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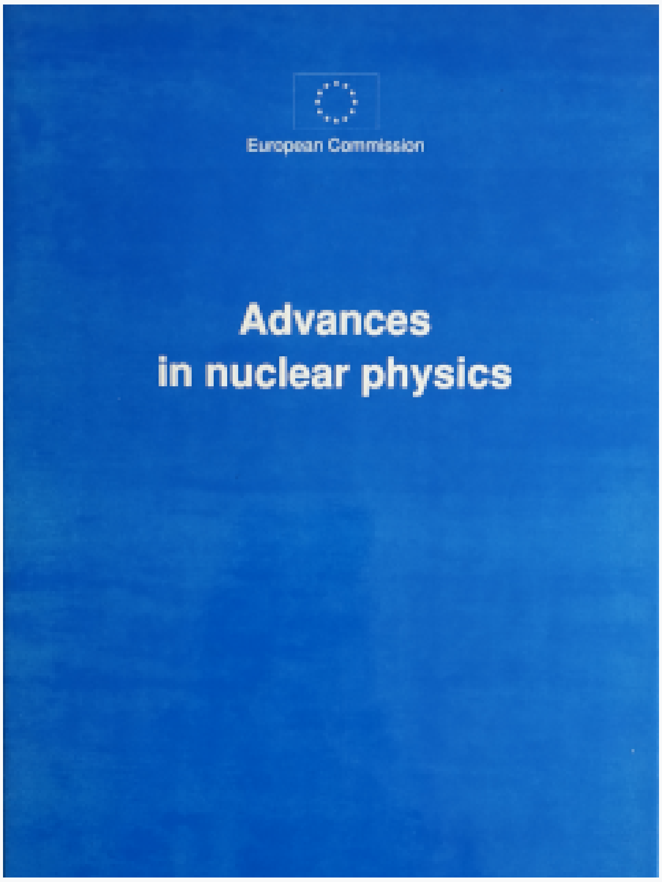

Relativistic Expressions for the r.m.s. Radii of the λ-Particle Orbits in Hypernuclei and of the Corresponding Potential Energies

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RELATIVISTIC EXPRESSIONS FOR THE R.M.S. RADII OF THE Λ-PARTICLE ORBITS IN HYPERNUCLEI AND OF THE CORRESPONDING POTENTIAL ENERGIES

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Abstract

The root mean square radii of the Λ-particle orbits in hypernuclei are calculated semi-analytically for every bound state, using the Dirac equation with a scalar potential $U_S(r)$ and the fourth component of a vector potential $U_V(r)$ in the case of rectangular shapes of these potentials with the same radius R.In addition an analytic expression of the expectation value of the corresponding potential energy operator is derived. For the above quantities, expressions of the energy eigenvalues in terms of the potential parameters are needed and approximate formulae may be used, in certain cases. The variation of these quantities with the mass number is also investigated and numerical calculations are performed.

1. Introduction

As is well known, various problems in nuclear and hypernuclear Physics are traditionally studied using the Schröndinger equation [1-6]. In the last decades, however, a trend has been developed in using also the Dirac equation for such studies, [7-16], which has certain advantages. Relativistic effects in nuclei and Λ-hypernuclei are known, however, in most cases to be very small.

In the present paper we follow a phenomenological relativistic treatment in order to calculate analytically the root mean square radii of the Λ-particle orbits in hypernuclei, for every bound state, assuming that the Λ-nucleus potential is made up of an attractive component $U_S(r)$ and a repulsive component $U_V(r)$, both of rectangular shape with the same radius. An analytic expression of the expectation value of the Λ-particle potential energy operator is also given, for every bound state. In all these calculations the knowledge of the binding energy B_Λ of the Λ particle in hypernuclei in every bound state is needed. A feature of the Dirac eigenvalue problem with such a potential is that it can be solved "semi-analytcally". The large and small component wave functions are given analytically in terms of the mass of the particle, the potential parameters and the energy. The energy eigenvalue, however, is given implicitly, that is through the eigenvalue equation which has to be solved numerically. Nevertheless, approximate formulae for B_Λ in terms of the potential parameters may be obtained in certain cases, [15].

