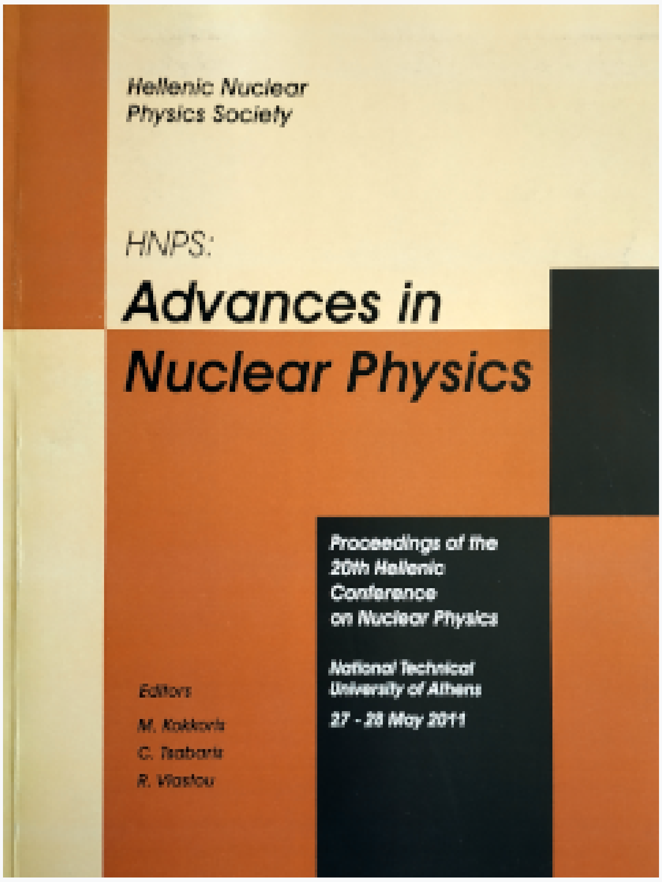
HNPS Advances in Nuclear Physics

Vol 19 (2011)

HNPS2011



Quantum phase transition phenomena in odd-A nuclei

D. Petrellis

doi: [10.12681/hnps.2509](https://doi.org/10.12681/hnps.2509)

Quantum phase transition phenomena in odd-A nuclei

D. Petrellis

Institute of Nuclear Physics, N.C.S.R. "Demokritos", GR-15310 Aghia Paraskevi, Greece

Abstract

Quantum phase transitions in even-even nuclei been extensively studied both in theory and experiment in the recent years. Odd-even nuclei can be considered as examples of mixed Bose-Fermi systems with a single fermion coupled to an even-even core. The effect of a fermion with angular momentum j on quantum phase transitions of a bosonic system comprised of s (L=0) and d (L=2) bosons is investigated.
The analysis is based on an Interacting Boson Fermion Model (IBFM) Hamiltonian and its classical limit and it demostrates the changes in the energy level structure as well as the classical and quantum order parameters involved. Some experimental evidence is also presented and compared with theoretical calculations.

Key words: Quantum shape-phase transitions; Interacting Boson Fermion Model (IBFM); Odd-A nuclei

1 Introduction

Quantum phase transitions in even-even nuclei are very well known and excellent review articles about them exist in the literature [1,**?**]. The effect of a single odd-fermion coupled to an even-even boson core, as in odd-mass nuclei, has also been studied recently [3–6]. A more extensive treatment of the work presented here is included in a recent publication [7].

