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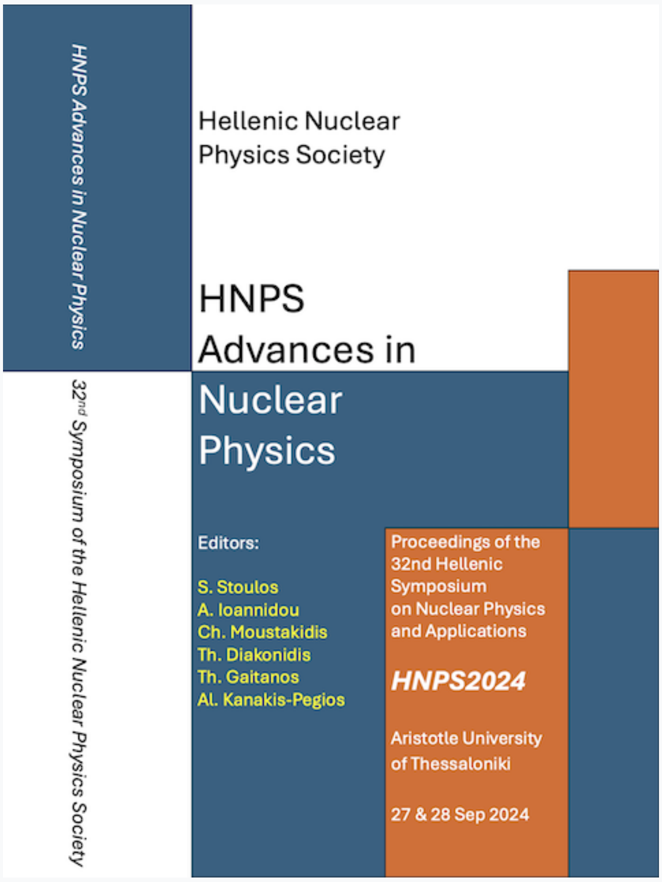

Symmetry energy effects on the quark deconfinement phase transition

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Symmetry energy effects on the quark deconfinement phase transition

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Abstract The nuclear symmetry energy plays an important role on the structure of finite nuclei, as well as on the bulk properties of neutron stars. In the present study we study the effects of the symmetry energy on some properties of hybrid stars, which consist mainly of hadron and quark matter. To this end, for the hadron matter of hybrid stars, we parameterize the equation of state which describes the asymmetric and symmetric nuclear matter with the help of the parameter $\eta = (K_0L^2)^{1/3}$, where K_0 is the incompressibility and L is the slope parameter.

Keywords symmetry energy, hybrid stars, quark matter, phase transition

INTRODUCTION

The Nuclear Symmetry Energy (NSE) is one of the most fundamental quantities relevant to the study of both neutron- rich finite nuclei and neutron stars. The uncertainty that exists in the knowledge of the symmetry energy, especially at low densities of nuclear matter, similar to those found in finite nuclei, can be partly addressed by terrestrial experiments. However, its values at high densities, which are encountered in astrophysical objects such as neutron stars, are completely uncertain and the corresponding empirical data have a large error. In this work, we study some effects of the symmetry energy on hybrid stars, which consist mainly of hadron and quark matter.

HADRON MATTER

The key quantity in our calculations is the energy per particle of asymmetric nuclear matter, where in good approximation, at least for densities close to the saturation density, is given by the expression:

$$\boldsymbol{E(n,\alpha) = E_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + S(n)\alpha^2}$$

where $E_0 = E(n_0,0)$ is the energy per particle at the saturation density n_0, K_0 is the incompressibility and S(n) is the symmetry energy. The parameter α, given by $\alpha = (n_n - n_p)/n$, is the asymmetry parameter with n_n and n_p being the neutron and proton number densities, respectively, and n is the total number density ($n=n_n+n_p$).

The nuclear symmetry energy (NSE) S(n) can be developed in a series around the saturation density, where, expanding up to the 1st order term we have:

$$S(n) = J + \frac{L}{3n_0}(n-n_0)+\ldots$$

where $J=S(n_0)$. The slope parameter is related to the second derivative of the NSE according to the definition

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$$L = 3n_0 \left(\frac{dS}{dn}\right)_{n=n_0}.$$