The arrangment of this paper, in which we give our preliminary results of the treatment described earlier, is as follows. In the next section, the basic formulae used are exhibited and the derived analytic expressions for the above mentioned quantities are given. In the final section, numerical results are given and disscussed.

2. Basic formalism and analytic results

It is assumed that the average potential between the Λ-particle and the nucleus is made up of an attractive scalar relativistic single particle potential $U_S(r)$ and of a repulsive relativistic single particle potential $U_V(r)$ which is the fourth component of a vector potential and that the differential equation describing the motion of the Λ-particle in hypernuclei is the Dirac equation

$$[c\vec{\alpha}\cdot\vec{p}+\beta\mu c^2+\beta U_S(r)+U_V(r)]\psi=E\psi \tag{1}$$

where $\vec{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$, β are the Dirac matrices. E is the total energy (i.e. $E=-B_\Lambda+\mu c^2$, B_Λ being the binding energy of the Λ-particle) and ψ is the Dirac four-spinor.(We are using the formalism outlined in refs [8-10])

Instead of the potentials $U_S(r)$ and $U_V(r)$, the potentials

$$U_\pm(r)=U_S(r)\pm U_V(r) \tag{2}$$

are used, which are both attractive. We consider the case in which $U_+(r)$ and $U_-(r)$ are rectangular wells having the same radius R and depths D_+ and D_-, respectivelly i.e.

$$U_\pm(r)=-D_\pm[1-\Theta(r-R)] \tag{3}$$

where Θ is the unit step function and $R=r_0A_c^{1/3}$. A_c is the mass number of the core system. In such a case the generalized Dirac equation with the rectangular well potentials we are discussing may be solved " semi-analytically", for every bound state [14]. We find it more convenient, however, to express the large and small component wave functions in terms of the spherical Bessel j_l and the spherical MacDonald functions k_l instead of the spherical Hankel functions. The expressions of G and F may then be written:

$$G(r) = Nnr\left[[1-\Theta(r-R)]j_l(nr)+\Theta(r-R)\frac{j_l(nR)}{k_l(n_0R)}k_l(n_0r)\right] \quad (4)$$

$$F(r) = Nnc\hbar\Big[[1-\Theta(r-R)]\frac{1}{-B_\Lambda+2\mu c^2-D_-}[nrj_{l-1}(nr)+(k-l)j_l(nr)]$$

$$+\Theta(r-R)\frac{1}{-B_\Lambda+2\mu c^2}\frac{j_l(nR)}{k_l(n_0R)}[-n_0rk_{l-1}(n_0r)+(k-l)k_l(n_0r)]\Big] \quad (5)$$

while the energy eigenvalue equation becomes

$$-\left[1-\frac{D_-}{2\mu c^2-B_\Lambda}\right]\frac{n_0Rk_{l-1}(n_0R)}{k_l(n_0R)} = \frac{(k-l)D_-}{2\mu c^2-B_\Lambda}+\frac{nRj_{l-1}(nR)}{j_l(nR)} \quad (6)$$

In these expressions the quantities n and n_0 are defined as follows:

$$n = \left[\frac{2\mu}{\hbar^2}(D_+-B_\Lambda)(1-(D_-+B_\Lambda)(2\mu c^2)^{-1})\right]^{1/2} \quad (7)$$

$$n_0 = \left[\frac{2\mu}{\hbar^2}[B_\Lambda(1-B_\Lambda(2\mu c^2)^{-1})]\right]^{1/2} \quad (8)$$

The quantum numbers in G, F,B_Λ and N have been suppressed.

The normalization constant N is calculated using the following normalization condition

$$\int_0^\infty [G^2(r)+F^2(r)]dr = 1 \quad (9)$$

After using the expressions for the radial components $G(r)$ and $F(r)$ of the wavefunction, which were given above, we find for the normalization constant the following formula

$$N = \frac{1}{n}\{\frac{j_l^2(nR)}{k_l^2(n_0R)}\frac{R^3}{2}k_{l-1}(n_0R)k_{l+1}(n_0R)-\frac{R^3}{2}j_{l-1}(nR)j_{l+1}(nR)+$$

