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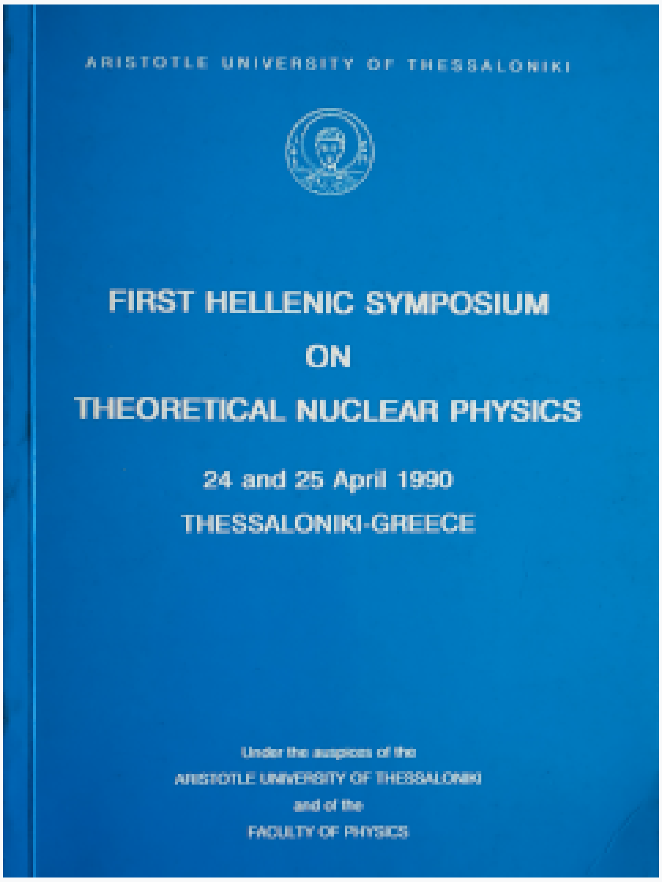

Nuclear form factors and closure approximation in the study of the (μ-,e-) conversion in nuclei

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Nuclear form factors and closure approximation in the study of the (μ^-, e^-) conversion in nuclei*

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ABSTRACT : The methods of studying the exotic (μ^-, e^-) conversion in nuclei are discussed. For the coherent process the dependence of the rate on the nuclear parameters is obtained by using shell model nuclear form factors. For the noncoherent processes the relevant matrix elements are calculated in the framework of the closure approximation. Finally the fraction of the transition rate of the coherent process throughout the periodic table is calculated.

1. Introduction

The anomalous process of converting the bound muon of a muonic atom into an electron :

$$\mu^- + (A, Z) \rightarrow e^- + (A, Z) \qquad (1),$$

has recently aroused a special experimental [1,2] and theoretical [3-8] interest among all the muon number violating processes. On the theoretical side muon number violation is predicted to occur due to neutrino mixing, s-lepton mixing et.c. in the framework of almost all the extensions of the standard weak interaction model. Since it has been suggested [4] that the muon conversion (1) is the best place to look for the muon number violation if it occurs, experimental efforts were made in searching for the limits of the branching ratio R_{eN} of the $\mu - e$ conversion rate to the total rate of the ordinary muon capture reaction [9]. Up to now the best limit has been set for the ^{48}Ti nucleus [2] in the value $R_{eN} \leq 4.6 \times 10^{-12}$ and it is expected to be improved further by new experimens.

Of special interest is the coherent process of the (μ^-, e^-), i.e. when the nucleus (A,Z) remains in its ground state. In this process only ground state to ground state transitions $(gs \rightarrow gs)$ are present and they have been discussed in detail previously [5,6]. Transitions to all excited states were only recently studied [7] by calculating the total rate, sum of the partial rates for all the excited states $| f >$ of the nucleus in the process considered, by using closure approximation. In the recent survey [8]

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of the muon number violation in the (μ^-, e^-) the (A,Z) dependence of the total $\mu - e$ rate for all the periodic table has been extensively studied.

In this talk, after stating briefly the main conclusions of the related theoretical works we shall focus our attention on the calculation of the $gs \longrightarrow gs$ transitions which exaust the main part of the total (μ^-, e^-) rate [5,6].

