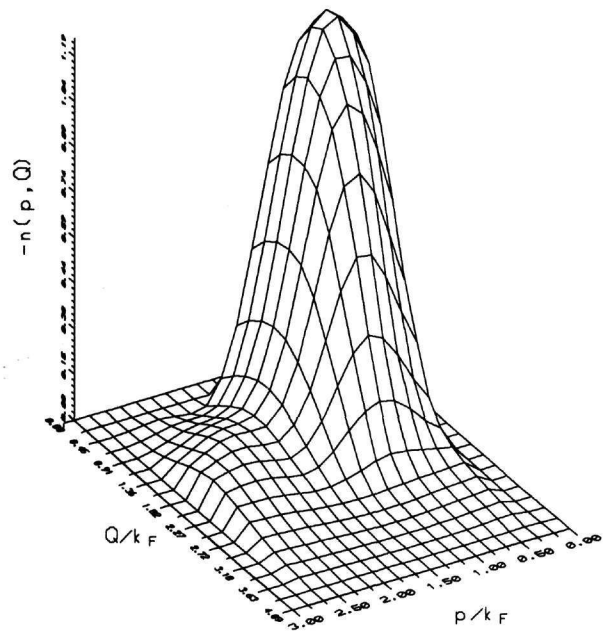

Fig. 2. Generalized momentum distribution $n(\boldsymbol{p},\boldsymbol{Q})$ as a function of momentum variables p and Q (>0) for $\boldsymbol{p}$ parallel to $\boldsymbol{Q}$, calculated by the FHNC/0 procedure based on MC correlations and nucleon density $\rho = 0.182\,\mathrm{fm}^{-3}$.

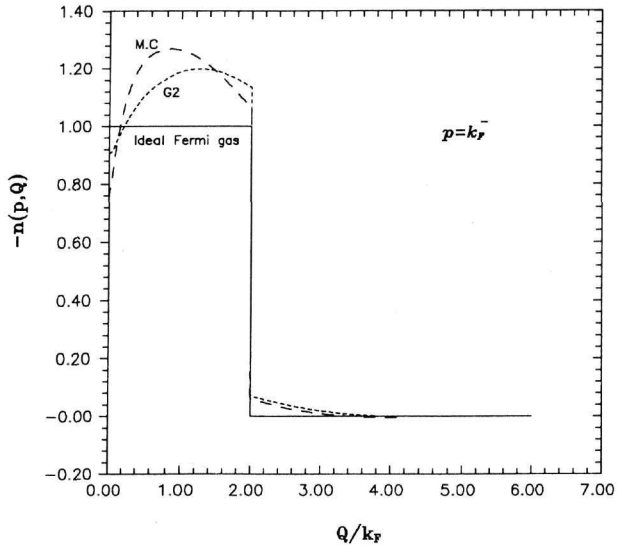

Fig. 3. Generalized momentum distribution $n(\boldsymbol{p},\boldsymbol{Q})$ as a function of Q (>0) for $\boldsymbol{Q}//\boldsymbol{p}$ and $p = k_F^-$, calculated by the FHNC/0 procedure based on the MC and G2 models and nucleon density $\rho = 0.182\,\mathrm{fm}^{-3}$. The result for the ideal Fermi gas (solid curve) is included for comparison.

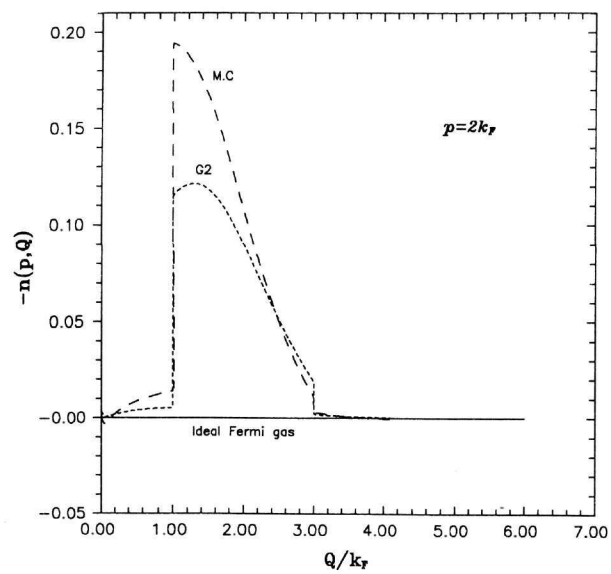

Fig. 4. As in Fig. 3 but at $p = 2k_F$.

