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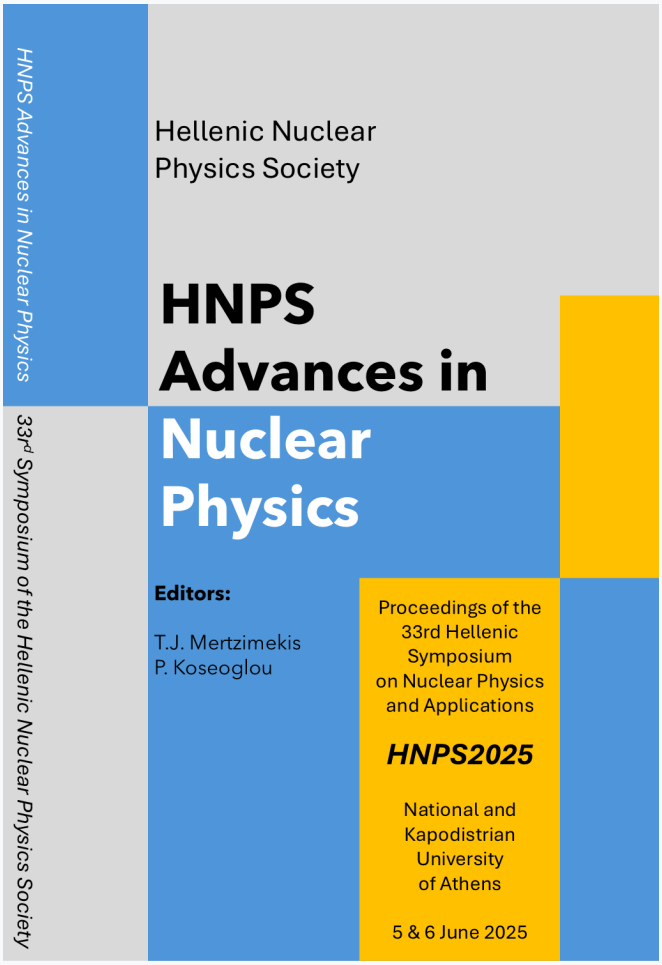

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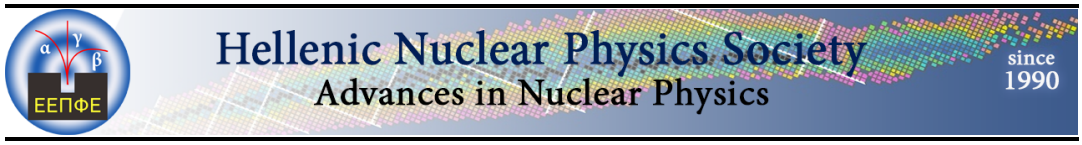

ARTICLE

Investigation of the Yb and W Isotopic Chains using the Confined β-Soft Rotor Model

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Abstract
The Confined β-soft (CBS) rotor model provides a simple collective approach in the region between the X(5) critical point and the axially deformed rotor limit, SU(3), for axially symmetric prolate ($\gamma = 0°$) shapes. Even-even isotopes of ytterbium and tungsten serve as a good testing ground for exploring fundamental aspects of nuclear structure, as their $R_{4/2} = E(4+)/E(2+)$ ratio lies between 2.9 and 3.333. The ratio $R_{4/2} = 2.9$ corresponds to the phase transition between spherical vibrational nuclei and well-deformed rotational systems, while the value $R_{4/2} = 3.33$ is the prediction of the rigid rotor model, where the nucleus behaves as an axially symmetric rotor with a fixed moment of inertia and energy levels following $E(J) \propto J(J+1)$. In the present study, the CBS formalism is applied to calculate the energies of the ground-state band, the associated B(E2) transition rates, and the β-band excitations, directly comparing these predictions to available experimental data. The results demonstrate that CBS succeeds in reproducing the observed spectra and transition strengths for the case of axially symmetric isotopes of Yb and W.