2 A transitional IBFM Hamiltonian

We begin by considering a typical IBFM Hamiltonian [8], with a boson, fermion and boson-fermion part, but with the boson part represented by a transitional IBM Hamiltonian [9], which contains two control parameters ξ and χ that determine the nature of the phase transition:

$$
\begin{aligned}
H &= H_B + H_F + V_{BF} \\
H_B &= \varepsilon_0 \left[(1-\xi)\, \hat{n}_d - \frac{\xi}{4N} \hat{Q}^\chi \cdot \hat{Q}^\chi \right] \\
H_F &= \varepsilon_j \, \hat{n}_j \\
V_{BF} &= V_{BF}^{MON} + V_{BF}^{QUAD} + V_{BF}^{EXC} ,
\end{aligned}
\tag{1}
$$

with the superscripts MON, $QUAD$ and EXC denoting the monopole, quadrupole and exchange terms, respectively.

$$
\begin{aligned}
V_{BF}^{MON} &= A \hat{n}_d \, \hat{n}_j \\
V_{BF}^{QUAD} &= \Gamma \, \hat{Q}^\chi \cdot \hat{q}_j \\
V_{BF}^{EXC} &= \Lambda \sqrt{2j+1} : [(d^\dagger \times \tilde{a}_j)^{(j)} \times (\tilde{d} \times a_j^\dagger)^{(j)}]^{(0)} : \ ,
\end{aligned}
\tag{2}
$$

with coefficients $A \equiv A_j / \sqrt{5(2j+1)}$, $\Gamma \equiv \Gamma_{jj}/\sqrt{5}$ and $\Lambda \equiv \Lambda_{jj}^{j} / \sqrt{2j+1}$ as in [8], and

$$
\begin{aligned}
\hat{n}_d &= d^\dagger \cdot \tilde{d} \\
\hat{Q}^\chi &= (d^\dagger \times s + s^\dagger \times \tilde{d})^{(2)} + \chi (d^\dagger \times \tilde{d})^{(2)} \\
\hat{n}_j &= -\sqrt{2j+1} (a_j^\dagger \times \tilde{a}_j)^{(0)} \\
\hat{q}_j &= (a_j^\dagger \times \tilde{a}_j)^{(2)} \ .
\end{aligned}
\tag{3}
$$

where, as usual, dots $\cdot$ denote scalar products, $\times$ denote tensor products and : denotes normal ordering, while $s^\dagger, d_\mu^\dagger$ (s, d_μ) $(\mu = 0, \pm 1, \pm 2)$ denote creation (annihilation) operators for s, d bosons and $a_{j,m}^\dagger (a_{j,m})$ $(m = \pm \frac{1}{2}, ..., \pm j)$ creation (annihilation) operators for fermions with angular momentum j. The adjoint operators are $\tilde{d}_\mu = (-)^\mu d_{-\mu}, \tilde{a}_{j,m} = (-)^{j-m} a_{j,-m}$. It should be noted that the transitional IBM Hamiltonian above, describes the (first order) U(5)-SU(3) transition for $\chi = -\sqrt{7}/2$ and $\xi : 0 \to 1$, and the (second order) U(5)-SO(6) transition when $\chi = 0$ and $\xi : 0 \to 1$. The interaction strengths Γ and Λ also play the role of control parameters, however we put $\Gamma \propto \xi$ as in [3]. The purely fermionic part and the monopole part of the B-F interaction are not of interest in our study and are not taken into account.

3 Semiclassical analysis

The next step is to take the expectation value of the full Hamiltonian in the boson condensate

$$|N;\beta,\gamma\rangle = \frac{1}{\sqrt{N!}} \left[b_c^\dagger(\beta,\gamma)\right]^N |0\rangle \tag{4}$$

with

$$b_c^\dagger(\beta,\gamma) = \frac{1}{(1+\beta^2)^{1/2}}[\beta\cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_2^\dagger + d_{-2}^\dagger) + s^\dagger]. \tag{5}$$