A comparison of the FHNC/0 results with the cluster-truncation results is presented elsewhere [26]. It is found that higher-order contributions present in the FHNC/0 treatment have a net large positive effect at low Q or at low $Q-k_F$ which greatly reduces the amplitude of the correlation correction to the Fermi-gas limit. Also, for all cases examined, use of the FHNC/0 algorithm in place of the low-order cluster prescriptions leads to dramatic improvement toward satisfaction of the sequential relation.

We pause in the presentation of the results for $n(\boldsymbol{p},\boldsymbol{Q})$ to provide a view of some of its ingredients calculated within the FHNC/0 approximation for the MC model. Fig. 5 illustrates the momentum distribution $n(k)$, the modified momentum distribution $n_{DI}(k)$ and the circular-circular form factor $F_{Qcc}(k)$.

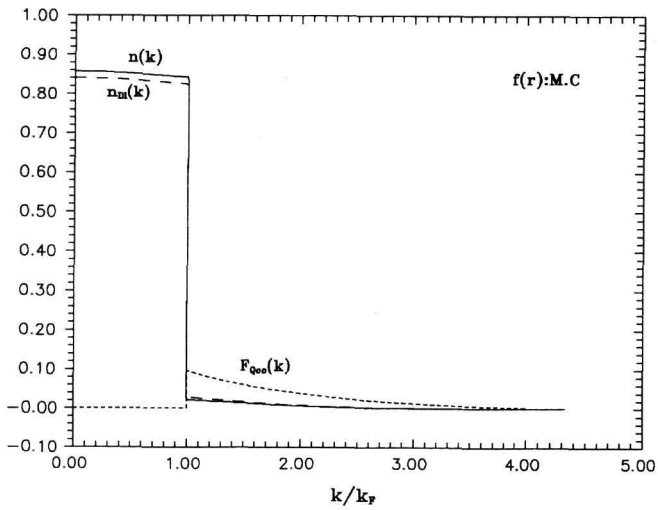

Fig. 5. Momentum distribution $n(k)$, modified momentum distribution $n_{DI}(k)$ and the circular-circular $F_{Qcc}(k)$ form factor , as functions of k, calculated in the FHNC/0 approximation for the MC model and nucleon density $\rho = 0.182\,\mathrm{fm}^{-3}$

We shall now present a test of the quality of the simple formula (11) employed for the generalized momentum distribution by Silver [17], by comparing it with our FHNC/0 evaluation. Silver's formula (11) corresponds to the following replacements

$$n(p) \cong n_{DI}(p) \tag{20}$$

$$F_{Qdd}(Q) + F_{Qde}(Q) \cong S(Q) - 1 \tag{21}$$

$$\theta(k_F - p) - F_{Qcc}(p) \cong 0 \tag{22}$$

Figs. 5-7 provide the results at the FHNC/0 level for the MC model which allow us to judge approximation (11) in terms of its ingredients (20)-(22). Fig. 5 exhibits the merits and demerits of approximation (20). The two momentum distribution functions *n(p)* and $n_{DI}(p)$ are seen to have very similar behaviour; their magnitudes differ typically by less than 2-3%. The poor quality of the estimate (21) is revealed in Fig. 6, which shows *S(Q)-1* and the sum $F_{Qdd}(Q) + F_{Qde}(Q)$.