Keywords: CBS calculations, rigid-rotor, Yb, W

1. Introduction

The Confined β-soft rotor model [1] is a theoretical framework in nuclear structure physics designed to describe the collective motion of nuclei undergoing shape-phase transitions. Starting from the well known Bohr–Hamiltonian, we consider a potential $V(\beta, \gamma) = v(\beta) + u(\gamma)$ for axially symmetric prolate ($\gamma \approx 0°$) nuclei. The wave functions are separated into

$$\Psi(\beta, \theta_i) = \xi_L(\beta) \mathcal{D}^L_{M,\kappa}(\theta_i), \tag{1}$$

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where $\mathcal{D}$ are the Wigner functions and θ_i the Euler angles of the intrinsic system. By focusing on the radial part of the Bohr–Hamiltonian, we get the following expression

$$-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\left(\beta^4\frac{\partial}{\partial\beta}\right)-\frac{L(L+1)}{3\,\beta^2}+u(\beta)\right]\xi_L(\beta)=E\,\xi_L(\beta). \tag{2}$$

By assuming an infinite square-well potential in β (β_m<β<β_M) the radial equation becomes a Bessel differential equation, with general solutions superpositions of Bessel functions of the first and second kind. For a state labeled by angular momentum L, the energy eigenvalues are given as

$$E_{L,1}=\frac{\hbar^2}{2B\beta_M^2}z^2_{\beta_{L,1}}, \tag{3}$$

where s=1 represents the ground-state quantum number and $z_{L,1}$ is the appropriate zero determined by both L and the boundary ratio r_b. As a result, the ratio $R_{4/2}$, defined as the ratio of the excitation energies of the first 4^+ and first 2^+ states within the ground band, becomes

$$R_{4/2}=\frac{E(4_1^+)}{E(2_1^+)}=\frac{z^2_{\beta_{4,1}}}{z_{\beta^2_{2,1}}}. \tag{4}$$

The CBS model depends on a single parameter, r_β, which represents the relative position of the inner and outer boundaries of the β-potential well. Here, $r_\beta=\beta_m/\beta_M$ defines the ratio of these boundaries and determines the stiffness of the β degree of freedom [1]. For each isotope, r_β is determined by fitting the calculated ground-state energy ratios to the experimental ones. In practice, a trial value of r_β fixes the corresponding Bessel zeros $z_{L,1}(r_\beta)$, from which the normalized excitation energies are computed. The value of r_β is adjusted until the deviation between calculated and experimental ratios is minimized. Smaller r_β values reproduce softer, transitional behavior near the X(5) limit ($R_{4/2}=2.9$), whereas larger r_β correspond to a stiffer β potential approaching the rigid-rotor limit ($R_{4/2}=3.33$), associated with the SU(3) limit, depicted in the Casten triangle (see Fig. 1). Once r_β is fixed, all remaining CBS predictions follow without further adjustment.

