Cross section calculations for neutrino-nucleus reactions at low and intermediate energies.

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Abstract

Inelastic neutrino-nucleus reaction cross sections are studied focusing on the neutral current processes. Particularly, we investigate the angular and initial neutrino-energy dependence of the differential and integrated cross sections for low and intermediate energies of the incoming neutrino (or antineutrino). Contributions coming from both, the vector and axial-vector components of the corresponding hadronic currents have been included. The initial and final state nuclear wave-functions have been calculated in the context of the Quasi-particle Random Phase Approximation (QRPA) tested on the reproducibility of the low-lying energy spectrum (up to about 5 MeV) of the studied nuclei. The results presented here refer to the nuclear isotopes $^{16}O$ and $^{98}Mo$. As it is well known, $O$ plays a significant role in supernova evolution phenomena and $Mo$ is used as a target in the MOON neutrino experiment at Japan.

1 Introduction

Recently, neutrinos and their interactions with nuclei have attracted a great deal of attention, since they play a fundamental role to nuclear physics, cosmology and to various astrophysical processes, especially in the dynamics of core-collapse supernova-nucleosynthesis [1, 2, 3, 4, 5, 6]. Moreover, neutrinos proved to be interesting tools for testing weak interaction properties, by examining nuclear structure, and for exploring the limits of the standard model [7]. In spite of the important role the neutrinos play in many phenomena in nature, numerous questions concerning their properties, oscillation-characteristics, their role in star evolutions and in the dark matter of the universe, etc., remain still unanswered. The main goal of experimental [8, 9] and theoretical studies [10, 11, 12, 13, 14, 15] is to shed light on the above open problems to which neutrinos are absolutely crucial.

Among the probes which involve neutrinos, the neutrino-nucleus reactions possess prominent position [16, 17, 19, 20, 24, 22]. For example, the investigation of neutrino scattering off nuclei is a good way to detect or distinguish neutrinos of different flavor and to explore the basic structure of the weak interactions. Also, specific neutrino-induced transitions between discrete nuclear states allows us to study the structure of the weak hadronic currents. Furthermore, experiments performed or planned to detect astrophysical neutrinos (solar, supernova neutrinos, etc.) and neutrino-induced nucleosynthesis could be interpreted through the neutrino-nucleus interactions within several theories [7].
From the four categories of neutrino–nucleus processes (see Fig. 1): the two types of charged-current (CC) reactions of neutrinos, $\nu_\ell$, ($\ell = e, \mu, \tau$), and antineutrinos, $\bar{\nu}_\ell$, and the two types of neutral-current (NC) ones, in the present work we address the neutral-current reactions (neutrino scattering). In these processes the neutrinos (anti-neutrinos) interact via exchange of neutral $Z^0$ bosons (see Fig. 1b) with the nucleus as

$$\nu + Z A N \rightarrow Z A^* N + \nu', \quad \bar{\nu} + Z A N \rightarrow Z A^* N + \bar{\nu}' ,$$  

(1)

where $\nu$ ($\bar{\nu}$) denote neutrinos (anti-neutrinos) of any flavor. Both coherent and incoherent channels could occur in the reactions of Eq. (1). In the coherent channel, the nucleus remains in its ground state and in the incoherent the nucleus is excited. Mostly, the neutrino-induced reactions leave the final nucleus in an excited state below or above particle-emission threshold. The first transitions constitute the semi-inclusive processes (radiochemical-type neutrino-detection experiments) [14, 15], while the latter often decay by particle emission and supply light particles that can cause further nuclear reactions [15]. From the nuclear structure perspective, the main task is the calculation of the nuclear matrix elements of various multipole operators between the initial and final nuclear many-body states, which could be achieved by following detailed prescriptions such those described in Refs. [14, 19, 20, 24]. These matrix elements enter the neutrino-nucleus cross sections formalism which is briefly discussed in the following sections (see also Refs. [1, 19]).

In the present work we investigate the angular and energy dependence of differential and integrated cross sections for exclusive neutral-current neutrino-nucleus processes leading to final nuclear states with well-defined angular momentum, spin and parity. The starting point of our calculations is the Walecka-Donnelly [1] formalism which describes in a unified way electromagnetic and weak semi-leptonic processes in nuclei by taking advantage of the multipole decomposition of the relevant hadronic current density operators. This formalism has recently been improved [19], by constructing compact analytic expressions for all nuclear matrix elements of the basic multipole expansion operators entering these cross sections.