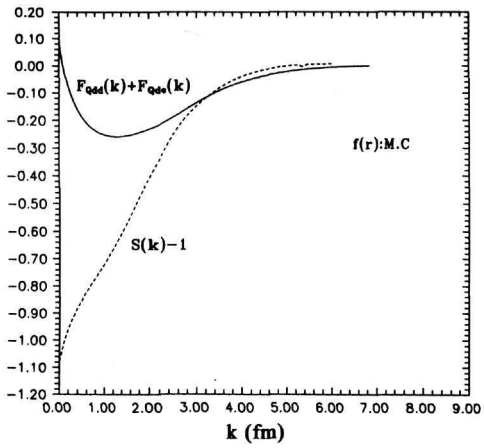

Fig. 6. Comparison of the FHNC/0 results for the quantities $F_{Qdd}(k) + F_{Qde}(k)$ and $S(k) - 1$, computed for the MC model and $\rho = 0.182\,\mathrm{fm}^{-3}$.

The shortcomings of assumption (21) are clearly exposed and are particularly apparent at small momenta. It is interesting to consider a coordinate-space view of Fig. 6, in relation to the assumption $F_{Qdd}(r) + F_{Qde}(r) \cong g(r) - 1$ (the Fourier transform of Eq.(21)). The quantities $F_{Qdd}(r) + F_{Qde}(r)$ and $g(r) - 1$ are compared in Fig. 7, where we note that the former function has a significantly smaller correlation hole than the latter at short distance, corresponding to a significantly smaller excluded volume. The same behaviour is seen for G2 model (Fig.8) as

well as for liquid ^{3}He [13]. The Pauli exclusion corrections to $n(\boldsymbol{p},\boldsymbol{Q})$ of the circular type, included in $\theta(k_F - p) - F_{Qcc}(p)$, are omitted in Silver's approximation; even the trivial kinematic statistical effect of the first term is ignored. Fig. 5 shows the significance of the circular-circular form factor $F_{Qcc}(k)$. This function vanishes inside the Fermi sea, jumps to a height of about 0.1 at the Fermi surface, and decreases slowly in magnitude with further increase of the wave number p.

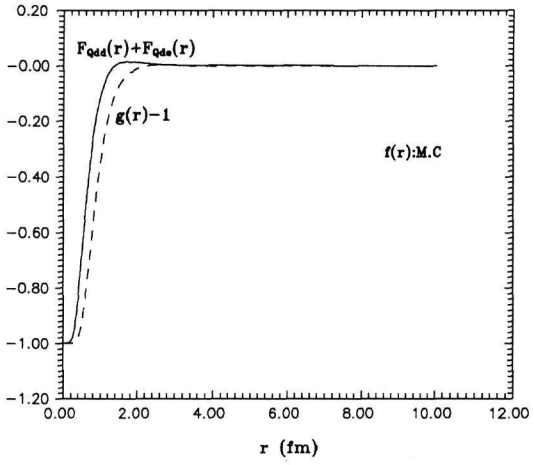

Fig. 7. The comparison of Fig. (6) is repeated in coordinate space, $g(r)$ being the radial distribution function corresponding to the static structure function.

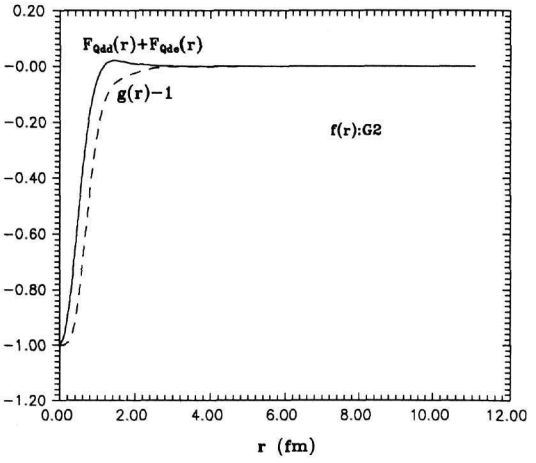

Fig. 8. As in Fig. (7) but for the G2 model.

