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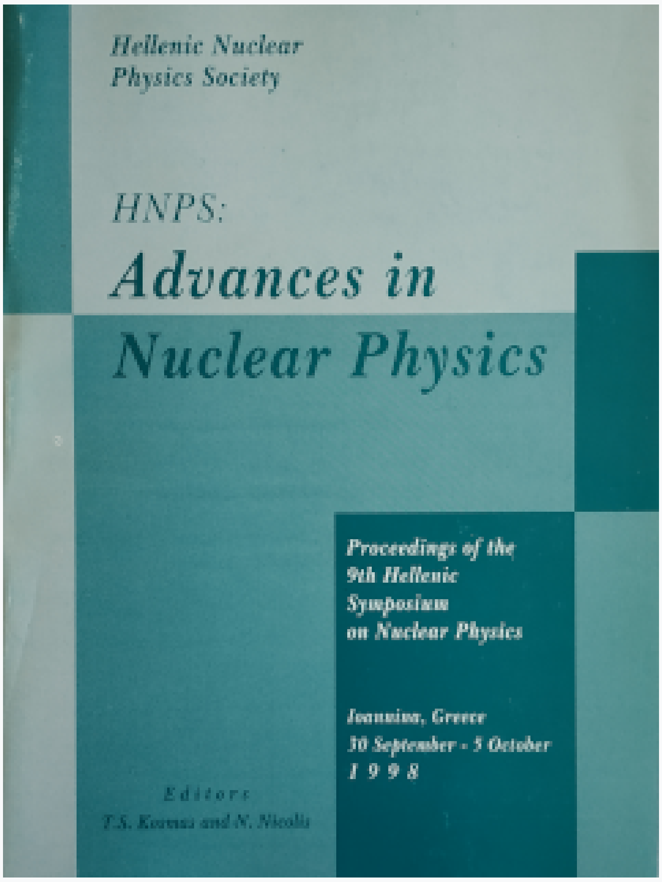

Study of the exotic μ-e conversion in nuclei using RQRPA

Zhongzhu Ren, A. Faessler, T. S. Kosmas

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Study of the exotic $\mu - e$ conversion in nuclei using RQRPA

Zhongzhou Ren [a,1], Amand Faessler [a] and T.S. Kosmas [b]

[a] *Institute für Theoretische Physik der Universität Tübingen, D-72076 Tübingen, Germany*

[b] *Theoretical Physics Division, University of Ioannina, GR-45110 Ioannina, Greece*

Abstract

The neutrinoless muon-to-electron conversion in nuclei is studied by using the renormalized quasiparticle random-phase approximation (RQRPA). This generalization of RPA is more reliable for the extremely small (μ^-, e^-) transition matrix elements than the ordinary QRPA because it restores the Pauli principle to a large extent. We apply the method to a set of nuclei throughout the periodic table, but we specifically investigate the ^{48}Ti and ^{208}Pb nuclei which are currently used as stopping targets at the PSI $\mu - e$ conversion experiments with the SINDRUM II spectrometer.

Key words: Lepton flavor violation, rare muon decays, $\mu - e$ conversion, muon capture, RPA, renormalized quasiparticle RPA, ground state correlations, transition matrix elements.
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1 Introduction

The investigation of the neutrinoless $\mu^- \to e^-$ conversion in muonic atoms,

$$\mu_b^- + (A, Z) \to e^- + (A, Z)^* \tag{1}$$

is especially interesting research both from an experimental [1]-[3] and theoretical [4]-[7] physics point of view. The importance of this process is due to the fact that its observation would signal a breakdown of the separate lepton number conservation for electrons and muons. Up to now no μ-e conversion

[1] Perm. address: Department of Physics, Nanjing University, Nanjing 210008, China

events have been seen but instead upper limits for the observation of this process have been well established by several experiments [1]-[3]. The best limit of the $\mu^- \to e^-$ branching ratio to the ordinary μ^- capture, $R_{\mu e}$, has been set by the SINDRUM II collaboration using ^{48}Ti [1,2] as

$$R^{Ti}_{\mu e} \leq 6.1 \times 10^{-13} \tag{2}$$

This limit provides very severe constraints for muon number non-conservation compared to other similar processes [8]-[11]. This is mostly due to the possibility of the coherent effect, a distinct feature of process (1) which constitutes the signature of the $\mu - e$ conversion in the relevant experiments. The coherent channel is expected to appear as a single peak at momentum $q \approx m_\mu - \epsilon_b$ in the measured electron-spectrum (m_μ is the muon mass and ϵ_b its binding energy in the muonic atom).