The limit $N \to \infty$ of the expectation value of the boson part of the Hamiltonian gives the potential energy surface

$$\begin{aligned} \bar{E}_B(\beta,\gamma) &= \lim_{N\to\infty} E_B(N;\beta,\gamma) \qquad (6) \\ &= \varepsilon_0 N \left\{ \left(\frac{\beta^2}{1+\beta^2}\right)(1-\xi) - \frac{\xi}{4}\frac{1}{(1+\beta^2)^2}\left[4\beta^2 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos 3\gamma + \frac{2}{7}\chi^2\beta^4\right]\right\} \end{aligned}$$

while for the full Hamiltonian (eq. (2)) the expression is

$$\begin{aligned} H(N;\beta,\gamma) &= E_B(N;\beta,\gamma) \qquad (7) \\ &+ \sum_{m_1,m_2} [\varepsilon_j\,\delta_{m_1,m_2} + g_{m_1,m_2}(N;\beta,\gamma)]\left(a_{j,m_1}^\dagger a_{j,m_2} + a_{j,m_2}^\dagger a_{j,m_1}\right)(1+\delta_{m_1,m_2})^{-1} \end{aligned}$$

with g_{m_1,m_2} the elements of a real, symmetric matrix, whose eigenvalues, if we diagonalize it in the single particle basis:

$$|j,m\rangle = a_{j,m}^\dagger |0\rangle \qquad (m = \pm j, \pm(j-1), ..., \pm\frac{1}{2}) \ . \tag{8}$$

are the single particle energies given as functions of β, γ and the control parameters for a certain j (fig. (1)). The case of $\gamma = 0°$ has been worked analytically in [10]. All the examples in the present study concern an odd-particle that occupies a $h_{11/2}$ ($l = 5,\ j = 11/2$) shell which means that we limit our study to the negative parity states. The core consists of ten bosons ($N = 10$). It should be noted that in the SU(3) limit the states are characterized by K, the projection of j on the symmetry axis of the deformed core.

By minimizing the total (boson + boson-fermion) energy surfaces with respect to β and γ one can use the β_e and γ_e as classical order parameters and study

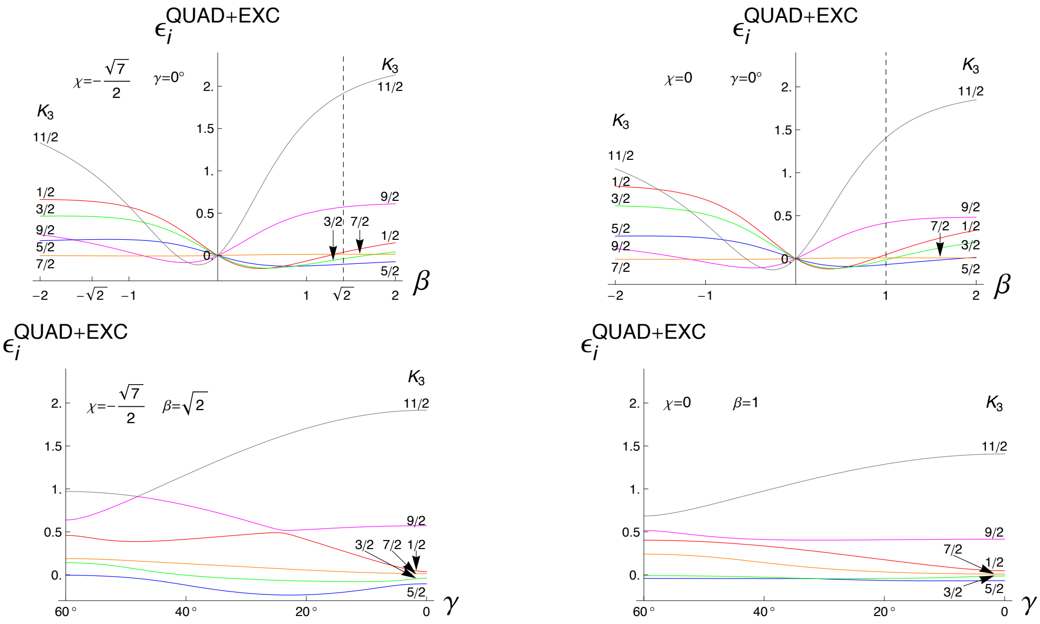


Fig. 1. Single particle energies of a $j = 11/2$ particle as functions of β and γ separately, in a field of quadrupole and exchange strengths such that $\Gamma/\Lambda = 3$ ($\Gamma = -0.0958,\ \Lambda = -0.287$), for the two cases U(5)-SU(3) (left) and U(5)-SO(6) (right).

their behaviour with ξ and χ. Next, by replacing them in the expressions of the single particle energies one obtains the minimum single particle energies (fig. (2)) and similarly the minimum total energies .