Further, we compare the FHNC/0 evaluation of the generalized momentum distribution with results obtained within Silver's approximation scheme. Silver's Ansatz (11) is constructed from inputs $n(p)$ and $S(Q)$ calculated in FHNC/0 approximation. In table I, selected results for $n(\boldsymbol{p},\boldsymbol{Q})$ are compared with results from the current FHNC/0 evaluation of this quantity based on the Ristig-Clark theory. In Figs. 9 and 10 the comparison is made for Q//p at $p = k_F^-$ and $p = 2k_F$ respectively. For $p = k_F^-$, the Silver estimate of $-n(\boldsymbol{p},\boldsymbol{Q})$ lies considerably below the FHNC/0 result in the "Fermi gas" regime specified by $p \le k_F$ and $|\boldsymbol{p}-\boldsymbol{Q}| \le k_F$ and misses the discontinuities in the Q dependence implied by the Pauli kinematic effect. This behaviour is also exposed in Fig. 10.

TABLE I. Values of the generalized momentum distribution $n(\boldsymbol{p},Q\,\boldsymbol{p}/p)$ of nuclear matter at nucleon density $\rho = 0.182\,\mathrm{fm}^{-3}$ obtained for the MC choice of correlations using the FHNC/0 method and using Silver's approximation.

Q/k_F	$p/k_F = 1.0^-$		$p/k_F = 2.0$	
	FHNC/0	Silver's	FHNC/0	Silver's
0.5	-1.2226	-0.6702	-0.0085	-0.0050
1.0^-	-1.2634	-0.5131	-0.0142	-0.0038
1.0^+			-0.1943	
1.5	-1.1876	-0.3208	-0.1664	-0.0024
2.0^-	-1.0633	-0.1559	-0.1078	-0.0012
2.0^+	-0.0610			
2.5	-0.0293	-0.0712	-0.0502	-0.0005
3.0^-	-0.0092	-0.0244	-0.0113	-0.0002
3.0^+			-0.0032	
3.5	0.0006	-0.0033	-0.0006	-0.0000
4.0	0.0043	0.0042	0.0003	0.0000

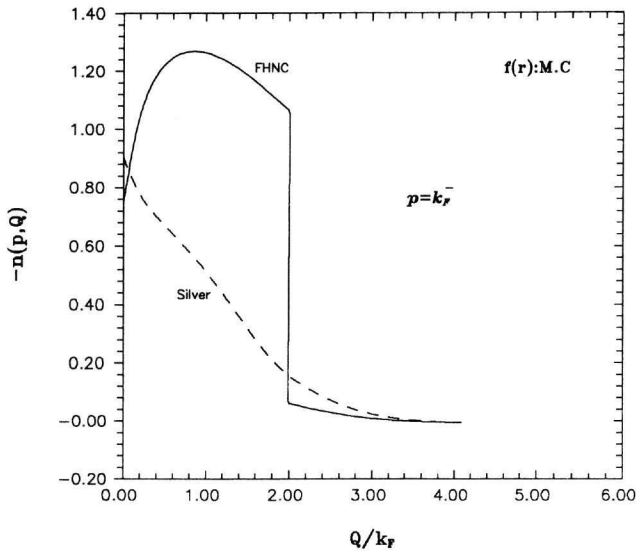

Fig. 9. Generalized momentum distribution $n(\boldsymbol{p},Q\,\boldsymbol{p}/p)$ as a function of Q (>0) at $p = k_F^-$, as calculated from Silver's formula and in FHNC/0 approximation, for the MC model and at $\rho = 0.182\,\mathrm{fm}^{-3}$.