The advantages of the $\mu - e$ conversion motivated some of the most sensitive experiments [1]-[3] performed the last decades to look for flavor violation events (see Ref. [5]). The SINDRUM II experiment, which is at present the only operating $\mu - e$ conversion experiment, is using ^{48}Ti as target and is aiming to increase the sensitivity on the branching ratio $R_{\mu e}$ by about two orders of magnitude. Recently, a new $\mu - e$ conversion experiment, the so called MECO experiment at Brookhaven, has been designed on ^{27}Al target [3] with the aim to push the best limit below 10^{-16} and search for new physics. For these experiments the knowledge of reliable nuclear physics inputs for all possible $\mu^- \to e^-$ conversion channels of the targets used is an essential prerequisite.

In previous works [7,8] the renormalized quasiparticle random phase approximation (RQRPA) appropriate for the $\mu - e$ conversion has been formulated. The influence of the renormalized quasi-boson approximation, on which the RQRPA relies, to the coherent matrix elements was also studied [7]. As has been shown, the improved quasi-boson approximation takes reliably into account the nucleon-nucleon ground state correlations which are important in evaluating accurately the sensitive (μ^-, e^-) rates. The purpose of this work is to investigate the incoherent channels and calculate all possible transition matrix elements of the reaction (1) within the context of the RQRPA. The correlated ground state, on which the excited states reached by the incoherent channels are built, is derived in the same manner as in Ref. [7] and plays important role in our calculations, because it includes more exactly the Pauli-exclusion principle than the previously used normal QRPA.

The relevant $\mu - e$ conversion operators refer to the photonic and some non-photonic mechanisms (see Ref. [12]-[16]) and calculations are performed for a set of nuclei throughout the periodic table. We emphasize on the nuclei ^{48}Ti and ^{208}Pb which have recently been used as targets in the SINDRUM II experiment at PSI. We should note that, similar calculations for the odd-A ^{27}Al target employed by the Brookhaven experiment cannot be carried out with RQRPA (detailed shell model calculations for this target have been performed in Ref. [9]).

Our present results combined with the lowest experimental limits on $R_{\mu e}$ could determine bounds on the flavor violating parameters entering the branching ratio $R_{\mu e}$ in some extensions of the standard model (with finite neutrino masses) as well as in supersymmetric theories [7]. With these limits we can constrain the neutrino masses, mixing angles, etc. of a neutrino mixing scenario or the parameters of a SUSY model entering a $\mu-e$ conversion Lagrangian describing this process.

2 The renormalized QRPA for the $\mu^- \to e^-$ reaction

The refinement of the QRPA adopted in the present work is the so called charge-conserving Renormalized QRPA, since in the reaction (1) the charge of the target nucleus is not changed. This implies that, for the $\mu^- \to e^-$ reaction, the RQRPA with two-proton or two-neutron quasi-particle excitations is needed. This type of QRPA, goes beyond the quasi-boson approximation (QBA) in which the Fermion pairs are treated as bosons. The so called renormalized QBA [7], takes into account the exact commutation relations of a Fermion pair in the RPA ground state expectation value and includes in a good approximation the Pauli principle, which prevents to have too many quasi-particles in the ground state.

The ordinary QRPA, as is well known, relies on the assumption that the angular momentum coupled bifermion operators $A^\dagger$, A obey boson commutation relations in a correlated RPA ground state [10]. This assumption is a reliable approximation if the correlated ground state does not appreciably differ from the uncorrelated one. As has been recently shown [7], the normal QRPA overestimates the ground state correlations, a shortcoming which can be cured by rewriting the commutation relations for $A^\dagger$, A as

$$\langle 0^+_{RPA}|[A_\tau(kl, JM), A^\dagger_{\tau'}(k'l', JM)]|0^+_{RPA}\rangle \simeq \delta_{kk'}\delta_{ll'}\delta_{\tau\tau'}\mathcal{D}_{\tau\tau'}(kl, J)\,, \qquad (3)$$

where $\mathcal{D}$ is a renormalizing matrix defined as

$$\mathcal{D}_{\tau\tau'}(kl, J) =$$
$$1 - \hat{\jmath}_k^{-1} < 0^+_{RPA}|[a^\dagger_{\tau k} a_{\tau \tilde{k}}]^0_0|0^+_{RPA} > - \hat{\jmath}_l^{-1} < 0^+_{RPA}|[a^\dagger_{\tau' l} a_{\tau' \tilde{l}}]^0_0|0^+_{RPA} > \qquad (4)$$