The pressure is given by

$$P_b = n^2 \frac{d(\varepsilon_b/n)}{dn}$$

where ε_b is the energy density. We also consider a lepton contribution for the hadron part of the star (electrons). The contribution to the total energy density and pressure by electrons is given by the well-known formula for the relativistic Fermi gas, that is:

$$\varepsilon_e = \frac{(m_e c^2)^4}{8\pi^2(\hbar c)^3}\left[z(2z^2+1)\sqrt{1+z^2} - ln\left(z+\sqrt{z^2+1}\right)\right]$$

$$P_e = \frac{(m_e c^2)^4}{24\pi^2(\hbar c)^3}\left[z(2z^2-3)\sqrt{1+z^2} + 3ln\left(z+\sqrt{z^2+1}\right)\right]$$

where

$$z = \frac{(\hbar c)(3\pi^2 n_e)^{1/3}}{m_e c^2}, \qquad n_e = x_p n$$

and x_p is the proton fraction. The density dependence of the proton fraction for the hadron part of the star, is calculated by the following equation, which results from beta equilibrium:

$$4(1-2x_p)S(n) = \hbar c(3\pi^2 n_e)^{1/3} = \hbar c(3\pi^2 n x_p)^{1/3}$$

Now the total energy density and total pressure for matter that is charge neutral and in β-equilibrium are:

$$\varepsilon_{tot} = \varepsilon_b + \varepsilon_e$$

$$P_{tot} = P_b + P_e$$

We parameterize the different EoS that we construct, with the help of the parameter η, where $\eta=(K_0L^2)^{1/3}$. This parameter describes the compound effect of both the incompressibility K_0 and the slope parameter L.

QUARK MATTER MODEL

For the quark matter part of the hybrid star, we consider in our study, three flavor, color superconducting quark matter. In particular, we use the quark matter model used by D. Blaschke et al [1], where for u, d, s massless quarks we have for the pressure and the energy density:

$$P_Q(\mu) = \frac{3}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2}\Delta^2 \left(\frac{\mu}{3}\right)^2 - B_{eff},$$

$$\varepsilon_Q(\mu) = \frac{9}{4\pi^2} a_4 \left(\frac{\mu}{3}\right)^4 + \frac{3}{\pi^2}\Delta^2 \left(\frac{\mu}{3}\right)^2 + B_{eff}$$

where a_4 is a dimensionless parameter, Δ is the diquark pairing gap (in MeV) and B_{eff} is the effective Bag pressure of the model (in MeV^4).

HADRON TO QUARK MATTER PHASE TRANSITION

We employ in our study the Maxwell construction, in which the phase transition from hadron to quark matter inside the hybrid star, occurs when the pressures and the chemical potentials of the two phases become equal, i.e. when

$$P_H = P_Q = P_0$$

and

$$\mu_H = \mu_Q = \mu_0$$

where P_0 and μ_0 are the critical pressure and critical chemical potential respectively. The chemical potentials for the hadron and quark phases, for T=0, are calculated by the relations [2]:

$$\mu_H = \frac{\varepsilon_H + P_H}{n_H},$$

$$\mu_Q = \frac{\varepsilon_Q + P_Q}{n_Q}$$

CALCULATION OF ASTROPHYSICAL OBSERVABLES

In order to extract some astrophysical observables, namely the mass M and the radius R of the hybrid star, we have to solve the well-known Tolman-Oppenheimer-Volkoff (TOV) system of differential equations, given by

$$\frac{dP(r)}{dr} = -\frac{G\varepsilon(r)M(r)}{c^2r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi P(r)r^3}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{c^2r}\right)^{-1},$$

$$\frac{dM(r)}{dr} = \frac{4\pi r^2}{c^2}\varepsilon(r).$$

An important and well measured quantity by the gravitational wave detectors is the dimensionless tidal deformability Λ. During the inspiral phase of binary neutron star systems that are in the process of merging, tidal effects can be detected. The tidal number k_2 describes the response of the neutron star (in our work the hybrid star) to the tidal field. This quantity depends on the hybrid star mass and also on the applied EoS. The tidal love number k_2 is given by

$$\begin{gathered} k_2 = \frac{8\beta^5}{5}(1-2\beta)^2[2-y_R+(y_R-1)2\beta] \\ \times\left[2\beta\left(6-3y_R+3\beta(5y_R-8)\right)\right. \\ +4\beta^3\left(13-11y_R+\beta(3y_R-2)+2\beta^2(1+y_R)\right) \\ \left.+3(1-2\beta)^2[2-y_R+2\beta(y_R-1)]ln(1-2\beta)\right]^{-1} \end{gathered}$$

where β=GM/Rc^2 is the compactness of the hybrid star. The parameter y_R=y(R) is determined by solving the following differential equation:

$$r\frac{dy(r)}{dr} + y^2(r) + y(r)F(r) + r^2Q(r) = 0.$$

The functions F(r) and Q(r) are functions of the energy density ε(r), pressure P(r), and mass M(r), and are defined by the following equations:

$$F(r) = \left[1 - \frac{4\pi r^2 G}{c^4}\left(\varepsilon(r) - P(r)\right)\right]\left(1 - \frac{2M(r)G}{rc^2}\right)^{-1},$$

$$r^2 Q(r) = \frac{4\pi r^2 G}{c^4}\left[5\varepsilon(r) + 9P(r) + \frac{\varepsilon(r) + P(r)}{\partial P(r)/\partial\varepsilon(r)}\right]$$
$$\times\left(1 - \frac{2M(r)G}{rc^2}\right)^{-1} - 6\left(1 - \frac{2M(r)G}{rc^2}\right)^{-1}$$
$$-\frac{4M^2(r)G^2}{r^2c^4}\left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right)^2\left(1 - \frac{2M(r)G}{rc^2}\right)^{-2}$$

Now the dimensionless deformability is given by the equation

$$\Lambda = \frac{2}{3}k_2\left(\frac{c^2 R}{GM}\right)^5$$

The differential equation for y(r) must be solved numerically and self consistently with the TOV equations, under the following boundary conditions, which hold for the center of the star (i.e. r=0):
y (0)= 2, P(0)= P_c (where P_c is the central pressure of the star), and M(0)=0.

For the EoS of the core of the hybrid star, we use the piecewise function:

$$\varepsilon(P) = \begin{cases} \varepsilon_{tot}(P), & P^m{}_c < P \le P_0 \\ \varepsilon_Q(P), & P > P_0 \end{cases}$$

where $P^m{}_c$ is the maximum pressure which holds for the crust. As we can see, this EoS is in accordance with the Maxwell construction, where the phase transition occurs at constant pressure (equal to the critical pressure P_0), but with an energy density jump.

RESULTS AND DISCUSSION

By constructing the pressure P – chemical potential μ curves for the hadron and quark matter cases, we can extract the critical pressure P_0 and critical chemical potential μ_0, by finding the intersection points of the two curves. Then we can construct the EoS of the core of the hybrid star and then solve the aforementioned differential equations to extract the M-R and Λ-M diagrams. Another important quantity we can compute is the critical hadron matter number density, in saturation density units, n_c/n_0.

Below we present our results for two quark matter cases: for a_4=0.7, Δ=0 MeV, $B_{eff}^{1/4}$ = 160 MeV and for a_4=0.7, Δ=50 MeV, $B_{eff}^{1/4}$ = 160 MeV.

For both sets of quark matter parameters, we note firstly that M_{max} is a decreasing function of η, whereas R_{Mmax}, $R_{1.4}$ are increasing functions of η. For the second case of parameters (Δ=50 MeV), $\Lambda_{1.4}$ obtains a minimum value. For the first set of parameters (Δ=0 MeV), due to the phase transition occurring for various η cases in the range $P_0 \in [21.0603, 47.1477]$ MeV fm^{-3}, the values of $R_{1.4}$ are relatively higher, compared to the second case of parameters (Δ=50 MeV), where the corresponding range for the critical pressure is $P_0 \in [3.15892, 5.78213]$ MeV fm^{-3}. It appears that an increase in Δ has a decisive effect on the critical pressure P_0 , leading to smaller values for P_0, and consequently for $R_{1.4}$.

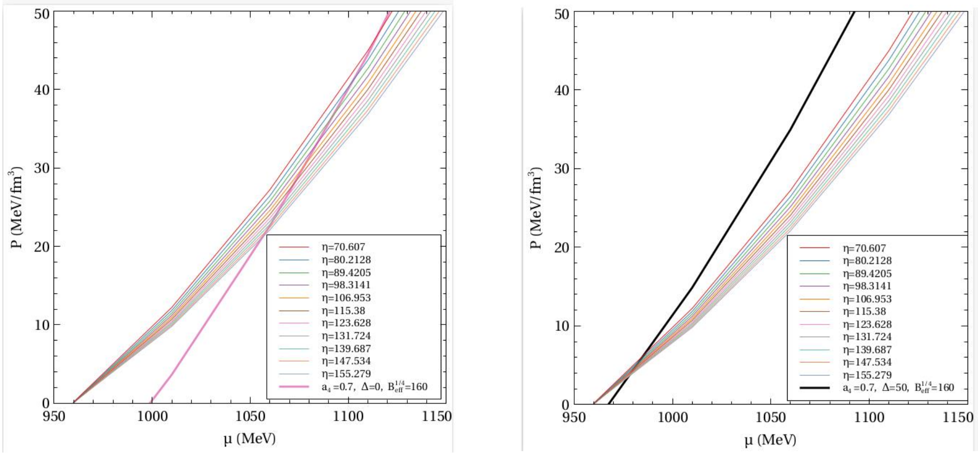

Figure 1. *P-μ diagrams for 11 cases of η. The quark matter characteristics used are: a_4 = 0.7, Δ=0 MeV, $B_{eff}^{1/4}$ = 160 MeV (left panel) and a_4 = 0.7, Δ=50 MeV, $B_{eff}^{1/4}$ = 160 MeV (right panel).*