$$\frac{c^2\hbar^2}{(2\mu c^2 - B_\Lambda - D_-)^2} \cdot [-\frac{n^2R^3}{2} j_{l-2}(nR) j_l(nR) +$$
$$\frac{n^2R^3}{2} j_{l-1}^2(nR) - \frac{(k-l)^2}{2l+1} R j_l^2(nR)] +$$
$$\frac{c^2\hbar^2}{(2\mu c^2 - B_\Lambda)^2} \frac{j_l^2(nR)}{k_l^2(n_0R)} [\frac{n_0^2R^3}{2} k_{l-2}(n_0R) k_l(n_0R) -$$
$$\frac{n_0^2R^3}{2} k_{l-1}^2(n_0R) + \frac{(k-l)^2}{2l+1} R k_l^2(n_0R)]\}^{-1/2} \quad (10)$$

It is interesting to note that the normalization constant in the non-relativistic limit, that is omitting terms of the order $(\mu c^2)^{-1}$ and higher reduces to the expression

$$N_{nr} = \frac{1}{n} \frac{2^{1/2}}{R^{3/2}} [\frac{j_l^2(nR)}{k_l^2(n_0R)} k_{l-1}(n_0R) k_{l+1}(n_0R) - j_{l-1}(nR) j_{l+1}(nR)]^{-1/2} \quad (11)$$

given by Sitenko [17] for the nonrelativistic square well case.

The root mean square radii of the Λ-particle orbits in hypernuclei for every bound state are obtained by means of the expression

$$< r^2 >^{1/2} = \{\int_0^\infty r^2 [G^2(r) + F^2(r)] dr\}^{1/2} \quad (12)$$

since G and F are normalized, by means of condition (9). Calculating the above integral,(using expression (10) for the normalization constant), the following lengthy formula for $< r^2 >^{1/2}$ is derived:

$$< r^2 >^{1/2} = \frac{R}{3^{1/2}} \{\frac{(2l+3)(2l-1)}{2n^2R^2} (-j_{l-1}(nR) j_{l+1}(nR) + (1/2) j_l^2(nR)) +$$
$$((1/2)(j_{l-1}(nR) - j_{l+1}(nR)) - \frac{j_l(nR)}{nR})^2 +$$
$$\frac{D_+ - B_\Lambda}{2\mu c^2 (1 - (B_\Lambda/2\mu c^2) - (D_-/2\mu c^2))} \cdot$$
$$[\frac{(2l+1)(2l-3)}{2n^2R^2} (-j_{l-2}(nR) j_l(nR) + (1/2) j_{l-1}^2(nR)) +$$
$$((1/2)(j_{l-2}(nR) - j_l(nR)) - \frac{j_{l-1}(nR)}{nR})^2 + j_{l-1}^2(nR) +$$

$$\frac{3(k-l)^2}{n^2R^2}(-j_{l-1}(nR)j_{l+1}(nR)+j_l^2(nR))+$$

$$\frac{3(k-l)}{4n^2R^2}[(2l+1)(j_{l-2}(nR)-j_l(nR))^2+$$

$$(4-\frac{(2l-1)(2l+1)}{n^2R^2})(2l-1)j_{l-1}^2(nR)]]+$$

$$\frac{j_l^2(nR)}{k_l^2(n_0R)}[\frac{(2l+3)(2l-1)}{2n_0^2R^2}(-k_{l-1}(n_0R)k_{l+1}(n_0R)+(1/2)k_l^2(n_0R))+$$

$$((1/2)(k_{l-1}(n_0R)+k_{l+1}(n_0R))+\frac{k_l(n_0R)}{n_0R})^2]+$$

$$\frac{B_\Lambda}{2\mu c^2(1-(B_\Lambda/2\mu c^2))}\frac{j_l^2(nR)}{k_l^2(n_0R)}[\frac{(2l+1)(2l-3)}{2n_0^2R^2}(-k_{l-2}(n_0R)k_l(n_0R)+$$

$$(1/2)k_{l-1}^2(n_0R))+((1/2)(k_{l-2}(n_0R)+k_l(n_0R))+\frac{k_{l-1}(n_0R)}{n_0R})^2-$$

$$k_{l-1}^2(n_0R)+\frac{3(k-l)^2}{n_0^2R^2}(k_{l-1}(n_0R)k_{l+1}(n_0R)-k_l^2(n_0R))-$$