The required initial and final nuclear wave-functions are obtained in the context of a version of the Quasi-particle Random Phase Approximation (QRPA) which has not been employed previously for such processes (see [17]). The corresponding differential cross sections are, then, evaluated state-by-state for all excitations produced in our chosen model space and, subsequently, the total cross sections are obtained by summing-over-
partial-rates. The results presented here refer to currently interesting nuclei like $^{98}$Mo, which has been proposed as target in the MOON neutrino-experiment at Japan [18], and $^{16}$O which plays a key role in the supernova neutrino phenomena (see e.g. Refs. [16, 17]).

2 Neutral current neutrino-nucleus interaction hamiltonian

In the present work we consider weak neutral current interactions in which a low and intermediate energy neutrino (or antineutrino) is scattered elastically or inelastically from a nucleus $(A,Z)$. The initial nucleus is assumed to be spherically symmetric and to reside in its ground state with angular momentum and parity $J^\pi = 0^+$. The standard model effective Hamiltonian governing the neutral current interactions (1) can effectively be written in current-current form as

$$
\mathcal{H} = \frac{G_F}{\sqrt{2}} (j_\lambda J_\lambda + h.c.)
$$

(2)

where $j_\lambda$ and $J_\lambda$ denote the leptonic and hadronic currents, respectively, $G_F = 1.1664 \times 10^{-5}\text{GeV}^{-2}$ is the weak coupling constant. According to V-A theory and flavor universality of the weak interactions the leptonic currents take the general form

$$
j_\lambda = \sum_l \bar{\Psi}_l(x) \gamma_\lambda (1 - \gamma_5) \Psi_\nu_l(x), \quad l = e, \mu, \tau
$$

(3)

where $\Psi_l$ and $\Psi_\nu_l$ are the lepton spinors. In the present work we consider neutrinos and antineutrinos of electron type only $\nu_e$ and $\bar{\nu}_e$. The structure of the hadronic current describing the neutral current processes, by keeping only vector and axial-vector components (the pseudo-scalar contributions are usually neglected) is written as

$$
J_\lambda = \bar{\Psi}_N \{ F^Z_1 \gamma_\lambda + F^Z_2 \frac{\sigma_\lambda q^\nu}{2M} + G_A \gamma_5 \gamma_\lambda \} \Psi_N
$$

(4)

(the index $Z$ comes from the vector boson $Z^0$ mediating the neutral current neutrino-nucleus processes, see Fig. 1, and $M$ stands for the nucleon mass). The weak form factors entering the neutral hadronic currents are given by

$$
F^Z_{1,2} = \left( \frac{1}{2} - \sin^2 \theta_W \right) \left[ F^p_{1,2} - F^n_{1,2} \right] \tau_0 - \sin^2 \theta_W \left[ \frac{F^p_{1,2} + F^n_{1,2}}{2} \right]
$$

(5)

$$
G_A = -\frac{1}{2} g_A \left( 1 - \frac{q^2}{M_A^2} \right)^{-2} \tau_0
$$

(6)

Here $F^p_{1,2}$, $F^n_{1,2}$ denote the charge and electromagnetic form factors of proton and neutron, respectively, $\theta_W$ is the Weinberg angle ($\sin^2 \theta_W = 0.2325$), $M_A = 1.05 \text{ GeV}$ is the dipole mass and $g_A = 1.258$ is the axial charge. In the convention used in the present work $q^2$ is written as

$$
q^2 = q^0 q_\mu = q^0 - q^2 = (\varepsilon_i - \varepsilon_f)^2 - (p_i - p_f)^2
$$

(7)

$p_i$, $p_f$ are the 4-momentum for the incoming and outgoing neutrino, respectively, and $\tau_0$ represents the nucleon isospin operator, which, according to our convention, by acting on a pure proton state $|p\rangle$ or neutron state $|n\rangle$ gives

$$
\tau_0 |p\rangle = +|p\rangle, \quad \tau_0 |n\rangle = -|n\rangle.
$$
3 The formalism for neutrino-nucleus cross section calculations