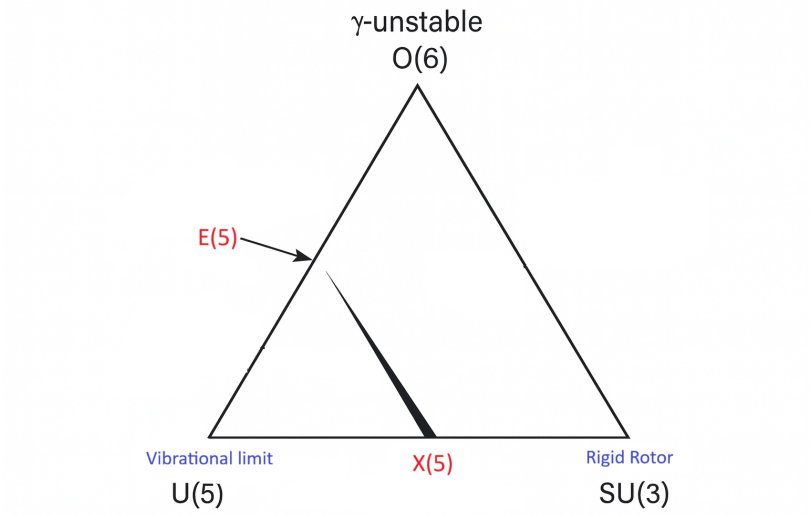
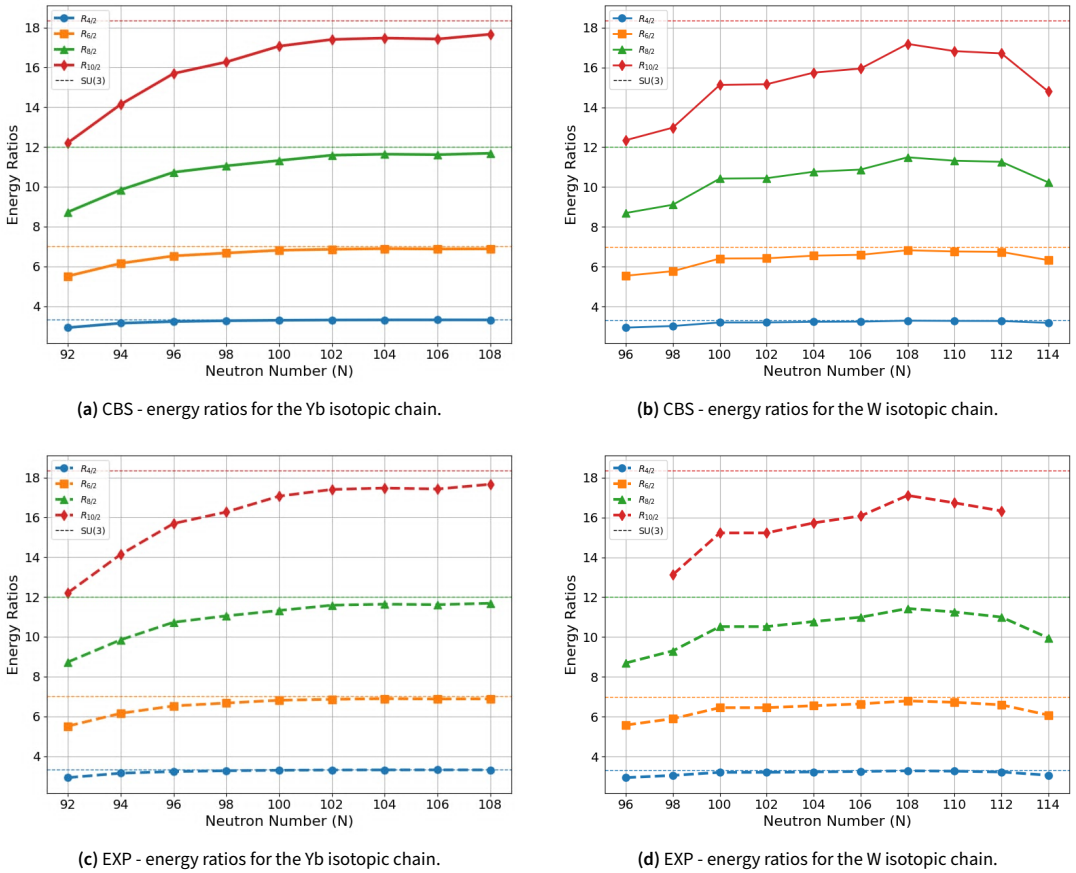


Figure 1. The Casten triangle. At its three vertices lie the dynamical symmetries, while the critical points E(5) and X(5) represent shape–phase transitions. Figure inspired by Ref. [2]

2. Results and Discussion

Using the Confined β-soft (CBS) rotor model program, which allows to calculate CBS predictions, developed by M. Reese [3], precise calculations were performed for various isotopes of the isotopic chains of ytterbium and tungsten. These nuclei, like most elements in the rare-earth region of the nuclear chart, are well known for their strong collective and rotational behavior, making them an excellent testing ground for the model. In this study, the main objective is to computationally reproduce the energies of the ground state band and the B(E2) transition strengths, evaluating the ability of the model to follow the experimental values. In addition, the β-band level energies are examined, where the experimental data are rather limited. The most effective way to illustrate these results and directly compare them with the experimental data is to plot the energy ratios of each state.



(a) CBS - energy ratios for the Yb isotopic chain.

(b) CBS - energy ratios for the W isotopic chain.

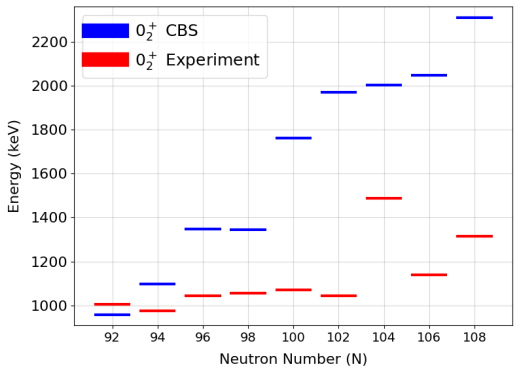
(c) EXP - energy ratios for the Yb isotopic chain.