($\hat{\jmath} \equiv \sqrt{2\jmath+1}$). Thus, the commutation relations (3) take into account the fact that the nucleon pair consists of two Fermions. In the above equations, k and l denote the set of quantum numbers (n_k, l_k, j_k) and (n_l, l_l, j_l) for the single particle levels that can be coupled to a total angular momentum J and parity π (J^π). The indices τ, τ' denote the charge of the nucleons (protons or neutrons).

The RQRPA method proceeds by rewriting the two-quasiparticle operators in terms of $A^\dagger$, A as

$$\begin{aligned}\overline{A}^\dagger_\tau(kl, JM) &= \mathcal{D}^{-1/2}_{\tau\tau}(kl, J) A^\dagger_\tau(kl, JM)\,,\\ \overline{A}_\tau(kl, JM) &= \mathcal{D}^{-1/2}_{\tau\tau}(kl, J) A_\tau(kl, JM)\end{aligned} \tag{5}$$

Thus, $\overline{A}$ and $\overline{A}^\dagger$ commute like bosons and restore the Pauli principle to a large extent. It should be noted that, the definition of the matrix $\mathcal{D}$ in Eq. (4) takes into account only the diagonal part of the exact fermion commutation relations, and consequently we can still write down an eigenvalue problem by defining new variational amplitudes $\overline{X}$ and $\overline{Y}$ in terms of the old ones X and Y as

$$\overline{X} = \mathcal{D}^{1/2} X, \qquad \overline{Y} = \mathcal{D}^{1/2} Y, \tag{6}$$

and new RPA matrices $\overline{\mathcal{A}}$, $\overline{\mathcal{B}}$ in terms of the ordinary-QRPA matrices $\mathcal{A}$, $\mathcal{B}$ as

$$\overline{\mathcal{A}} = \mathcal{D}^{1/2} \mathcal{A} \mathcal{D}^{-1/2}, \qquad \overline{\mathcal{B}} = \mathcal{D}^{1/2} \mathcal{B} \mathcal{D}^{-1/2}. \tag{7}$$

It is worth noting that, the replacement of the free variational amplitudes X, Y and the bifermion operators $A^\dagger$, A by their renormalized ones, Eqs. (5) and (6) respectively, does not change the form of the phonon-operator $Q^{m\dagger}_{J^\pi M}$ [7,8]. This means that in the renormalized quasiparticle RPA the m^{th} excited state having total angular momentum J projection M and parity π, $|J^\pi_m M\rangle$, is created by acting with the phonon-operator $Q^{m\dagger}_{J^\pi M}$ on the correlated RQRPA vacuum $|\tilde{0}\rangle_{RQRPA}$ as

$$|J^\pi_m M\rangle = Q^{m\dagger}_{J^\pi M} |\tilde{0}\rangle_{RQRPA}. \tag{8}$$

The ground state $\left|\tilde{0}\right\rangle_{RQRPA}$ can be deduced as usually from the Thouless theorem and the uncorrelated ground state $|0\rangle$ as [10]

$$\left|\tilde{0}\right\rangle_{RQRPA} = \overline{N}_0 \exp\left\{\frac{1}{2}\sum_{\substack{a,b,\tau\\ \lambda\mu}} \frac{1}{\hat{\lambda}} \overline{C}^{(\lambda,\tau)}_{ab} \overline{A}^\dagger_\tau(a, \lambda\mu) \overline{A}^\dagger_\tau(b, \overline{\lambda\mu})\right\} |0\rangle\,. \tag{9}$$

where $a \equiv (kl)$, $b \equiv (k'l')$ run over the configurations coupled to momentum λ. The matrix $\overline{C}$ is a new correlation matrix derived as in the case of the ordinary QRPA [8] but now the new variational amplitudes $\overline{X}$ and $\overline{Y}$ should be used as

$$\overline{C}_{ab}^{(\lambda,\tau)} = \left\{ \overline{Y}_{\tau}^{\lambda} \left[\overline{X}_{\tau}^{\lambda} \right]^{-1} \right\}_{ab} . \tag{10}$$