In our J-projected nuclear structure calculations, the initial and final nuclear states have well-defined spins and parities, $|J_m^\pi\rangle$. Thus, a multipole analysis of the weak hadronic current at nuclear level can be performed. This has been at first carried out [10] in close analogy to electron scattering off nuclei within a general treatment of semi-leptonic weak interaction processes in nuclei. The neutrino-nucleus scattering differential cross section is written as

$$\left(\frac{d^2\sigma_{i\rightarrow f}}{d\Omega d\omega}\right)_{\nu/\bar{\nu}} = \frac{G_F^2 |k_f| \varepsilon_f}{\pi (2J_i + 1)} \left( \sum_{J=0}^\infty \sigma_{CL}^J + \sum_{J=1}^\infty \sigma_T^J \right) \left(\int_0^\infty \frac{d\omega}{2}\right)$$

where $\omega = \varepsilon_i - \varepsilon_f$ is the excitation energy of the nucleus and $\varepsilon_i$, denotes the energy of the incoming neutrino while $\varepsilon_f$ ($k_f$) represent the energy (momentum) of the outgoing lepton. The summations in Eq. (8) contain the contributions of Coulomb $\hat{M}_J$, longitudinal $\hat{L}_J$, transverse electric $\hat{T}_{el}^J$ and transverse magnetic $\hat{T}_{mag}^J$ multipole operators as defined in [1]. The contributions $\sigma_{CL}^J$, for the Coulomb and Longitudinal components, and $\sigma_T^J$, for the transverse components, are written as

$$\sigma_{CL}^J = (1 + \cos \Phi) \left| \langle J_f||\hat{M}_J(q)||J_i\rangle \right|^2 + \left(1 + \cos \Phi - 2b \sin^2 \Phi \right) \left| \langle J_f||\hat{L}_J(q)||J_i\rangle \right|^2$$

$$+ \left[ \frac{\omega}{q} (1 + \cos \Phi) \right] 2\text{Re} \langle J_f||\hat{M}_J(q)||J_i\rangle \langle J_f||\hat{L}_J(q)||J_i\rangle^*$$

$$\sigma_T^J = \left(1 - \cos \Phi + b \sin^2 \Phi \right) \left[ \left| \langle J_f||\hat{T}_{mag}^J(q)||J_i\rangle \right|^2 + \left| \langle J_f||\hat{T}_{el}^J(q)||J_i\rangle \right|^2 \right]$$

$$+ \frac{(\varepsilon_i + \varepsilon_f)}{q} (1 - \cos \Phi) 2\text{Re} \langle J_f||\hat{T}_{mag}^J(q)||J_i\rangle \langle J_f||\hat{T}_{el}^J(q)||J_i\rangle^*$$

where $\Phi$ is the lepton scattering angle and $b$ are given by

$$b = \frac{\varepsilon_i \varepsilon_f}{q^2}$$

Figure 2: Low energy spectrum of $^{98}\text{Mo}$ obtained with the QRPA (right). The experimental spectrum of this isotope is also shown (left).
and in our convention ($\hbar = c = 1$) $|\mathbf{k}_f| = \varepsilon_f$. The magnitude of the three momentum transfer $q$ is given by

$$q = |\mathbf{q}| = \left[\omega^2 + 2\varepsilon_i\varepsilon_f (1 - \cos \Phi)\right]^{1/2} \quad (12)$$

### 4 Results

In this work, we present realistic state-by-state calculations for inelastic neutrino-nucleus scattering on $^{16}\text{O}$ and $^{98}\text{Mo}$ nuclei. Both Fermi and Gamow-Teller like contributions have been calculated using a version of the quasi-particle RPA (QRPA) method. As field interaction we employed a Coulomb corrected Woods-Saxon potential and as two-body residual interaction we used the Bonn-C meson exchange potential. The determination of the parameters of the model was achieved in two levels: (i) in the BCS level, the pairing parameters $g^{\text{pair}}_{\text{np}}$, for neutron pairs, and $g^{\text{pair}}_{\text{pp}}$, for proton pairs, were determined by solving the BCS equations iteratively, and (ii) in the QRPA level the strength parameters $g_{\text{ph}}$, for the particle-hole channel and $g_{\text{pp}}$, for the particle-particle channel, were fixed so as the QRPA energies, originating from the solutions of the QRPA-equations, to reproduce the low-lying nuclear spectrum up to about 5 MeV (as an example see the spectrum of the $^{98}\text{Mo}$ isotope in Fig. 2). We mention that an alternative fixing of the parameters $g_{\text{ph}}$ and $g_{\text{pp}}$ could be done on the energy of the giant dipole resonance of the studied nucleus [23].

The main purpose of the present calculations, is to focus on the angular dependence as well as on the initial neutrino-energy dependence of the neutral current $\nu$-nucleus cross sections at low and intermediate energies (up to $\varepsilon_i = 100$ MeV). This neutrino-energy range covers solar, supernova and the low-energy stellar-collapse neutrinos. Total differential cross sections are evaluated by summing over partial rates for various sets of multipole states included in our truncated model space up to $J^\pi = 6^+$. For integrated (total) cross-sections we used numerical integration techniques to integrate the aforementioned differential cross sections. Our results are in good agreement with previous Continuum RPA calculations of the Gent group [17] in the case of $^{16}\text{O}$.

