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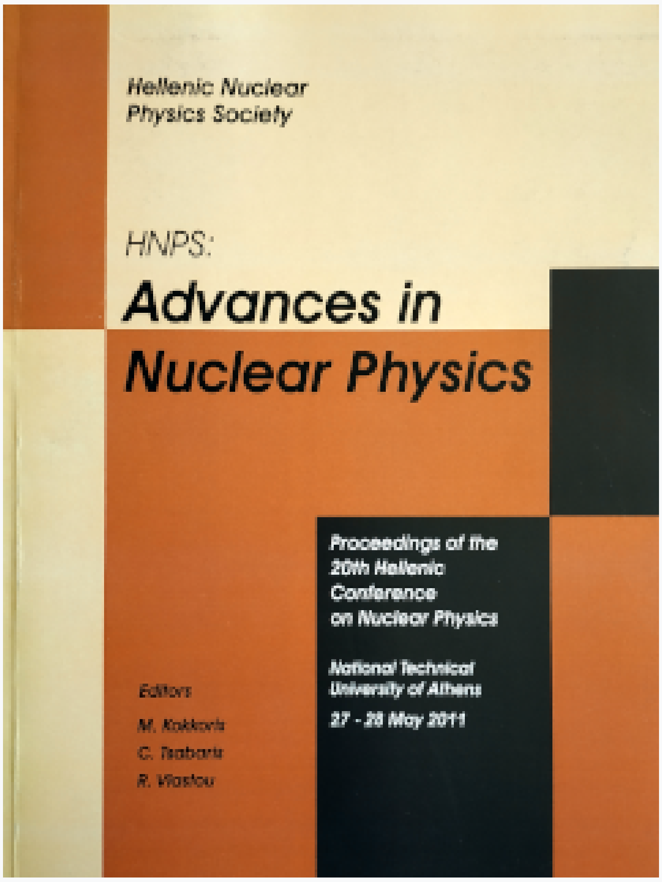

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Line of approximate SU(3) symmetry inside the symmetry triangle of the Interacting Boson Model

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Abstract

The U(5), SU(3), and O(6) symmetries of the Interacting Boson Model (IBM) have been traditionally placed at the vertices of the symmetry triangle, while an O(5) symmetry is known to hold along the U(5)–O(6) side of the triangle. We construct [1] for the first time a symmetry line in the interior of the triangle, along which the SU(3) symmetry is preserved. This is achieved by using the contraction of the SU(3) algebra to the algebra of the rigid rotator in the large boson number limit of the IBM. The line extends from the SU(3) vertex to near the critical line of the first order shape/phase transition separating the spherical and prolate deformed phases. It lies within the Alhassid–Whelan arc of regularity, the unique valley of regularity connecting the SU(3) and U(5) vertices amidst chaotic regions, thus providing an explanation for its existence.

Key words: IBM, Alhassid–Whelan arc of regularity, contraction of SU(3) algebra

1 Introduction

The study, carried out by Alhassid and Whelan, of chaotic properties at the symmetry triangle [2] of the IBM [3], brought to the surface a region [4–6] of nearly regular behavior, inside the symmetry triangle, connecting the U(5) and SU(3) vertices, called the Alhassid-Whelan arc of regularity. The symmetry underlying this has remained an open question. The presence of (near) regularity presupposes the existence of some underlying (approximate) symmetry. A well–known hallmark of SU(3) symmetry, is the degeneracy within sets of bands comprising a given irreducible representation (irrep), such as those between the levels of the β band and those (with the same L) of the γ band. At a recent study [7], by imposing the $2^+_{\gamma_1} = 2^+_{\beta_1}$ degeneracy in the IBM framework, a line was found inside the symmetry triangle of the IBM, starting from the SU(3) vertex, reaching the shape/phase coexistence region [8], lying very close to the arc of regularity. The analysis of Ref. [7] was limited to the low-lying part of the spectrum, while the study of Alhassid and Whelan [4–6] took into consideration the whole spectrum.