$$\frac{3(k-l)}{4n_0^2R^2}[(2l+1)(k_{l-2}(n_0R)+k_l(n_0R))^2-$$

$$(4+\frac{(2l-1)(2l+1)}{n_0^2R^2})(2l-1)k_{l-1}^2(n_0R)]]\}^{1/2}.$$

$$\{\frac{j_l^2(nR)}{k_l^2(n_0R)}k_{l-1}(n_0R)k_{l+1}(n_0R)-j_{l-1}(nR)j_{l+1}(nR)+$$

$$\frac{D_+-B_\Lambda}{2\mu c^2(1-(B_\Lambda/2\mu c^2)-(D_-/2\mu c^2))}\cdot[-j_{l-2}(nR)j_l(nR)+$$

$$j_{l-1}^2(nR)-\frac{(k-l)^2}{2l+1}\frac{2}{n^2R^2}j_l^2(nR)]+$$

$$\frac{B_\Lambda}{2\mu c^2(1-(B_\Lambda/2\mu c^2))}\frac{j_l^2(nR)}{k_l^2(n_0R)}[k_{l-2}(n_0R)k_l(n_0R)-k_{l-1}^2(n_0R)+$$

$$\frac{(k-l)^2}{2l+1}\frac{2k_l^2(n_0R)}{n_0^2R^2}]\}^{-1/2} \qquad (13)$$

In view of the complexity of the above expression, approximate formulae for the root mean square radii of the Λ-particle orbits in hypernuclei for every bound state were obtained by using the following asymptotic forms for the spherical Bessel and MacDonald functions

$$j_l(x) \simeq \frac{1}{x} cos(x - (1/2)(l+1)\pi), \tag{14a}$$

$$k_l(x) \simeq -\frac{i^{l-1}}{x}\frac{\pi}{2} e^{-(x+(1/2)il\pi+(i/2)\pi)} \tag{14b}$$

One of these expressions is

$$< r^2 >^{1/2} \simeq \frac{R}{3^{1/2}} + \frac{1}{2n_0 3^{1/2}} - \frac{1}{2n_0 3^{1/2}}[cos(2nR - l\pi) + sin(2nR - l\pi)\frac{n_0}{n}] \tag{15}$$

If we set

$$\varphi = arccot\frac{n_0}{n} \tag{16}$$

and write R as a function of A_c in the first term, we have

$$< r^2 >^{1/2} \simeq \frac{r_0 A_c^{1/3}}{3^{1/2}} + \frac{1}{2n_0 3^{1/2}} - \frac{1}{2n_0 3^{1/2}} \frac{sin(2nR - l\pi + \varphi)}{sin\varphi} \tag{17}$$

Observing that the ratio

$$\frac{sin(2nR - l\pi + \varphi)}{sin\varphi} \tag{18}$$

can be approximated by a constant C_{Nl}, (where N is the principal quantum number $N = 1, 2, ...$), one obtains the simple approximate formula

$$< r^2 >^{1/2} \simeq \frac{r_0}{3^{1/2}} A_c^{1/3} + \frac{(1 - C_{Nl})}{2n_0 3^{1/2}} \tag{19}$$

for the lower bound states. From this formula one can deduce immediately the almost linear behaviour of the curves $< r^2 >^{1/2}$ versus $A_c^{1/3}$ for the heavier hypernuclei. (We note by, that in the above formula if $l = 0$, $C_{1l} = -1.2$. If $l = 1$, $C_{1l} = -1.4$, e.t.c.)

Another approximate expression which may be derived from (15) is the following

$$< r^2 >^{1/2} \simeq \frac{r_0}{3^{1/2}}[1 + sin(\frac{l\pi}{4}) - cos(\frac{l\pi}{4})(\frac{D_+}{B_\Lambda} - 1)^{1/2}(1 - \frac{D_-}{4\mu c^2})]A_c^{1/3} +$$

$$\frac{c\hbar}{2(3^{1/2})(2\mu c^2)^{1/2}}[B_\Lambda^{-1/2}(1+sin(\frac{l\pi}{4})+(\frac{3\pi}{2}-\frac{l\pi}{4}+l\pi)cos(\frac{l\pi}{4})-$$

$$(D_+-B_\Lambda)^{-1/2}((\frac{3\pi}{2}-\frac{l\pi}{4}+l\pi)sin(\frac{l\pi}{4})-cos(\frac{l\pi}{4})(1+\frac{D_-}{4\mu c^2}))] \qquad (20)$$

For the ground state ($l=0$) all the expressions for the root mean square radii go over to those given in ref [16].