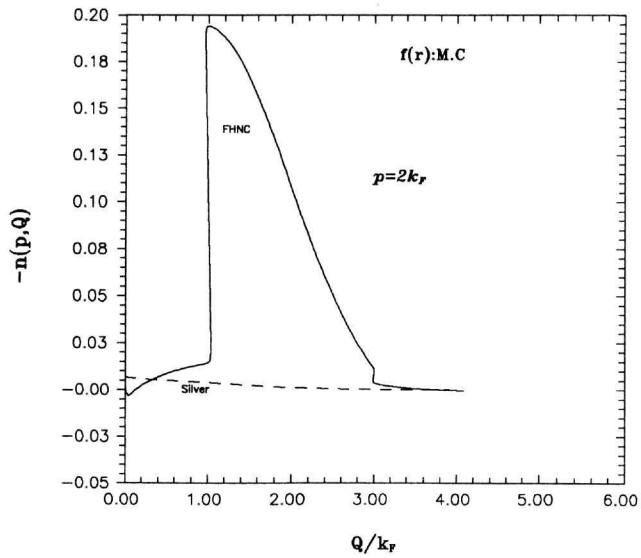

Fig. 10. As in Fig. (9) but at $p = 2k_F$.

The conclusions drawn from Fig. 7 and 8 are also relevant to a recent study of FSI in inclusive (e,e') scattering from nuclear matter carried out within Glauber correlated theory by Benhar et al. [3], which showed that short-range correlations produce an effect qualitatively similar to that of color transparency [27-29]. However, they have approximated the effects of short-range correlations in essentially the same manner as Silver, who replaces $F_{Qdd}(r)+F_{Qde}(r)$ by $g(r)-1$. Therefore, it is evident that if convincing conclusions are to be drawn from experiment regarding the quantitative importance of color transparency in inclusive scattering of GeV electrons, it will be necessary to make an accurate accounting of the analogous effect of short-range nucleon-nucleon correlations, with the half-diagonal two-body density matrix as the natural descriptor.

4.Conclusions

In summary, we have performed a quantitative microscopic determination of a momentum-space transform $n(\boldsymbol{p},\boldsymbol{Q})$ of the half-diagonal two-body density matrix of nuclear matter within the Fermi hypernetted-chain scheme. The calculations were restricted to the case of a ground-state trial function containing only state-independent, central, two-body correlations. The results exhibit interesting features that reflect the interplay of statistical and geometrical correlations and serve to test the validity of Silver's approximation. Results for $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$ in FHNC/0 and the test of the validity of the corresponding Silver's approximation will be published elsewhere [30]. Further investigations of $\rho_{2h}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_1')$ and $n(\boldsymbol{p},\boldsymbol{Q})$ in nuclear matter should take into account realistic, state dependent correlations. Some progress in this direction has been made recently by Gearhart [31]. A second important direction for future work is the microscopic determination of these quantities in finite nuclei. To this end, an extension of the local-density-approximation proposed in Refs. [32,33] for the one-body density matrix might be developed.

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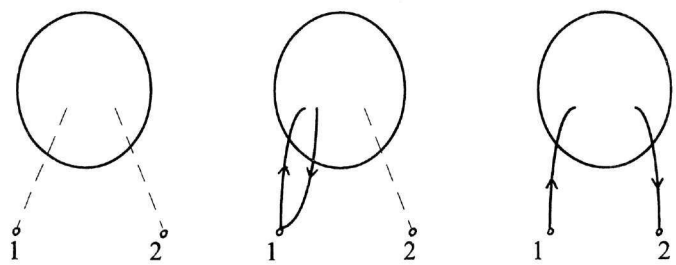
Appendix

Graphical representation of the various quantities adapted to a Fermi system described by Jastrow-Slater wave function is performed in terms of the Ursell-Mayer diagrammatic representation [22,34]. The elements of this representation consist of root (or external) points, field (or internal) points, direct bonds and exchange bonds.

A root point represents a particle coordinate which is not integrated over whereas a field point implies an integration. Bonds representing dynamical and statistical correlations join pairs of coordinate points. A dashed-direct (dashed-wavy) line represents a dynamical direct bond and corresponds to the function $f^2(r)-1$ $(f(r)-1)$. A solid line bearing an arrow represents an exchange bond and corresponds to the function $\ell(k_F r)$.

Following the above notation, dd, de and cc type $F_{Qxy}(r)$'s are of the form:



dd de cc

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