The above way of constructing the correlated RQRPA ground state preserves only linear terms of the new correlation matrix $\overline{C}$ in the series expansion of Eq. (9) and consequently we have for evaluation only bifermion operators acting on the uncorrelated ground state. Under these conditions, the normalization factor entering Eq. (9) is written as

$$\overline{N}_0 = \left[1 + |\overline{C}|^2 \right]^{-1/2} \tag{11}$$

This factor measures the effect of the nucleon-nucleon ground state correlations [7].

3 The μ-e conversion transition matrix elements

Theoretically, the $\mu - e$ conversion can proceed in many models (common extensions of the standard model as well as supersymmetric theories) via photonic and non-photonic mechanisms [4,5,11,16]. In the photonic mechanism the photon γ is virtual coupling the leptons to the nucleus. There are many types of non-photonic mechanisms which occur through the exchange of various particles [5,16].

The expression for the branching ratio $R_{\mu e}$ of the $\mu^- \to e^-$ conversion contains the square of the nuclear matrix elements of tensor operators resulting from the hadronic currents describing the above mechanisms [16] in the non-relativistic approximation (nuclear level) [5,6].

3.1 The matrix elements for the multipole expansion operators

In the coordinate space, the nuclear operators of the $\mu^- \to e^-$ conversion are obtained via the multipole expansion procedure [10]. This gives tensor operators of the form $T_M^{(l,S)J}$ (J is the operator angular momentum rank and

M its projection) which are given by

$$T_M^{(l,0)J} = \tilde{g}_V \delta_{lJ} \sqrt{4\pi} \sum_{i=1}^{A} (3 + \beta \tau_{3i}) j_l(q r_i) Y_M^l(\hat{\mathbf{r}}_i) , \tag{12}$$

for the polar-vector component of the hadronic current (Fermi type or spin independent operators), and

$$T_M^{(l,1)J} = \tilde{g}_A \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} (\xi \beta'' + \beta' \tau_{3i}) j_l(q r_i) \Big[Y^l(\hat{\mathbf{r}}_i) \otimes \sigma_i \Big]_M^J , \tag{13}$$

for the axial-vector component of the hadronic current (Gamow-Teller type or spin dependent operators). $j_l(qr)$ are the spherical Bessel functions resulting from the plane-wave, $e^{i\mathbf{q}\cdot\mathbf{r}}$, representation of the outgoing electron. The magnitude of the momentum transfer $\mathbf{q}$ is given by $q = m_\mu - \epsilon_b - E_x$ where E_x the excitation energy of the daughter nucleus.

For our calculations the operators T_M^J of Eqs. (12) and (13) must be first rewritten in the quasi-particle basis, i.e. in terms of the operators $A^\dagger$, A [10]. To this aim we start from their second quantization form

$$\hat{T}_M^J = \sum_{j_2, j_1, \tau} \frac{\langle j_2 \,||\, \hat{T}^J \,||\, j_1 \rangle}{\hat{J}} \Big[\, c_{j_2}^\dagger(\tau) \; c_{\,\bar{j}_1}(\tau) \, \Big]_M^J , \tag{14}$$

where $c^\dagger$ (c) is a particle (hole) operator, and then we use the well known Bogolubov-Valatin transformations

$$c_{jm}^\dagger = U_j \; a_{jm}^\dagger - V_j \; a_{\overline{jm}}, \qquad c_{\overline{jm}} = U_j \; a_{\overline{jm}} + V_j \; a_{jm}^\dagger \, . \tag{15}$$

In these transformations, $V_j^{(\tau)}$ and $U_j^{(\tau)}$ represent the probability amplitudes for the single particle states to be occupied and unoccupied, respectively, and $c_{\overline{jm}} = (-)^{j-m} c_{j-m}$. The operators $\hat{T}_M^J$ take the form

$$\begin{aligned} \hat{T}_M^J = \sum_{j_2 j_1, \tau} \Big[& s_\tau(j_2 j_1, J) B^\dagger(j_2 j_1, JM) \; + \; \tilde{s}_\tau(j_2 j_1, J) B^\dagger(j_1 j_2, JM) \\ & + p_\tau(j_2 j_1, J) A^\dagger(j_2 j_1, JM) + \tilde{p}_\tau(j_2 j_1, J) A(j_2 j_1, \overline{JM}) \Big] \\ & - \delta_{M0} \delta_{J0} \sum_{\nu, \tau} \hat{j}_\nu \, W_\tau^J(\nu\nu) \Big(V_\nu^{(\tau)} \Big)^2 \end{aligned} \tag{16}$$

