Abstract

We present several possibilities offered by nuclear structure, the dynamics of intermediate energy heavy ion collisions and neutron stars to investigate the nuclear matter equation of state (EoS) beyond the ground state. In particular the high density nuclear EoS of asymmetric matter, i.e. the symmetry energy, is discussed.

Keywords: Nuclear matter, equation of state, symmetry energy, finite nuclei, heavy ion collisions, strangeness ratio, neutron stars, maximum mass constraint, proton fraction, Direct Urca process

1. Introduction

The investigation of the physical properties of isospin asymmetric nuclear and hadronic matter over a wide range in the phase space diagram, is crucial in extrapolating structure calculations beyond the valley of stability and for the understanding of astrophysics such as the physical mechanism of supernovae explosions and the neutron star structure [1,2,3]. Studies on the behavior of asymmetric matter at ground state and under extreme conditions of density and temperature has been recently initiated, theoretically [4,5] and experimentally [6]. Whereas finite nuclei provides information on the symmetry energy around saturation, heavy ion collisions (HIC) and neutron star studies explore its density dependence beyond the ground state. Here we present recent results from studies on finite nuclei, HIC and neutron stars and their relations to the still unknown and widely debated density behavior of the symmetry energy.

2. The Equation of State (EoS)

The EoS of nuclear matter characterizes the relation of the binding energy per particle or the pressure density with respect to the baryon density or volume and the temperature. The ground state is determined by vanishing pressure and temperature at the so called saturation density $\rho_{\text{sat}}=0.16 \text{ fm}^{-3}$ and binding energy of -16 MeV [7]. In the general case of isospin asymmetric matter, i.e. different densities of protons and neutrons, an additional parameter appears, the bulk asymmetry parameter $a_4$ of the Bethe-Weizaecker mass formula with an saturation value of about 30-35 MeV.

Theoretically nuclear matter is traditionally studied within the so-called Relativistic Mean-Field (RMF) approach of the Quantumhadrodynamics (QHD) [7]. Hereby the baryons (protons and neutrons), determined by the Dirac equation, interact through a classical mean...
field which is given in terms of mesons with distinct Lorentz properties. In the iso-scalar sector these are the Lorentz-scalar $\sigma$ and Lorentz-vector $\omega$ classical fields with the first one being responsible for the attractive and the second one for the repulsive part of the NN-interaction, respectively. Finite nuclei studies show that these two fields are very strong and comparable with the nucleon mass of 939 MeV [8,9]. Their different Lorentz structure (cancellation effects between attractive and repulsive character) leads to a very small value of the binding energy per nucleon of ca. –16 MeV, but to a strong spin-orbit potential, which is proportional to the sum between them. This is one (among many others which cannot be listed here) of the reasons why a relativistic description is meaningful. The iso-vector part of the EoS is characterized by the symmetry energy $E_{\text{sym}}$ defined as the second derivative of the energy density with respect to the asymmetry parameter $\alpha=(\rho_n-\rho_p)/\rho$ ($\rho_n$, $\rho_p$, $\rho$ being the neutron-, proton and total density, respectively) [5]. The asymmetry term at saturation density is well known from the mass formula. It can be described by an iso-vector, Lorentz-vector $\rho$-field, or by the presence of an additional iso-vector, Lorentz-scalar $\delta$-field. As in the iso-scalar case, the $\rho$-field is responsible for the repulsive and the $\delta$-field for the attractive character of the iso-vector potential. However, for the description of the bulk asymmetry parameter $a_4$ there are several possibilities, in contrast to the iso-scalar case: the inclusion of the $\rho$-meson, or of both, the consideration of the $\rho$- and $\delta$-fields. In the later case one has to increase the coupling of the $\rho$-field to describe the same value of $a_4$. The different Lorentz-structure of the two mesons will influence the high density behaviour of $E_{\text{sym}}$ [4].

Fig. 1. Density dependence of the EoS in terms of the binding energy per nucleon (left) and the symmetry energy (right) for different models of nuclear structure (see [1] for details).

The description of ground state nuclear matter and finite nuclei has been widely studied within non-relativistic and covariant models in the spirit of phenomenological Hartree-Fock and more sophisticated Brueckner-Hartree-Fock approaches. It is not the aim of this contribution to go into further details. The situation is summarized in Fig.1 in terms of the

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nuclear matter EoS (the binding energy per nucleon on the left and the symmetry energy on the right). In summary, all models can describe the saturation properties, i.e. the binding energy of ca. −16 MeV, the compression modulus of ca. 200–250 MeV and the asymmetry parameter of about 30 MeV. However, they differ significantly at supra-normal densities (ρ>0.3 fm⁻³). They can be classified into two groups with a moderate (strong) compressional energy at high densities respectively a “soft” (“hard”) nuclear EoS. Therefore it turns out that in HiC the high density behavior is tested, and not necessarily the compression modulus. In the following we particularly discuss the density dependence of the symmetry energy.