4 Quantal analysis

In the microscopic treatment of the problem the quadrupole and exchange interaction strengths are written in terms of BCS occupation probabilities and are proportional to the strengths used in the classical analysis (see [11,7]). Therefore, one can study how the spectrum evolves when moving from a particle-like to a hole -like picture.

The behaviour of the ground state energy and its derivatives is another indicator of a phase transition (see fig. (3), left), even though the effect is smoothed out due to the finiteness of the system. Other quantities, such as the expectation value of the number of d-bosons can be used as order parameters [12]:

$$\nu_i^{(1)} = \frac{\langle \psi_i \left| \hat{n}_d \right| \psi_i \rangle}{N}, \tag{9}$$

were the expectation values are taken with respect to each of lowest states of total angular momentum $J = 11/2, ..., 1/2$ (fig. (3), right).

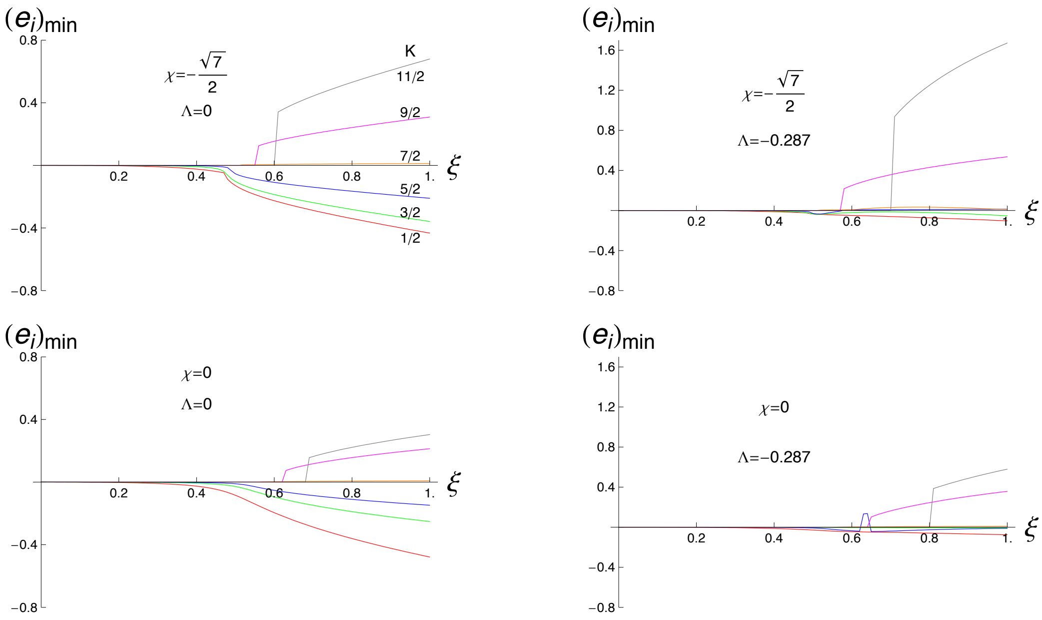


Fig. 2. Minimum single particle energies normalized to $(-N\Gamma)$ as functions of the control parameter ξ with exchange interaction (right) and without (left) , for the U(5)-SU(3) (top) and U(5)-SO(6) (bottom) transitions.