We have calculated also the potential energy of the Λ-particle in every bound state.The potential energy operator is of the form

$$V(r)=\beta U_S(r)+U_V(r) \qquad (21)$$

The expectation value of $V(r)$ for the rectangular potentials considered here is

$$<V(r)>=\frac{-D_+\int_0^R G^2(r)dr+D_-\int_0^R F^2(r)dr}{\int_0^\infty[G^2(r)+F^2(r)]dr} \qquad (22)$$

Using the normalization condition and the normalization constant we find for the potential energy the expression

$$<V(r)>=\{-D_+[-j_{l-1}(nR)j_{l+1}(nR)+j_l^2(nR)]+$$

$$D_-\frac{D_+-B_\Lambda}{2\mu c^2(1-(B_\Lambda/2\mu c^2)-(D_-/2\mu c^2))}\cdot$$

$$[-j_{l-2}(nR)j_l(nR)+j_{l-1}^2(nR)-\frac{(k-l)^2}{2l+1}\frac{2}{n^2R^2}j_l^2(nR)]\}\cdot$$

$$\{\frac{j_l^2(nR)}{k_l^2(n_0R)}k_{l-1}(n_0R)k_{l+1}(n_0R)-j_{l-1}(nR)j_{l+1}(nR)+$$

$$\frac{D_+-B_\Lambda}{2\mu c^2(1-(B_\Lambda/2\mu c^2)-(D_-/2\mu c^2))}\cdot$$

$$[-j_{l-2}(nR)j_l(nR)+j_{l-1}^2(nR)-\frac{(k-l)^2}{2l+1}\frac{2}{n^2R^2}j_l^2(nR)]+$$

$$\frac{B_\Lambda}{2\mu c^2(1-(B_\Lambda/2\mu c^2))}\frac{j_l^2(nR)}{k_l^2(n_0R)}[k_{l-2}(n_0R)k_l(n_0R)-k_{l-1}^2(n_0R)+$$

$$\frac{(k-l)^2}{2l+1}\frac{2}{n_0^2R^2}k_l^2(n_0R)]\}^{-1} \qquad (23)$$

In the non-relativistic limit the above expression goes over to the expression

$$< V(r) >= -D_+ + \{D_+ + j_l^2(nR)(\frac{k_{l-1}(n_0R)k_{l+1}(n_0R)}{k_l^2(n_0R)} - 1)\}\cdot$$

$$\{j_{l-1}(nR)j_{l+1}(nR) + j_l^2(nR)\frac{k_{l-1}(n_0R)k_{l+1}(n_0R)}{k_l^2(n_0R)}\}^{-1} \tag{24}$$

From expression (23) one can obtain also the following simple approximate expression for the potential energy.

$$< V(r) >\simeq -D_+ + (D_+ - D_-)(D_+ - B_\Lambda)(2\mu c^2 - 2B_\Lambda - D_- + D_+)^{-1} \tag{25}$$

This is expected to hold, however, for very large values of A_c (and lower states).

3. Numerical results and comments

In this section we present our results concerning the root mean square radii of the Λ-particle orbits in hypernuclei and also its potential energy. These results were obtained using the expressions (13), (15), (23), and the following potential parameters

$$D_- = 300MeV\ ,\ \ D_+ = 25.74MeV,\ \ r_0 = 1.22fm$$

derived from an "overall fitting" that is a least squares fitting to the experimental binding energies B_Λ^{ex} of all states of a number of hypernuclei. In the above fitting the potential parameter D_- was kept fixed to the value $D_- = 300MeV$(see ref [14]).

The results for the root mean square radii of the Λ-particle orbits in its ground and excited states are given in tables I and II, respectively. In table I the results derived with the exact expression (13) are given while in table II those derived with the approximate expression (15). (Note that in table II only the lower states are tabulated.)

The results for the potential energy of the Λ-particle in all bound states, calculated using the exact expression (23) are given in table III . It is seen that the variation of $< V(r) >$ with A_c is increasing, as expected.