In Eq. (16), in addition to the two quasi-particle operators $A^{+}(j_2 j_2, JM)$ and $A(j_2 j_2, JM)$, there also appear the scattering operators B^{+} and B defined as

$$B^{\dagger}(j_2 j_1, JM) = \left[a^{\dagger}_{j_2} a_{\bar{j}_1}\right]^{J}_{M}, \qquad B(j_2 j_1, JM) = [B^{\dagger}(j_2 j_1, JM)]^{\dagger} \tag{17}$$

The quantities s, p, $\tilde{s}$ and $\tilde{p}$ in Eq. (16) are given in Ref. [8] and the quantities $W^{J}_{\tau}(j_2 j_1)$ are

$$W^{J}_{\tau}(j_2 j_1) \; = \; Q \; (2J+1)^{-1} \; \langle j_2 \; || \; \hat{T}^{(l,S)J} \; || \; j_1 \rangle \tag{18}$$

with

$$Q \; = \; (3 + \beta\tau), \qquad \text{for Fermi operators}$$
$$Q \; = \; (\xi\beta'' + \beta'\tau), \qquad \text{for Gammow-Teller operators}\,.$$

The single-particle reduced matrix elements $\langle j_2 || T^J || j_1 \rangle$ in Eq. (18), for harmonic oscillator (h.o.) wave functions, have been written in a compact way as

$$\langle j_2 \; || \; \hat{T}^{(l,S)J} \; || \; j_1 \rangle \; = \; e^{-\chi} \sum_{\kappa=0}^{\kappa_{max}} \theta^{S}_{\kappa} \, \chi^{\kappa + l/2}, \qquad \chi = (qb)^2/4 \tag{19}$$

with b the h.o. size-parameter and $\theta^{S}_{\kappa}(j_1 j_2; J)$ given in Ref. [8]. Equation (19) permits the computation of the coefficients θ^{S}_{κ}, which are independent of the momentum transfer q, once and for all the necessary configurations. Then, the reduced matrix elements $\langle j_2 \; || \; \hat{T}^{J} \; || \; j_1 \rangle$ needed for a given nucleus are readily evaluated for every value of the momentum transfer q.

3.2 Inclusive $\mu - e$ conversion matrix elements.

The inclusive (μ^-, e^-) conversion rate is evaluated by summing over the partial contributions for all possible final states $\mid f \rangle$ induced by the Fermi and Gammow-Teller type operators of Eqs. (12) and (13). The coherent contribution in RQRPA has been calculated and discussed in Ref. [7]. The corresponding incoherent matrix elements are written as [8]

$$S_{\alpha} = \sum_{f} \left(\frac{q}{m_{\mu}}\right)^{2} \sum_{l,J} \mid \langle f \; || \; \hat{T}^{J} \; || \; \tilde{0} \rangle_{RQRPA} \mid^{2}, \; f \equiv (J^{\pi}_{\lambda} M), \; \alpha = V, A \tag{20}$$

(S_V is the Fermi-type and S_A the Gammow-Teller-type contributions). The reduced matrix elements $\langle f || \hat{T}^{J} || \tilde{0} \rangle_{RQRPA}$ for a given multipole RQRPA state $|f\rangle \equiv |J^{\pi}_{\lambda}\rangle$ take the form

$$\langle J^\pi_\lambda || \hat{T}^J || \tilde{0} \rangle_{RQRPA} = \sum_{j_2 \geq j_1, \tau} \overline{W}^J_\tau(j_2 j_1) \left[\overline{X}^\lambda_\tau(j_2 j_1, J) U^{(\tau)}_{j_2} V^{(\tau)}_{j_1} + (-)^\Theta \overline{Y}^\lambda_\tau(j_2 j_1, J) V^{(\tau)}_{j_2} U^{(\tau)}_{j_1} \right] \quad (21)$$

where

$$\overline{W}^J_\tau(j_2 j_1) = \mathcal{D}^{1/2}_{\tau\tau}(j_2 j_1, J) W^J_\tau(j_2 j_1)$$

Eq. (21) shows that the incoherent matrix elements in RQRPA have the same structure as those of the ordinary QRPA [see Eq. (15) of Ref. [10]], i.e. the first can be obtained by substituting in the normal QRPA the free variational amplitudes X and Y and the matrix elements W^J_τ with their renormalized counterparts $\overline{X}$, $\overline{Y}$ and $\overline{W}^J_\tau$.