5 Experimental evidence

The presence of discontinuities in the two-neutron separation energies of the isotopes of $_{61}$Pm, $_{63}$Eu, $_{65}$Tb [13] is evident as can be seen in fig. (4, (left)), much like their even-even counterparts $_{60}$Nd, $_{62}$Sm, $_{64}$Gd that are representative examples of a U(5)-SU(3) transition. By subtracting the smooth contribution linear in N in the two-neutron separation energies one can isolate the effect due to the deformation (fig. (4, (right))). A study of the negative parity states of the $_{61}$Pm, $_{63}$Eu, $_{65}$Tb isotopes has been conducted [7] based on previous studies [14,?].

The author would like to thank F. Iachello and A. Leviatan whose contribution and guidance have been instrumental for this project.

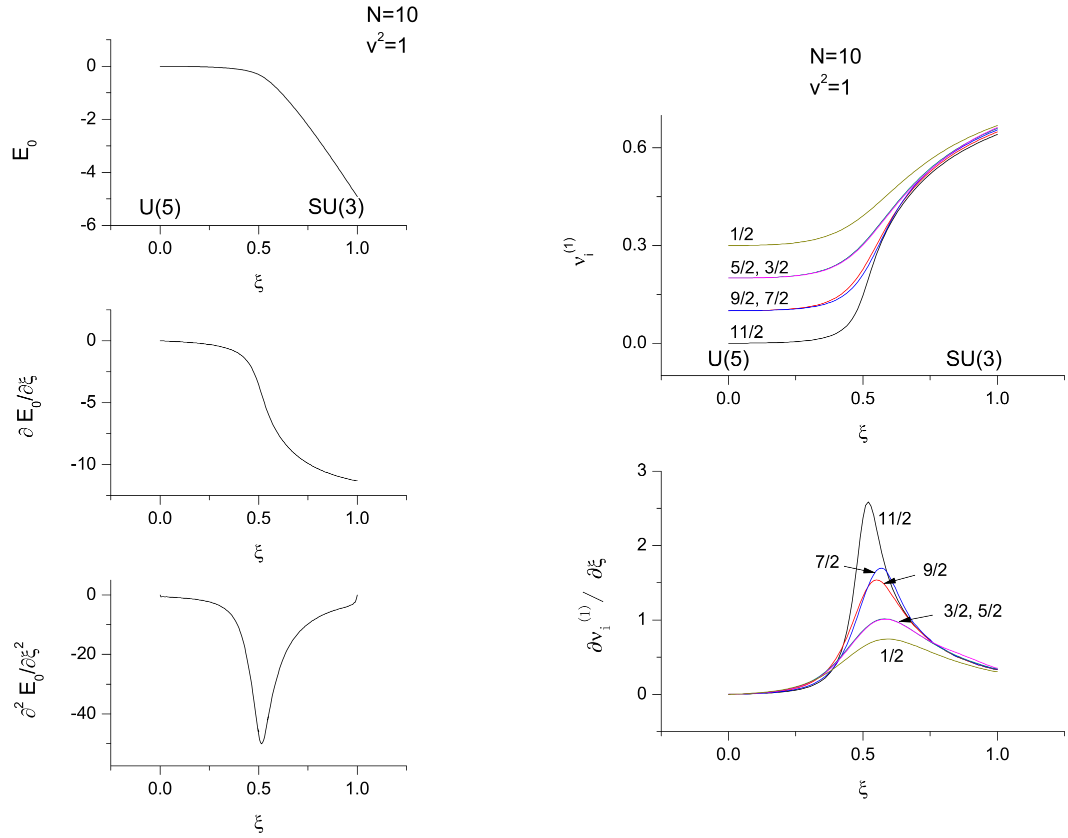


Fig. 3. Ground state energy E_0 and its derivatives (left) and quantum order parameter $\nu_i^{(1)}$ (right) as functions of ξ for a $j = 11/2$ particle coupled to a core of 10 bosons undergoing a U(5)-SU(3) transition. No exchange interaction- particle like spectra ($v^2 = 1$)

References

[1] F. Iachello and M.A. Caprio, in *Understanding Quantum Phase Transitions*, L. Carr ed., Taylor and Francis (2010).