In figure 1 the results given in table I (i.e the $< r_\Lambda^2 >^{1/2}$) are plotted versus $A_c^{1/3}$. It is interesting to note that the behaviour is rather similar to that reported in ref. [3]. The almost linear behaviour of the curves $< r_\Lambda^2 >^{1/2}$

versus $A_c^{1/3}$, for the heavier hypernuclei, is fairly well understood on the basis of expression (19) which predicts such a behaviour for large A_c.

The main advantage of the rectangular potential used is the possibility of solving the corresponding eigenvalue problem semi-analytically. This facilitates the analysis carried out and gives rise to fairly simple approximate analytic expressions of the quantities discussed, as function of the mass number.

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References

1. R.H. Dalitz, Proceedings of Summer School, Les Houches , Nuclear Physics, edited by C. de Witt and V. Gillet (Gordon-Breach, New York, 1969), p. 701.

2. A. Gal, Adv. Nucl. Phys.**8** (1975) 1.

3. M. Rayet, 1976 Ann.Phys. **102** (1976),226.

4. C.B. Daskaloyannis,M.E. Grypeos, C.G. Koutroulos, S.E. Massen and D.S. Saloupis, Phys.Lett. **134B** (1984),147.

5. D.J. Millener, C.B. Dover and A. Gal, Phys. Rev C **38** (1988) 2700.

6. G.A. Lalazissis, M.E. Grypeos and S.E. Massen, Phys.Rev. C **37** (1988), 2098.
G.A. Lalazissis, J. Phys. G, Nucl. and Part. Phys. **19** (1993) 695.
G.A. Lalazissis, Phys. Rev. C **49** (1994) 142.

7. M Rufa et. al. Phys. Rev. C **42** (1990) 2469.
B.K. Zennings, Phys. Let. **246B** (1990) 325.
J. Mares and J. Zofka, Phys. let. **249B** (1990) 181.

8. R.Brockmann and W. Weise, Phys.Lett.**69B** (1977),167.

9. R.Brockmann, Phys.Rev. **C18** (1978),1510

10. R.Brockmann and W.Weise, Nucl. Phys. **A 355** (1981), 365

11. C.G. Koutroulos and M.E. Grypeos, Phys. Rev. C **40** (1989), 275

12. C.G. Koutroulos, J.Phys.G:Nucl. Part. Phys. **15** (1989),1659
C.G. Koutroulos, Z.Naturforsch. **45a** (1990),191. C.G. Koutroulos Few-Body Systems, Suppl. **5** (1992) 430.

C.G. Koutroulos, J.Phys.G:Nucl. Part. Phys.**17** (1991), 1069
13. J. Mares and B.K. Jennings, Phys. Rev. C **49** (1994) 2472.
14. G.J. Papadopoulos, C.G. Koutroulos and M.E. Grypeos, International Journal of Modern Physics E, **3** (1994) 939.
15. M.E. Grypeos, C.G. Koutroulos and G.J. Papadopoulos, Physical Review A **50** (1994) 29
16. C.G. Koutroulos and G.J. Papadopoulos, Progress of Theoretical Physics, **90** (1993) 1039
17. A.G. Sitenko and V.K. Tartakovskii, translated and edited by P.J. Shepherd, Pergamon Press, Oxford, (1975).