The necessary matrix elements for the total (μ^-, e^-) rate in RQRPA are then easily computed by adding the vector and axial vector contributions of the coherent and incoherent rates (see below).

4 Results and Discussion

Using the formalism of the renormalized QRPA developed before (see also Ref. [7,8]) we carried out a detailed study of the inclusive (μ^-, e^-) conversion rates. For comparison with previous works we employed the same set of nuclear isotopes and use the same inputs as in Ref. [7]. In order to illustrate the difference between RQRPA and QRPA results and estimate the influence of the ground state nucleon-nucleon correlations on the incoherent $\mu - e$ conversion matrix elements, we have also listed previous QRPA results [12].

As has been stressed before, the effect of ground state correlations is of particular significance for the very small $\mu^- \to e^-$ transition matrix elements and they must be accurately incorporated in structure calculations. In principle all RPA methods take into consideration to some extent the ground state correlations. However, as has been pointed out [7], the RQRPA describes them in an explicit and more reliable way. For the $\mu - e$ conversion this is important not only because the final states are built on the correlated ground state [see Eq. (8)], but also because the $gs \to gs$ transitions dominate the $\mu^- \to e^-$ process [5]-[7]. The coherent RQRPA results and the extracted conclusions have been comprehensively discussed in Ref. [7].

The results for the incoherent (μ^-, e^-) matrix elements obtained as described before for photonic and non-photonic mechanisms are quoted in Table 1. For the photonic mechanism there is no axial vector contribution and the incoherent matrix elements are equal to S_V. For the non-photonic case, S_V and $3S_A$ refer to the vector and axial vector contributions of the W-boson exchange diagrams of Ref. [8], but the conclusions discussed below hold qualitatively also

Table 1
Incoherent (μ^-, e^-) conversion matrix elements in RQRPA for the photonic and the non-photonic diagrams of Fig. 1(c). S_V stands for the contribution of the vector component and S_A for the contribution of the axial vector component. For comparison we have also listed the QRPA results of Ref. [7]. All matrix elements are purified from the spurious unphysical contributions [12].

	Photonic Mechanism		Non-photonic Mechanism			
	RQRPA	QRPA	RQRPA		QRPA	
Nucleus	S_V	S_V	S_V	S_A	S_V	S_A
^{48}Ti	4.60	5.51	6.22	2.44	7.69	3.20
^{60}Ni	3.84	4.48	4.42	4.05	4.79	5.36
^{72}Ge	5.54	6.94	7.24	3.82	9.26	5.05
^{112}Cd	6.48	8.14	8.64	5.17	11.37	6.67
^{162}Yb	9.63	13.52	13.17	6.25	20.12	8.28
^{208}Pb	7.26	8.97	11.52	5.60	14.00	6.77

for other non-photonic diagrams [7]. As it is seen from Table 1, the incoherent $\mu - e$ conversion matrix elements calculated with RQRPA are smaller than the corresponding normal QRPA ones, but the differences are not very large. Despite the fact that for the coherent mode the RQRPA matrix elements are larger than those of QRPA [7], for the incoherent rate our calculations show the opposite trend. This can be justified by remembering that the ordinary QRPA ground state contains much more quasi-particles than the renormalized QRPA. Thus, in the normal QRPA the probability to excite quasi-particles is larger than that in RQRPA where the restored Pauli principle prevents them. This event can also be explained by the formalism described in Sects. 2 and 3, as follows. The magnitude of the coherent matrix elements is mainly governed by the square of the correlation matrix $|C|^2$ for QRPA and $|\overline{C}|^2$ for RQRPA [see Eq. (11)]. For the currently interesting nuclei ^{48}Ti and ^{208}Pb this is illustrated in Fig. (2) of Ref. [7] from which it is clearly concluded that the QRPA overestimates the ground state correlation due to the omission of the Pauli principle. Using the RQRPA, this principle is appreciably restored and the ground state correlations have been more correctly calculated. In contrast, the magnitude of the incoherent matrix elements [see Eq. (21)] is determined not only from the normalization coefficient $\overline{N}_0^2$, which contains the correlation matrix, but also from the renormalizing matrix $\mathcal{D}$. Thus, the increase induced by the $\overline{N}_0^2$ in the incoherent rate is compensated by the reduction of the $\mathcal{D}$ matrix.