In Fig. 3, the differential cross sections of the reaction $^{16}\text{O}(\nu, \nu')^{16}\text{O}^*$ are shown for multipole states which offer dominant contribution ($J^\pi = 0^+, 1^-, 2^-$), and a typical scattering angle $\phi = 150^\circ$. For results in other values of $\phi$ ($150^\circ < \phi < 165^\circ$, with step 150$^\circ$) see Ref. [20, 21, 24]. In Fig. (4), we also show (curve labeled $g.s. \rightarrow f_{\text{exc}}$, the sum-over all multipole contributions up to $J^\pi = 6^+$. Also, we have evaluated the coherent contribution, i.e. the single channel of $g.s. \rightarrow g.s.$ transition, which, obviously, is possible only in neutral-current processes (see Fig. 4) [24].

We must note that, in the present calculations we have not taken into account the ground-state correlations which could be included in the QRPA on the basis of the Thoules theorem (see Ref. [22]).

The results presented in this work, are useful for astrophysical neutrino searches, since, as it is known, neutrino-nucleus reactions (neutral- and charged-current ones), specifically in the iron (Fe) mass range, play crucial role in the evolution and explosion process of stars. On the other hand, neutrino-induced reactions on nuclei can contribute to the nucleosynthesis in two ways: (i) through the so-called neutrino-nucleosynthesis, which involves mainly neutrino reactions on nuclei with $A < 60$ and (ii) through the particles emitted due to the neutrino-induced nuclear excitations at energies much higher than the particle emission thresholds (these particles can cause further nuclear reactions).
Figure 3: Total differential cross section for the neutral-current reactions $^{98}\text{Mo}(\nu, \nu')^{98}\text{Mo}^*$ and $^{16}\text{O}(\nu, \nu')^{16}\text{O}^*$ for the dominant multipole contributions. For the multipole states $0^+, 1^-, 2^-, 2^+$ and $3^-$ of $^{16}\text{O}$ and $0^+, 2^+, 3^+, 4^+$ and $5^-$ of $^{98}\text{Mo}$. The curve labeled "total" corresponds to the sum over partial rates for all J-projected states up to $J^\pi=6^+$. 

5 Summary and Conclusions

In the present work, we performed realistic state-by-state calculations for inelastic neutrino-nucleus scattering on $^{16}\text{O}$ and $^{98}\text{Mo}$ nuclei. Both Fermi and Gamow-Teller like contributions have been calculated. The J-projected final states (with well-defined angular momentum, parity and isospin) were obtained by using a refinement of the Quasi-particle RPA method which has been tested on the reproducibility of the low-lying energy spectrum of the studied isotopes.

Differential cross sections have been evaluated for a great number of low-lying nuclear excitations induced through the neutrino-nucleus interactions. Our results show that the contribution of the $0^+$ set of multipole states dominates the total differential cross in both nuclear systems. The angular dependence of the cross sections has been studied by increasing the scattering angle $\phi$ by a step of $\Delta\phi = 15^\circ$ from $\phi = 0$ to $\phi = \pi$. We found
that, the forward scattering is favored and there is a smooth decrease of the cross sections as the scattering angle increases for initial neutrino-energies in the range $\varepsilon_i \leq 40\text{MeV}$. But, for higher energies this effect tends to be reversed, especially in the region of $\varepsilon_i \geq 60\text{MeV}$.

For obtaining total integrated cross-sections, we used numerical integration methods (e.g. the numerical Gauss method based on the use of Legendre polynomials). Our results for the nucleus $^{16}\text{O}$ are in good agreement with previous calculations performed by the Gent group [17]. For the case of $^{98}\text{Mo}$ isotope which is contained in large abundance in the target of the MOON experiment at Japan [18], there no similar results to compare with.

Before closing, it is worth mentioning that we have also calculated differential and integrated inelastic scattering cross sections of electron-neutrino and -antineutrino off other even-even nuclear systems. Results obtained for the nuclear isotopes: $^{40}\text{Ar}$, the target of the ICARUS neutrino-experiment at Gran Sasso, and for $^{56}\text{Fe}$, which plays a key role in nucleosynthesis of heavy isotopes in the interior of stars, are going to be published elsewhere.
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References