3. Probing saturation in finite nuclei

The theoretical description of finite nuclei has been performed within the RMF approach of QHD, see Refs. [9] for details. Here the quantal Dirac equation for the baryons and the mesonic field equations for the classical mesons have been self consistently solved in the static limit for spherical nuclei, using different parameterizations for the symmetry energy (see right panel in Fig. 1).

![Fig. 1. Dependence of charge radii for proton closed shell nuclei on the mass number. Theoretical predictions (open and full triangles, solid curve) are compared with the data (full squares, taken from Ref. [10]).](image)

The results are shown in Fig. 2 in terms of the charge radii as function of the mass number for proton closed shell nuclei. All models reproduce the data appropriate well, which is an important check before starting to study the same approaches in other nuclear systems (see next section). It turns out that charge radii cannot disintegrate between the different density dependence of the symmetry energy at supra-normal densities, since a finite nucleus characterizes nuclear systems around saturation, and not beyond it. It would be helpful to study isospin radii and neutron thickness properties of highly isospin asymmetric exotic non-spherical nuclei far beyond the valley of stability, in order to better fix the asymmetry parameter.
4. **Probing the high density behavior of the EoS in heavy ion collisions and neutron stars**

The high density behavior of the EoS has been widely investigated in terrestrial laboratories by means of collisions of heavy nuclei such as Au+Au or Pb+Pb at intermediate relativistic beam energies up to view GeV per nucleon [11]. In such collisions a maximal compression of $\rho_{\text{max}} = (2-3)\rho_{\text{sat}}$ can be reached. It turned out that the EoS of symmetric matter has to be soft at such high baryon densities [12]. Studies on asymmetric matter have been recently started and motivated by the plan of new experiments with radioactive ion beams, as e.g. the new FOPI experiment on Ru/Zr collisions at 1.5AGeV beam energies per particle at the GSI facility [6]. However, a universal picture on the density dependence of the symmetry energy can be better achieved if one goes to very high compressions than $\rho_{\text{max}}$. This can be done in theoretical studies on neutron star (NS) structure in which the matter is highly compressed. A selection of results of HIC and NS is therefore very appropriate to better constraint the high density behavior of the EoS by comparing the theoretical calculations with HIC and NS data [1].

Fig. 3 shows two examples of results in HIC (left panel) and NS (right panel). In the case of HIC the ratio of neutral to positive charged kaons, i.e. the strangeness ratio, as function of the laboratory energy per particle is shown. An important dependence of the strangeness ratio on the variable stiffness of $E_{\text{sym}}$ is observed. This due to the fact that kaons ($K^0,K^+$) are emitted very earlier during the high density pre-equilibrium stage of the collision without to undergo secondary interactions with the hadronic environment [4,12]. Future experiments will help hereby to constraint the high density ($\rho_{\text{max}}<3\rho_{\text{sat}}$ at $E_{\text{beam}}<1.8$ AGeV) dependence of the EoS. In NS calculations the proton fraction is a useful tool to constraint the high density EoS at even higher compressions, see right panel in Fig. 3. The upper limit for the proton fraction, after which the Direct Urca (DU) $n \rightarrow p + e^- + \nu_e$ becomes operative, excludes already three EoS’s, in which DU sets in already at very low densities. Together with other NS constraint data (see [1] for more details) a soft behaviour of the EoS of asymmetric matter turns out to be consistent at very high densities $\rho>>\rho_{\text{max}}$.

5. **Final remarks**

Several possibilities have been presented to constraint the nuclear EoS, in particular its isospin dependent part, starting from normal ground state (finite nuclei), going beyond saturation through moderately high densities ($\rho_{\text{max}}=(2-3)\rho_{\text{sat}}$, HIC) to very high values of $\rho$ ($\rho>>\rho_{\text{max}}$, NS). The common theoretical framework has been the RMF approach of the QHD, which has been applied to the different nuclear systems (finite nuclei, HIC, NS) by varying the stiffness of the symmetry energy. It turns out that the EoS of asymmetric matter seems to be moderately stiff at baryon densities up to $\rho_{\text{max}}=(2-3)$, as a preliminary conclusion of HIC studies [4,13]. At even higher densities $\rho>>\rho_{\text{max}}$ the degree of softness should increase in order to consistently fit new observations on maximum mass constraints and simultaneously not exceed the DU limit [1]. More systematic studies need to be done in arriving to more clear and definitive
conclusions, in particular, exotic nuclei far from the valley of stability and future comparisons with new experimental HIC data should be included in such studies.

Fig. 3. (Left) The strangeness ratio as function of the beam energy per nucleon in Au+Au collisions [4] (Right) Density dependence of the proton fraction in neutron star calculations [1]. The gray band $x_{DU}$ indicates the upper limit of the proton fraction after which the Direct Urca process sets in. In both figures theoretical calculations are shown using different parametrizations for the symmetry energy with its stiffness as indicated.

References