2. Theoretical formulation

Starting from an effective interaction hamiltonian constructed in the context of a gauge model predicting the process (μ^-, e^-) [3], we can write at nuclear level the single-particle operator responsible for this process as [8]

$$\Omega_0 = f_V \sum_{j=1}^{A} \Big(\frac{3+\beta\tau_{3j}}{3+\beta}\Big) e^{-i\mathbf{k}_f \cdot \mathbf{r}_j}, \qquad \mathbf{\Omega} = -f_A \sum_{j=1}^{A} \Big(\frac{3+\beta\tau_{3j}}{3+\beta}\Big) \frac{\sigma_j}{\sqrt{3}} e^{-i\mathbf{k}_f \cdot \mathbf{r}_j} \tag{2}$$

where Ω_0 is the vector and $\mathbf{\Omega}$ the axial vector component of the (μ^-, e^-) operator and σ_j (τ_j) the spin (isospin) operator of the nucleon with space coordinate $\mathbf{r}_j$. The parameter β is equal to the ratio of the isovector to the isoscalar component at the quark level and depends on the specific mechanism assumed for lepton flavour violation [3]. For simplicity in this talk we shall discuss the case of photonic mechanism ($\beta = 3$). Other mechanisms are discussed in ref. [8]. The magnitude of the momentum $\mathbf{k}_f$ of the outgoing electron is given approximately by

$$| \mathbf{k}_f | \approx m_\mu - (E_f - E_{gs}) \tag{3}$$

whith E_f, E_{gs} being the energies of the final and ground state of the nucleus, respectively.

The calculation of the total (μ^-, e^-) rate needs the evaluation of the vector and axial vector matrix elements of the operator (2) for all the intermediate states $| f >$ i.e.

$$S_\alpha = \sum_f \Big(\frac{k_f^2}{m_\mu^2}\Big) \int \frac{d\hat{\mathbf{k}}_f}{4\pi} |< f \mid \Omega \mid i >|^2, \qquad \alpha = V, A \tag{4}$$

In order to find the S_V and S_A, one can follow two general methods:

1) *Closure approximation*: With this method the contribution of each final state $| f >$ into the total rate is taken into account without constructing this final state explicitly. By assuming a mean excitation energy of the nucleus $\Delta E = E_f - E_{gs} \approx 20 MeV$ (or a mean momentum transfer $k \approx .435 fm^{-1}$), as is the

case in the ordinary muon capture process, the matrix elements (4) can be well approximated by the expectation value in the ground state of two-body operators. The result is written as

$$M_{tot}^2 = \frac{k^2}{m_\mu^2}\left[|<i \mid O_V \mid i>|^2 + 3|<i \mid O_A \mid i>|^2\right] \tag{5}$$

where the two-boby operators O_V (vector) and O_A (axial vector) result from the corresponding parts of the (μ^-, e^-) operator. Their exact expressions are given in eq. (23) of ref. [8].

2) *Summing over partial rates*: With this method we construct explicitly the final nuclear states $\mid f>$ in the context of a nuclear model e.g. shell model, random phase approximation et.c. Thus,by performing detailed calculations the validity of the closure approximation in the process (μ^-, e^-) can be checked. The method proceeds by expanding the exponential of the operators (2) in terms of spherical Bessel functions. Then the matrix elements (4) are written as

$$S_\alpha^2 = f_\alpha \sum_f \left(\frac{k_f^2}{m_\mu^2}\right) \sum_l |<f \mid\mid T^{(l,\sigma)J} \mid\mid i>|^2, \qquad \alpha = V, A \tag{6}$$

where the two types of the operators $T^{(l,\sigma)J}$, are given by

$$T_m^{(l,0)J} = \delta_{lJ} \sum_{all\ nucleons} (1+q\beta) j_l(kr) Y_l^m(\hat{\mathbf{r}}) \qquad (6a),$$

for the vector part $(\sigma = 0)$ and by

$$T_M^{(l,\sigma)J} = \sum_{all\ nucleons} (1+q\beta) j_l(kr) \left[Y_l^m(\hat{\mathbf{r}}) \times \sigma\right]_M^J \qquad (6b),$$

for the axial vector part $(\sigma = 1)$. The index q in eqs. (6a) and (6b) reffers to protons, $q = 1$, or neutrons, $q = -1$ and results because of the selection rules of the operator (2) (charge conserving).

In ref. [10] the quasi-particle random phase approximation (QRPA) is currently used to calculate the intermediate nuclear states. The reduced matrix elements of eq. (6) in terms of the QRPA method for a transition from the g.s. (0^+) to the state $\mid f>$ is written as:

$$<f \mid\mid T^{(l,\sigma)J} \mid\mid 0^+> = \sum_{\alpha \leq \beta, q} <\alpha \mid\mid T^J \mid\mid \beta>_q \left[X_{\alpha\beta}^{(f,q)J} u_\alpha v_\beta + Y_{\alpha\beta}^{(f,q)J} u_\beta v_\alpha\right]_q \tag{7}$$

The quantities X and Y are the forward and backward-going amplitudes and v, u represent the probability amplitutes for the single particle states to be occupied and unoccupied respectively. The indices α and β run over all configurations coupled to a given angular momentum J of the state $\mid f >$. Thus,the task of QRPA is to provide the values of X, Y amplitudes as well as the probability amplitudes u and v.