In the present work, by considering the large–boson–number limit of the IBA, we study Hamiltonians that approximately commute with the SU(3) generators. We also derive an analytic expression for the locus of points of these Hamiltonians inside the symmetry triangle of the IBM. This is achieved by using the contraction [9] of the SU(3) algebra to the $[\mathrm{R}^5]$SO(3) algebra [10,11] of the rigid rotator [12].

In Section 2, we describe the Hamiltonian of the model. In Section 3, the analytic expressions of the locus of points of the commutation of the Hamiltonian with the SU(3) generators are derived, using two different methods. The conclusions are presented in Section 4. Necessary commutation relations and matrix elements for the calculations are given in the Appendices of [1].

2 The IBA Hamiltonian and symmetry triangle

The IBA possesses an overall U(6) symmetry, with three dynamical symmetries, labeled by subgroups of U(6), namely, U(5), SU(3) and O(6), which correspond to vibrational, rotational and $\gamma-$unstable nuclei respectively. The IBA Hamiltonian used by Alhassid and Whelan is

$$\hat{H}(\eta,\chi) = \hat{H}_1 + \hat{H}_2 = c\left[\eta\hat{n}_d + \frac{\eta-1}{N}\hat{Q}^{(2)}_{\chi}\cdot\hat{Q}^{(2)}_{\chi}\right], \tag{1}$$

where N is the number of valence bosons, c is a scaling factor, $\hat{H}_1$ and $\hat{H}_2$ denote the first and second terms respectively, $\hat{n}_d = d^\dagger\cdot\tilde{d} = \sqrt{5}(d^\dagger\tilde{d})^{(0)}$ is the d boson number operator, $\hat{Q}_{\chi,\xi} = (s^\dagger\tilde{d} + d^\dagger s)^{(2)}_{\xi} + \chi(d^\dagger\tilde{d})^{(2)}_{\xi}$ is the quadrupole operator.

The above Hamiltonian contains two parameters, η and χ, with the parameter η ranging from 0 to 1 and the parameter χ ranging from 0 to $-\sqrt{7}/2$. The U(5) limit corresponds to $\eta = 1$, the SU(3) limit to $\eta = 0, \chi = -\sqrt{7}/2$ and the O(6) limit to $\eta = 0, \chi = 0$.

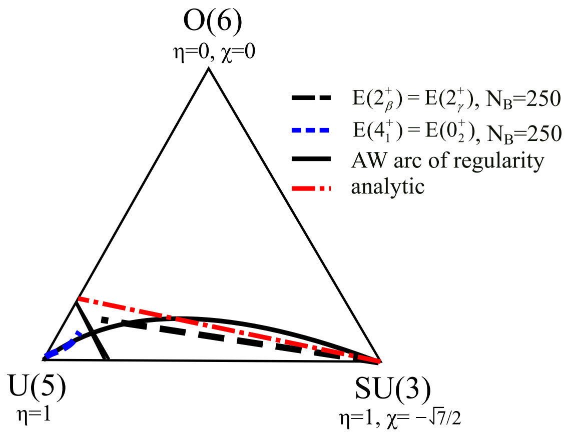

Fig. 1. IBA symmetry triangle with the three dynamical symmetries, the Alhassid–Whelan arc of regularity [Eq. (9)], the present line of Eq. (8) (labelled as analytic), the loci of the degeneracies $E(2^+_\beta)$=$E(2^+_\gamma)$ (dashed line on the right for N_B=250 [7]) and $E(4^+_1)$=$E(0^+_2)$ (dotted line on the left for N_B=250 [7]).

3 The SU(3) symmetry

3.1 Commutation relations

In order to see whether a system has an underlying SU(3) symmetry, the Hamiltonian of the system has to commute with the generators of SU(3). The generators of the SU(3) algebra [3] are the angular momentum operators

$$\hat{L}_\xi = \sqrt{10}(d^\dagger \tilde{d})^1_\xi, \tag{2}$$

and the quadrupole operators

$$\hat{Q}^{(2)}_{SU(3),\xi} = (s^\dagger \tilde{d} + d^\dagger s)^{(2)}_\xi - \frac{\sqrt{7}}{2}(d^\dagger \tilde{d})^{(2)}_\xi. \tag{3}$$

The Hamiltonian of Eq. (1) does commute with the angular momentum operators $\hat{L}_\xi$ by construction, since it is a scalar quantity. We will examine the special conditions under which the Hamiltonian also commutes (approximately) with the quadrupole operators.