(d) EXP - energy ratios for the W isotopic chain.

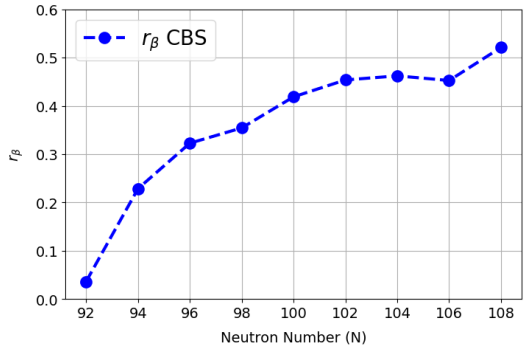
Figure 2. Comparison of ytterbium and tungsten CBS calculations and experimental data.

In Figs. 2a and 2b, we present the CBS results for the $R_{4/2}$, $R_{6/2}$, $R_{8/2}$, and $R_{10/2}$ energy ratios for the energy states of the ground-state band in the isotopic chains of Yb and W, where the dashed lines denote the predictions of the SU(3) symmetry for each ratio. The calculated data align with the experimental trends illustrated in Figs 2c and 2d. However, for heavier isotopes (e.g. ^{178}Yb) the β-band predicted by the CBS model lies at a significantly higher excitation energy than the first state 0^+ observed experimentally (see Fig. 3a). According to the NUDAT database, the experimentally measured 0^+_2 level in ^{178}Yb is located at 1315.3 keV, and no other 0^+ state is reported. For the case of

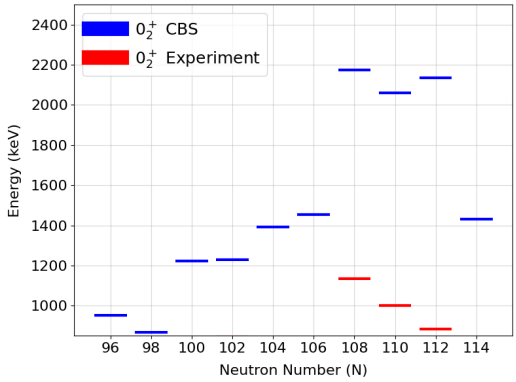
tungsten's isotopic chain, similar results are deduced, as the calculations seem to align well with the experimental data for the ground state levels. Additionally, we observe that both elements approach the rigid-rotor limit for isotopes with N=108. This behavior is depicted in both experimental and CBS data and can be explained with single-particle arguments, although the mechanism is not clear enough. In fact, $N = 108$ marks the point where the competition between pairing correlations and quadrupole collectivity reaches its maximum [4, 5]. From a microscopic perspective, this behavior can be traced to the filling of neutron orbitals near the Fermi surface, especially the $3p_{3/2}$ which enhances the collective behavior of the nuclei and its quadrupole deformation. Despite decades of research, a complete characterization of the β-vibrational motion and of the corresponding 0_2^+ states in deformed nuclei remains elusive [6]. Now that it has been established that the CBS approach reproduces the basic properties of the isotopes under investigation, we turn to the more complex question concerning the nature of these states. Within the CBS model, they are identified as β-excitations. Nevertheless, they do not always coincide with the first excited state 0^+ of the nucleus.



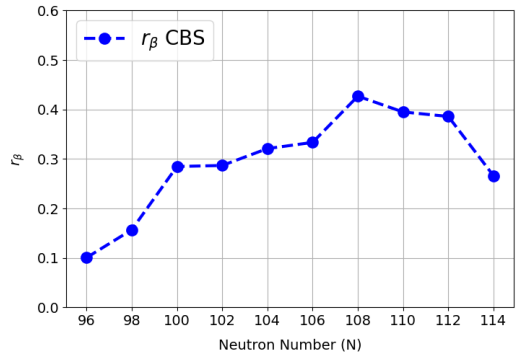
(a) First-excited 0^+ states for Yb isotopes



(b) r_β for Yb isotopes



(c) First-excited 0^+ states for W isotopes



(d) r_β for W isotopes

Figure 3. Ytterbium and tungsten: first-excited 0^+ energies and the r_β parameters.