In Fig. 1 we plot the individual contributions originating from various multipole components of the incoherent rate, in the non-photonic case, obtained with RQRPA for the two nuclear isotopes ^{48}Ti and ^{208}Pb. One can see that, the main contributions to the incoherent mode, come from the low-spin multipole states 1^- and 0^+, 1^+, 2^+, a result which is in agreement with previous

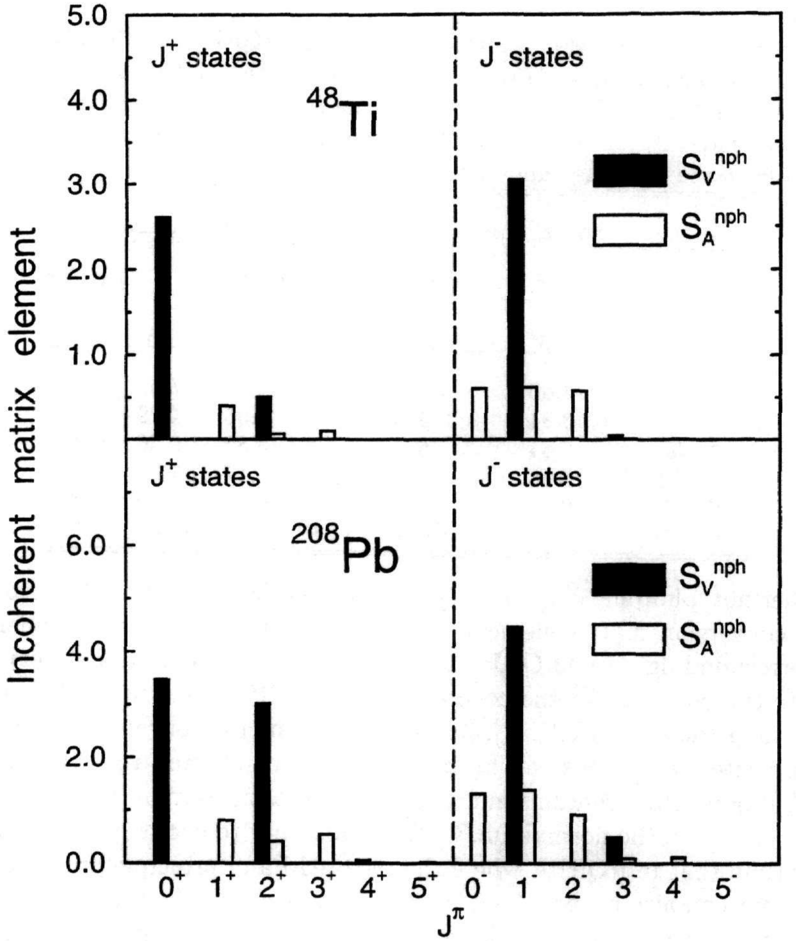

Fig. 1. The individual contributions of each multipolarity to the incoherent matrix elements for ^{48}Ti and ^{208}Pb in the nonphotonic case. We see that the low multipolarity components dominate the total transition probability.

QRPA calculations [7]. This means that the influence of the ground state correlations on the incoherent transition matrix elements does not show clear channel dependence. The picture of the dominance in the incoherent RQRPA strengths is the same as that of the normal QRPA. Thus, for example, the 1^- multipolarity is the most important in both methods. We mention that, the spurious center-of-mass admixures of the 1^- multipole states has been eliminated in both QRPA and RQRPA by utilizing the method of Ref. [12].