The computation of the reduced matrix elements in eq. (5) and (7) is straightforward. In the context of the shell model the matrix elements of eq. (5) for closed (sub)shell nuclei can be written in a simplified form as [7,8]

$$S = g(A,Z)\Big(1 - \sum_{\lambda=1}^{N_{max}} \xi_\lambda \, \alpha^{2\lambda}\Big) e^{-\alpha^2/4}, \quad \alpha = \sqrt{2}kb \tag{8}$$

where b is the harmonic oscillator parameter, N_{max} is the maximum number of quanta occupied by the nucleons in the considered nucleus and ξ_λ are appropriate coefficients. The functions $g(A,Z)$ describe the total rate for small momentum transfer ($k \approx 0$).

The partial rate for $gs \rightarrow gs$ 0^+ transitions has contribution only from the vector operator. In this case the corresponding matrix element takes the form

$$M^2_{gs\rightarrow gs} = f_V^2 \frac{k^2}{m_\mu^2} \Big[Z F_Z(k^2) + N \frac{3-\beta}{3+\beta} F_N(k^2) \Big]^2 \tag{9a}$$

where $F_Z(k^2)$ ($F_N(k^2)$) are the proton (neutron) nuclear form factors. For the coherent rate ($k \approx 0.53 fm^{-1}$) the shell model form factors $F_Z(k^2)$ for all closed (sub)shell nuclei take the simple form (point-like particles are assumed)

$$F_Z(k^2) = \frac{1}{Z} e^{-\alpha^2/4} \sum_{\lambda=1}^{N_{max}} \theta_\lambda \alpha^{2\lambda}, \qquad \alpha = kb \tag{9}$$

where the coefficients θ_λ are rational numbers.

3. Results and discussion

In the study of the (μ^-, e^-) two quantities are most important:

1. The branching ratio R_{eN} of the $\mu - e$ rate to the total muon-capture rate which can be written as [8]

$$R_{eN} = \frac{\Gamma(\mu^- \rightarrow e^-)}{\Gamma(\mu^- \rightarrow \nu_\mu^-)} = \rho\gamma \tag{10}$$

where the quantity ρ contains the lepton flavour violating parameters [3] and the function γ describes the nuclear dependence of R_{eN}. The function γ is given by

$$\gamma = \frac{M^2}{G^2 Z f(A,Z)} \tag{11}$$

$Nucleus(A,Z)$	F_Z	γ	$M^2_{gs\rightarrow gs}$	M^2_{tot}	$\eta\%$
(12, 6)	.763	3.64	21.0	26.8	78.4
(16,8)	.736	4.52	34.7	38.8	89.4
(28,14)	.639	5.95	80.0	100.6	79.5
(32,16)	.618	6.37	97.8	123.8	79.0
(40,20)	.582	7.05	135.5	169.8	79.8
(48,20)	.563	16.08	126.8	188.8	67.2
(60,28)	.489	9.24	187.5	289.0	64.9
(72,32)	.456	11.54	212.9	356.6	59.7
(88,38)	.412	12.98	245.1	446.8	54.9
(90, 40)	.406	11.41	263.7	471.1	56.0
(114,50)	.335	10.35	280.6	607.9	46.2
(156,64)	.263	11.96	283.3	783.0	36.2
(168,68)	.249	12.47	286.7	833.7	34.4
(176,70)	.242	13.75	287.0	861.7	33.3
(208,82)	.189	10.42	240.2	941.5	25.5

Table 1. Matrix elements for the photonic mechanism of the (μ^-, e^-) conversion in closed (sub)shell nuclei.

In eq. (11) $f(A,Z)$ represents the Primakoff's function [9], which describes the total muon capture rate, $G^2 \approx 6.0$ and M^2 are the nuclear matrix elements calculated as we stated previously.

In table 1 the proton nuclear form factors calculated at $k \approx .534 fm^{-1}$ by using eq. (9b) and taking into account the nucleon finite size are shown. Though this value of the momentum transfer k is relatively large for nuclear standards the agreement with the experimental elastic electron scattering data has been found [6] to be very good for all closed (sub)shell nuclei up to ^{208}Pb. In the above table the resulting values of function γ are also included. We see that the gross dependence of the $\mu - e$ rate on (A,Z) is relatively smooth. The variations reflect mainly the dependence of the Primakoff's function on the neutron excess (N-Z) [8].

2. The second usefull quantity of the (μ^-, e^-) reaction is the ratio of the coherent rate divided by the total $\mu - e$ rate i.e

$$\eta = M^2_{gs \to gs} / M^2 \tag{12}$$

Values of the ratio η and the needed matrix elements for stable nuclei throughout the periodic table are listed in table I. We see that for light nuclei the coherent channel dominates but η decreases for heavy nuclei.

4. Conclusions

The shell model nuclear form factors and total $\mu - e$ matrix elements obtained by assuming closure approximation, show that the behaviour of the rate as a function of the nuclear parameters is smooth and that the coherent $\mu - e$ rate for light nuclei dominates. For heavy nuclei other channels become signifficant and the coherent contribution is only about 30%.

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