The level energies suggested in Fig. 3, provide a new perspective and impose stricter constraints on these excitations in order to achieve a better understanding of the problem. Moreover, we display the values of r_β as a function of the neutron number of the nuclei. The correlation between the CBS fitting parameter r_β and the ratio $R_{4/2}$ is revealed, showing that, as the rigid-rotor limit is approached, the value of r_β increases. In fact, an increase in r_β moves the nuclei closer to the SU(3) symmetry point in the IBM triangle, which explains the trends in Figs. 2a and 2b. A recent comprehensive

review of the experimental 0^+ data [7] highlights the issue of determining the properties of these excited states. In Fig. 3a it is evident that the β-excitation does not coincide with the first excited 0^+ state for most isotopes. This implies that these isotopes are not purely β-soft but instead display a degree of γ-deformation [8]. Still, there is clear evidence that such β-vibrational states exist, although they correspond to higher-lying excitations of the nucleus. A representative case is ^{168}Yb (Fig. 4). The first excited 0^+ state and the rotational band built upon it predicted by the CBS model lie very close to the experimentally observed levels. However, according to the NNDC database, these levels correspond to the *third* excited 0^+ state rather than the first one, which occurs at approximately 1155 keV, as also shown in Fig. 4. This suggests that the lowest 0^+ excitations in most of these isotopes are not of pure β-vibrational character, but are likely dominated by two-particle–two-hole configurations [9], or by a mechanism not yet fully understood.

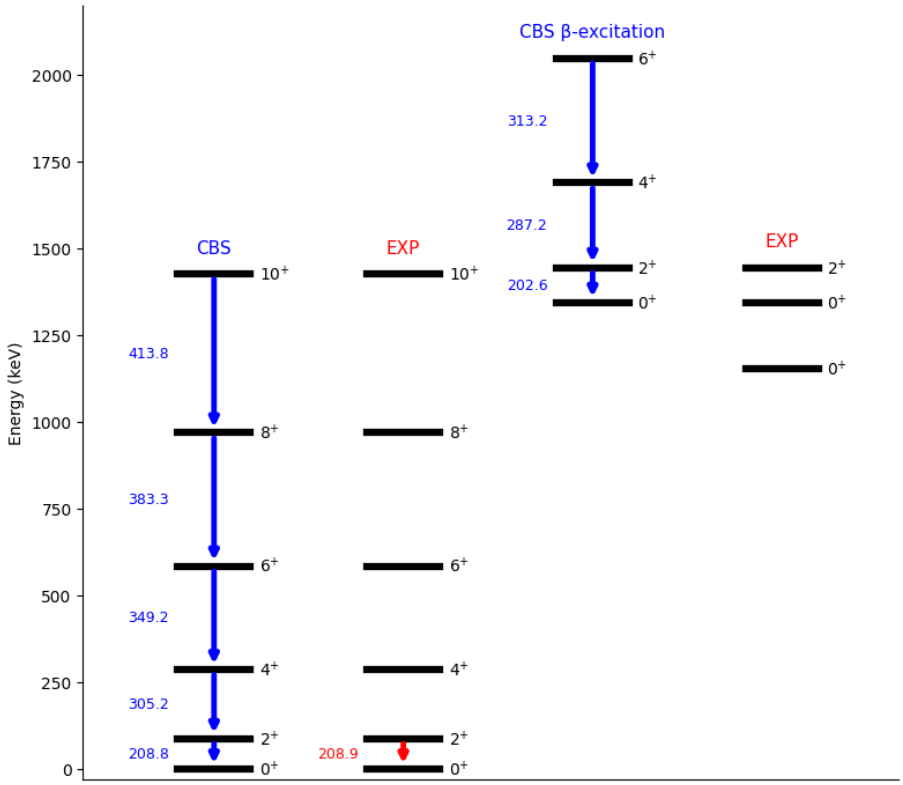


Figure 4. Level scheme of ^{168}Yb. The CBS results for the B(E2) transition strengths for the ground-state band and the β excitation are shown in blue in Weisskopf units (W.u), while known experimental B(E2) values are shown in red. With "EXP" are denoted the experimental data for the ground-state band, the first excited 0^+ state and the rotational band built on the third excited 0^+ bandhead.