Using the matrix elements of the coherent mode (M^2_{coh}) calculated in Ref. [7] and those of Table 1 (photonic mechanism) for the incoherent channels (M^2_{incoh}), we computed the total rate matrix elements (M^2_{total}) and the ratio η ($\eta = M^2_{coh}/M^2_{tot}$) of the coherent to total $\mu^- \to e^-$ matrix elements (see Table 2). It is seen that, the total matrix elements in RQRPA are slightly larger than

Table 2
Inclusive $\mu - e$ matrix elements for the photonic diagrams and ratio η of the coherent to total (μ^-, e^-) matrix elements within RQRPA. The notation is: coherent (M^2_{coh}) matrix elements, incoherent $(M^2_{total} = S_V + 3S_A)$ matrix, and total $(M^2_{total} = M^2_{coh} + M^2_{incoh})$ matrix elements. For comparison the corresponding results obtained with ordinary QRPA are quoted.

	RQRPA				QRPA			
Nucleus	M^2_{incoh}	M^2_{coh}	M^2_{total}	η	M^2_{incoh}	M^2_{coh}	M^2_{total}	η
^{48}Ti	4.60	127.2	131.8	97%	5.51	117.7	123.21	96%
^{60}Ni	3.84	171.1	174.94	97%	4.48	149.4	153.88	97%
^{72}Ge	5.54	199.1	204.64	97%	6.94	169.9	176.84	96%
^{112}Cd	6.48	285.7	292.18	98%	8.14	222.6	230.74	96%
^{162}Yb	9.63	393.3	402.93	98%	13.52	283.8	297.32	95%
^{208}Pb	7.26	415.6	422.86	98%	8.97	379.4	388.37	98%

those of ordinary QRPA although the incoherent RQRPA matrix elements are smaller than the corresponding QRPA ones. This is due to the dominance of the coherent channel for which the trend of the matrix elements in the two methods is reversed [7].

The ratio η of coherent to total $\mu^- \to e^-$ matrix elements, which is an interesting quantity for experiments [1]-[3], appears to be close to unity and this shows that the coherent transition exhausts about the entire total rate in the (μ^-, e^-) process. This result agrees very well with previous results [5]. We also note that, even though the absolute values of the $\mu^- \to e^-$ matrix elements calculated by RQRPA differ from those of ordinary QRPA both for the coherent and incoherent processes, the ratio η is not appreciably affected.

As has been emphasized in our previous work [7], by using the more reliable matrix elements obtained by the renormalized QRPA and adopting the limits on $R_{\mu e}$ extracted from the new run of SINDRUM II experiment for ^{48}Ti target, we can determine severe constraints for the fundamental lepton flavor violating parameters entering the branching ratio $R_{\mu e}$ as follows: Assuming that $R_{\mu e}$ can be written [see Eq. (18) of Ref. [7]] as a product $R_{\mu e} = \rho\,\gamma$, where γ contains the nuclear dependence of the branching ratio calculated with the aid of RQRPA matrix elements [7] and ρ contains the elementary sector dependence, we can put bounds on the parameter ρ. Even though, in principle, ρ is the only parameter one can constrain using the experimental sensitivity of $R_{\mu e}$, in elementary models where the dominance of specific terms in the $\mu - e$ Lagrangian is a reasonable assumption, one can extract limits [16] for some special parameters (or products of parameters) describing this exotic process e.g. isoscalar parameter, isovector parameter etc. [9].

5 Summary and Conclusions

We have formulated the renormalized quasiparticle random phase approximation (RQRPA) for the nuclear-charge conserving semi-leptonic reactions with the goal to investigate the matrix elements for the inclusive $\mu - e$ conversion process. This method goes beyond the usual quasi-boson approximation on which the ordinary QRPA relies and leads to the restoration of the Pauli principle. The nucleon-nucleon ground state correlations are suitably treated and therefore reliable results for the coherent and incoherent matrix elements of $\mu - e$ conversion are obtained.

We found that the ground state correlations affect the incoherent matrix elements in the opposite direction to that found previously for the coherent ones. As a result, the incoherent matrix elements calculated with the renormalized QRPA are smaller than those given by the ordinary QRPA. Reliable results for the very small $\mu^- - e^-$ transition matrix elements are of significant importance, since they can provide useful nuclear physics inputs for the PSI (SINDRUM II) and Brookhaven (MECO) $\mu - e$ conversion experiments which are some of the most sensitive current experiments seeking for events of muon number violation and new physics beyond the standard model.

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