The first term of the Hamiltonian gives

$$[\hat{H}_1, \hat{Q}^{(2)}_{SU(3),\nu}] = c\eta[\hat{n}_d, \hat{Q}^{(2)}_{SU(3),\nu}] = c\eta(d^\dagger s - s^\dagger \tilde{d})^{(2)}_\nu. \tag{4}$$

Using

$$\hat{Q}^{(2)}_{\chi,\xi} = \hat{Q}^{(2)}_{SU(3),\xi} + \left(\chi + \frac{\sqrt{7}}{2}\right)(d^\dagger \tilde{d})^{(2)}_{\xi}, \tag{5}$$

in the second term of the Hamiltonian one gets the intermediate result

$$[\hat{Q}^{(2)}_{\chi}\cdot\hat{Q}^{(2)}_{\chi}, \hat{Q}^{(2)}_{SU(3),\nu}] = \sum_{\xi}(-1)^{\xi}\Big\{[\hat{Q}^{(2)}_{SU(3),\xi}, \hat{Q}^{(2)}_{SU(3),\nu}]\hat{Q}^{(2)}_{\chi,-\xi} + \hat{Q}^{(2)}_{\chi,\xi}[\hat{Q}^{(2)}_{SU(3),-\xi}, \hat{Q}^{(2)}_{SU(3),\nu}]$$

$$+\left(\chi + \frac{\sqrt{7}}{2}\right)\Big\{[(d^\dagger\tilde{d})^{(2)}_{\xi}, \hat{Q}^{(2)}_{SU(3),\nu}]\hat{Q}^{(2)}_{\chi,-\xi} + \hat{Q}^{(2)}_{\chi,\xi}[(d^\dagger\tilde{d})^{(2)}_{-\xi}, \hat{Q}^{(2)}_{SU(3),\nu}]\Big\}\Big\}. \tag{6}$$

In the large N limit, Eq. (6) can be simplified. In this limit, the eigenvalue expression for the second order Casimir of SU(3) reduces to just the λ^2 term for SU(3) irreducible representations (irreps) (λ, μ) with $\lambda >> \mu$ and hence the ground state band [which belongs to the (2N,0) irrep] becomes energetically isolated from all other excitations. That is, SU(3) effectively reduces to a simple rigid rotator. This situation is formally known as the contraction of SU(3) to R^5[SO(3)] [10,11] and occurs when the $Q^{(2)}_{SU(3)}$ operators can be replaced by mutually commuting quantities. In the large N limit, where contraction occurs, the commutators in the first two terms in Eq. (6) will vanish and terms containing $(d^\dagger\tilde{d})^{(k)}$ can be omitted. Furthermore, $\hat{Q}^{(2)}_{\chi}$ can be replaced by $\hat{Q}^{(2)}_{SU(3)}$ and $\hat{Q}^{(2)}_{SU(3)}$ can be replaced by the intrinsic quadrupole moment (a scalar), which is $N\sqrt{2}$ in the present case. So, in the large N limit, the commutator for the second part of the Hamiltonian becomes

$$[\hat{H}_2, \hat{Q}^{(2)}_{SU(3),\nu}] = c(\eta-1)2\sqrt{2}\left(\chi + \frac{\sqrt{7}}{2}\right)(d^\dagger s - s^\dagger\tilde{d})^{(2)}_{\nu}. \tag{7}$$

However, we want a vanishing commutator, so the coefficients of $(d^\dagger s - s^\dagger\tilde{d})^{(2)}_{\nu}$ in Eqs. (6) and (7) should cancel, leading in the large N limit to the condition

$$\chi(\eta) = \frac{1}{2\sqrt{2}}\frac{\eta}{(1-\eta)} - \frac{\sqrt{7}}{2}. \tag{8}$$

The expression of the Alhassid–Whelan arc of regularity [4–6] is given by [13]

$$\chi(\eta) = \frac{\sqrt{7}-1}{2}\eta - \frac{\sqrt{7}}{2}. \tag{9}$$

The expressions of Eqs. (8) and (9) are visualized in Fig. 2. The two equations give very similar predictions for values of η between 0 and 0.6, i.e., from the

SU(3) vertex until quite close to the critical line.