For the case of the W isotopes, the scarcity of experimental data for most excited 0^+ states clearly underlines the need for new measurements, which would provide crucial input for constraining theoretical models and improving our understanding of β-vibrational excitations in deformed nuclei. Especially, for the cases of N=96 and N=98 the energies of the 0^+ states of the β-excitation lie close to where we usually expect the presence of the first-excited 0^+ states, as in the case of $^{162-164}$Yb (N=92-94). The neutron midshell at N=104 appears near the right end of the figure for Yb (2a), but in the middle of the figure for W (2b), where the maximum deformation appears around N=108. Then a

slight decrease starts, most probably due to Pauli saturation [10], leading to a transition from prolate to oblate shapes around N=114. Looking at the neutron number N, one can realize that the behavior of Yb and W is very similar up to N=108. The drop seen in W pertains to the region beyond N=108, which is not plotted for Yb. Comparing Figs. 3b and 3d, it is evident that the Yb isotopes exhibit a steady increase of the r_β parameter with mass number, suggesting a smooth evolution toward a more rigid rotor structure with increasing neutron number. In contrast, the W isotopes exhibit a less systematic behavior of r_β, with fluctuations and even a slight decrease for heavier isotopes, indicating a more complex shape evolution and possibly weaker or mixed β-vibrational character.

3. Conclusion

This study has primarily examined the collective properties and evolutionary trends of the even–even Yb and W isotopes as the number of neutrons increases. The investigation of collective behavior across these isotopic chains involved the calculation of various observables, such as excitation energies and reduced transition probabilities. The energy ratios calculated within the CBS framework exhibit good agreement with the available experimental data, reproducing the observed trends, and thereby confirming the effectiveness and applicability of the CBS approach in this mass region. Furthermore, the nature of the 0^+ states was addressed, and predictions were made for the existence of 0^+ bandheads, while also noting the lack of sufficient experimental data for their verification.

Acknowledgements

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References

[1] N. Pietralla and O. M. Gorbachenko. "Evolution of the "β excitation" in axially symmetric transitional nuclei". In: *Phys. Rev. C* 70 (2004, 011304). DOI: 10.1103/PhysRevC.70.011304.
[2] R. F. Casten. *Nuclear Structure from a Simple Perspective*. Oxford: Oxford University Press, 1990. ISBN: 0-19-504599-8.
[3] M. Reese. *CBS model program*. https://sourceforge.net/projects/cbsmodel/. Confined Beta-Soft (CBS) nuclear structure model code, version 1.0. 2011.
[4] J.B. Gupta and Vikas Katoch. "Review of nuclear structure N=86–118 of W isotopes". In: *Nucl. Phys. A* 1057 (2025, 123032). ISSN: 0375-9474. DOI: 10.1016/j.nuclphysa.2025.123032.
[5] A. Zyriliou, T. J. Mertzimekis, A. Chalil, et al. "A study of some aspects of the nuclear structure in the even–even Yb isotopes". In: *Eur. Phys. J. Plus* 137.3 (2022, 352). DOI: 10.1140/epjp/s13360-022-02414-2. URL: 10.1140/epjp/s13360-022-02414-2.
[6] P. E. Garrett. "Characterization of the β vibration and 0_2^+ states in deformed nuclei". In: *J. Phys. G* 27.1 (2001), R1. DOI: 10.1088/0954-3899/27/1/201.
[7] S. R. Lesher A. Aprahamian K. Lee and R. Bijker. "The nature of 0+ excitations in deformed nuclei". In: *Prog. Part. Nucl. Phys.* 143 (2025, 104173). ISSN: 0146-6410. DOI: 10.1016/j.ppnp.2025.104173.
[8] P. Cejnar and J. Jolie. "Quantum phase transitions in the interacting boson model". In: *Prog. Part. Nucl. Phys.* 62.1 (2009), p. 210. ISSN: 0146-6410. DOI: 10.1016/j.ppnp.2008.08.001. URL: 10.1016/j.ppnp.2008.08.001.
[9] K. Heyde, C. De Coster, J. L. Wood, and J. Jolie. "Proton 2p-2h intruder excitations and the modified vibrational intensity and selection rules". In: *Phys. Rev. C* 46 (1992), p. 2113. DOI: 10.1103/PhysRevC.46.2113.
[10] Da Hsuan Feng, Cheng-Li Wu, Mike W. Guidry, and Zhen-Ping Li. "Dynamical Pauli effects and the saturation of nuclear collectivity". In: *Phys. Lett. B* 205.2 (1988), pp. 156–162. ISSN: 0370-2693. DOI: 10.1016/0370-2693(88)91639-5.