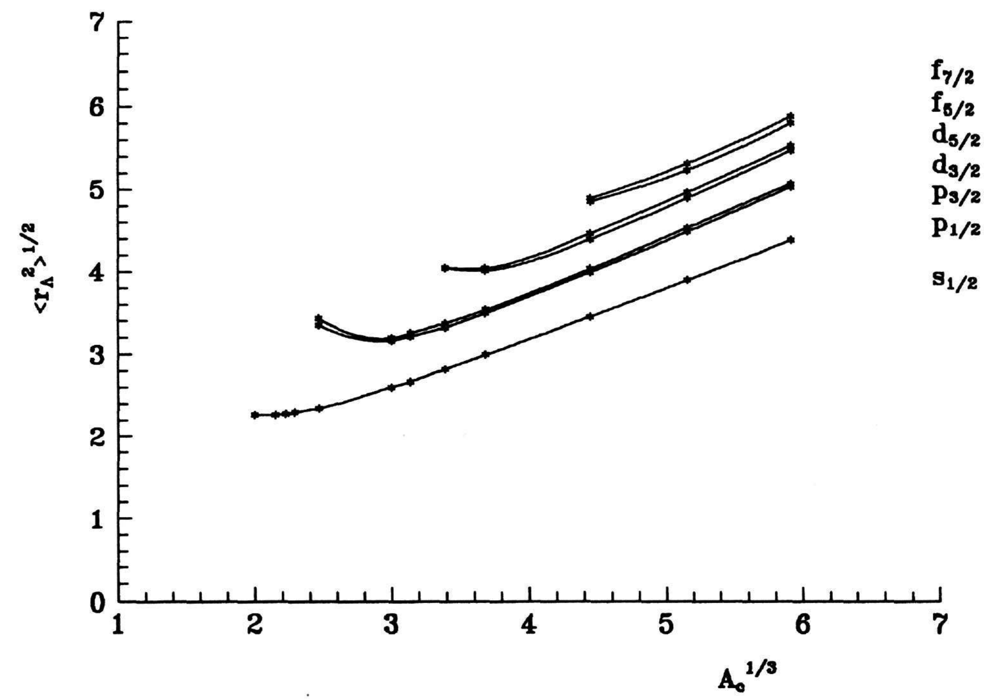

Fig. 1. Variation of the $<r^2>^{1/2}$ with $A^{1/3}$ for the ground and first excited states of Λ hypernuclei.

Table I

Root mean square radii $< r^2 >^{1/2}$ of the Λ-particle orbits in the ground and excited states for various hypernuclei obtained numerically using the definition (expression (13)). The potential parameters used are $r_0 = 1.22\ fm$, $D_+ = 25.74\ MeV$, $D_- = 300\ MeV$.

A_c	$s_{1/2}$ fm	$p_{3/2}$ fm	$p_{1/2}$ fm	$d_{5/2}$ fm	$d_{3/2}$ fm	$f_{7/2}$ fm	$f_{5/2}$ fm
8	2.27						
10	2.27						
11	2.28						
12	2.30						
15	2.35	3.44	3.96				
27	2.61	3.20	3.17				
31	2.68	3.26	3.22				
39	2.83	3.38	3.33	4.06	4.26		
50	3.00	3.54	3.50	4.05	4.02		
88	3.46	4.04	4.00	4.47	4.40	4.9	4.86
137	3.90	4.54	4.49	4.97	4.90	5.32	5.24
207	4.39	5.08	5.04	5.54	5.48	5.89	5.81

Table II

Root mean square radii $< r^2 >^{1/2}$ of the Λ-particle orbits in the ground and first excited states for various hypernuclei obtained using the approximate expression (15). The potential parameters used are the same as in table *I*.

A_c	$s_{1/2}$ fm	$p_{3/2}$ fm	$p_{1/2}$ fm
8	2.38		
10	2.40		
11	2.42		
12	2.44		
15	2.51	3.56	4.40
27	2.80	3.12	3.16
31	2.89	3.16	3.18
39	3.05	3.28	3.28
50	3.24	3.44	3.43
88	3.75	4.93	3.92
137	4.23	4.42	4.41
207	4.75	4.96	4.95

Table III

Potential energy $< V >$ of the Λ-particle in the ground and excited states for various hypernuclei obtained from expression (23). The parameters used are the same as in table I.

A_c	$s_{1/2}$ MeV	$p_{3/2}$ MeV	$p_{1/2}$ MeV	$d_{5/2}$ MeV	$d_{3/2}$ MeV	$f_{7/2}$ MeV	$f_{5/2}$ MeV
8	-17.44						
10	-18.82						
11	-19.32						
12	-19.75						
15	-20.70	-13.46	-11.03				
27	-22.64	-18.66	-17.91				
31	-22.81	-19.42	-18.81				
39	-23.26	-20.47	-20.03	-15.89	-14.14		
50	-23.67	-21.38	-21.07	-18.02	-17.04		
88	-24.35	-22.87	-22.72	-20.88	-20.47	-18.11	-17.12
137	-24.72	-23.65	-23.57	-22.26	-22.03	-20.48	-19.99
207	-24.98	-24.18	-24.13	-23.16	-23.02	-21.9	-21.62