[2] P. Cejnar, J. Jolie and R. F. Casten, Rev. Mod. Phys. 82 (2010) 2155.

[3] C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C 72 (2005) 061302(R).

[4] C.E. Alonso, J.M. Arias, and A. Vitturi, Phys. Rev. C 75 (2007) 064316.

[5] C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C 79, 014306 (2009).

[6] M. Boöyükata, C.E. Alonso, J.M. Arias, L. Fortunato and A. Vitturi, Phys. Rev. C 82, 014317 (2010).

[7] D. Petrellis, A. Leviatan and F. Iachello, Ann. Phys. (N.Y.) 326 (2011) 926.

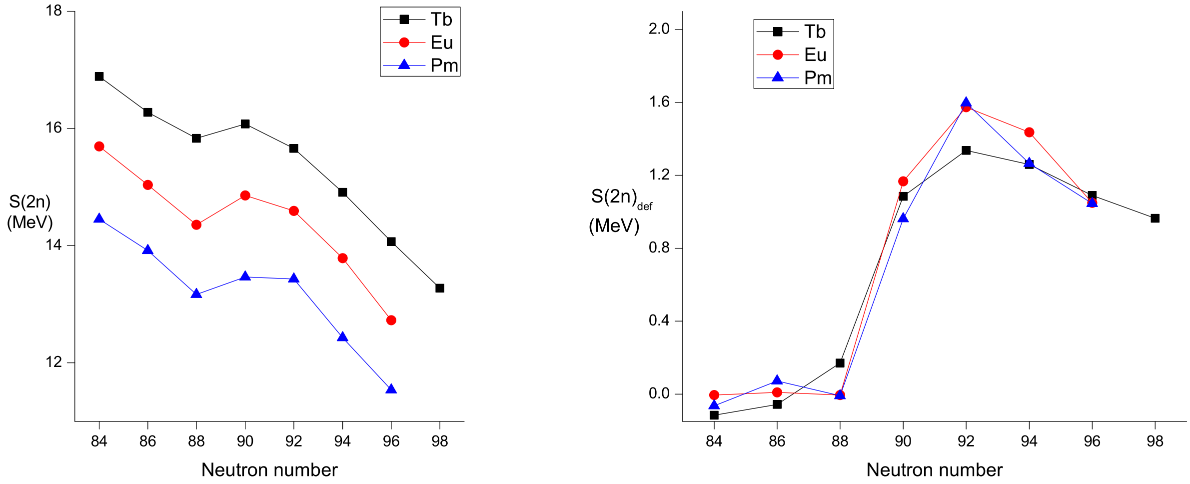


Fig. 4. Experimental two-neutron separation energies $S(2n)$ of the even-neutron isotopes of $_{61}$Pm, $_{63}$Eu, $_{65}$Tb (left) and the contribution of deformation $S(2n)_{def}$ (right).

[8] F. Iachello and P. van Isacker, *The Interacting Boson-Fermion Model* (Cambridge University Press, Cambridge, 1991).

[9] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).

[10] A. Leviatan, Phys. Lett. B 209, 415 (1988).

[11] O. Scholten, *The Interacting Boson Approximation Model and Applications*, Ph.D. Thesis, University of Groningen, The Netherlands (1980).

[12] F. Iachello and N.V. Zamfir, Phys. Rev. Lett. 92 (2004) 212501.

[13] LBNL Isotopes Project Nuclear Data Dissemination Home Page *http://ie.lbl.gov/toi2003/MassSearch.asp*

[14] O. Scholten and N. Blasi, Nucl. Phys. A 380 (1982) 509.

[15] O. Scholten and T. Ozzello, Nucl. Phys. A 424 (1984) 221.