3.2 Matrix Elements

At the previous subsection, we replaced the quadrupole operator $\hat{Q}^{(2)}_{SU(3)}$ by the intrinsic quadrupole moment. To justify this, we examine the conditions under which the matrix elements of $[\hat{H}_1, \hat{Q}^{(2)}_{SU(3),\nu}]$ and $[\hat{H}_2, \hat{Q}^{(2)}_{SU(3),\nu}]$ lead to a vanishing result, within the ground state band. The consideration of matrix elements within the ground state band is similar to the condition discussed earlier in the context of Eq. (6), where the $Q^{(2)}_{SU(3)}$ operators simplify when the ground state band becomes energetically isolated.

Using the standard formalism for treating matrix elements, one finds for the first and second terms of the Hamiltonian, after replacing $\hat{Q}^{(2)}_{\chi}$ by $\hat{Q}^{(2)}_{SU(3)}$

$$\begin{aligned} &\langle [N],(2N,0),\tilde{\chi}=0,L||[\hat{H}_1,\hat{Q}^{(2)}_{SU(3)}]||[N],(2N,0),\tilde{\chi}=0,L\rangle \\ &\qquad = -\frac{2}{3\sqrt{7}} c\eta N R_1 \sqrt{2L+1}, \end{aligned} \tag{10}$$

$$\begin{aligned} &\langle [N],(2N,0),\tilde{\chi}=0,L||[\hat{H}_2,\hat{Q}^{(2)}_{SU(3)}]||[N],(2N,0),\tilde{\chi}=0,L\rangle \\ &\qquad = \frac{1}{7\sqrt{2}} c(1-\eta)\left(\chi+\frac{\sqrt{7}}{2}\right) q_0 R_2 \sqrt{2L+1}, \end{aligned} \tag{11}$$

where $\tilde{\chi}$ is the Vergados quantum number [14] and R_1, R_2 are complicated expressions [1] of the number of bosons N and the angular momentum L. In the large N limit and for L not too small, we obtain $R_1 = 1$ and $R_2 = 16/3$. Replacing the intrinsic quadrupole moment q_0 by its value $(N\sqrt{2})$, we get vanishing matrix elements in the large N limit when

$$\chi(\eta) = \frac{\sqrt{7}}{8}\frac{\eta}{(1-\eta)} - \frac{\sqrt{7}}{2}. \tag{12}$$

The expression of Eq. (12) is also visualized in Fig. 2, where one can remark its similarity to Eq. (8).

4 Conclusions

In the present work, a line of SU(3) symmetry was determined analytically, in the limit of large boson numbers, taking advantage of the contraction of SU(3)

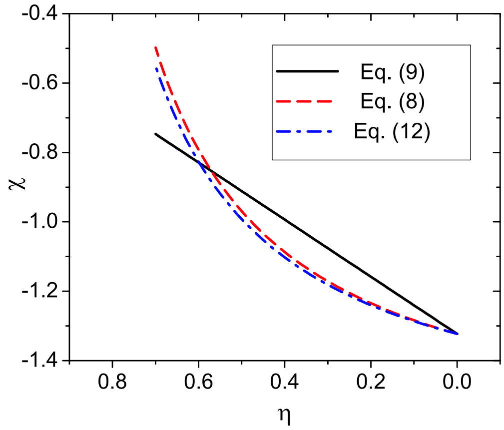

Fig. 2. (Color online) Location of the arc of regularity, as described by the original Eq. (9), and as predicted by the findings of the present work, Eqs. (8) and (12). The η axis has been reversed, in order to correspond directly to Fig. 1.

to [R^5]SO(3), the algebra of the rigid rotator. This line follows closely the Alhassid–Whelan arc of regularity, giving thus an explanation for its existence.

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