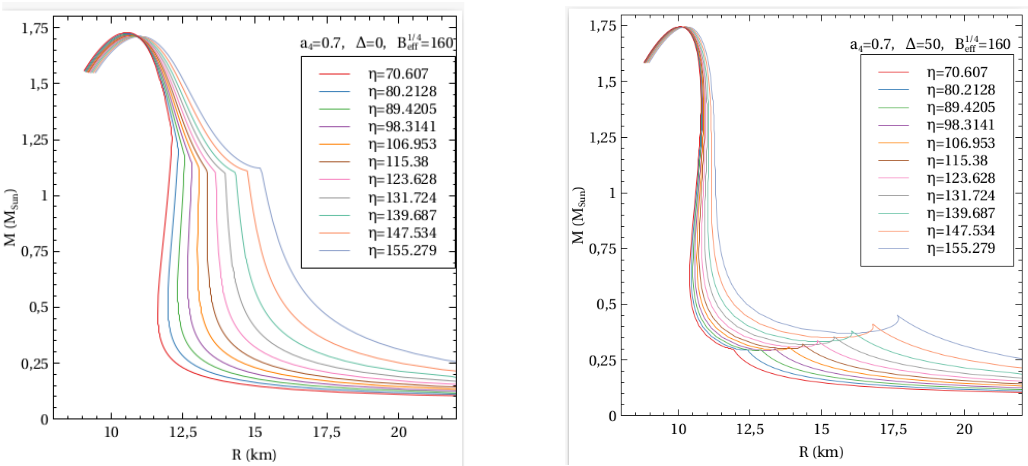

Figure 2. *M-R diagrams for 11 cases of η. The quark matter characteristics used are: a_4 = 0.7, Δ=0 MeV, $B_{eff}^{1/4}$ = 160 MeV (left panel) and a_4 = 0.7, Δ=50 MeV, $B_{eff}^{1/4}$ = 160 MeV (right panel).*

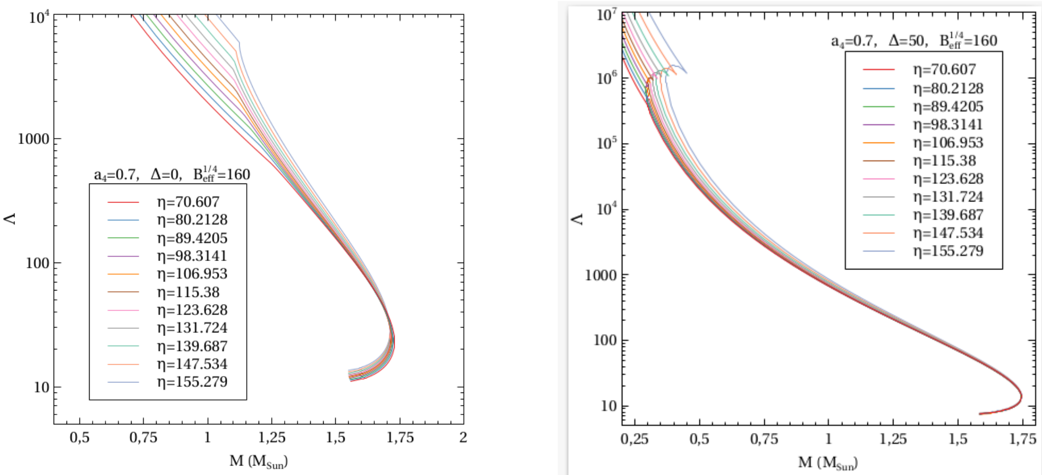

Figure 3. *Λ-M diagrams for 11 cases of η. The quark matter characteristics used are: a_4 = 0.7, Δ=0 MeV, $B_{eff}^{1/4}$ = 160 MeV (left panel) and a_4 = 0.7, Δ=50 MeV, $B_{eff}^{1/4}$ = 160 MeV (right panel).*

Table 1. The incompressibility K_0 (in MeV), the slope parameter L (in MeV), the parameter η (in MeV), R_{Mmax} (in Km), M_{max} (in $M_\odot$), $R_{1.4}$ (in Km), $\Lambda_{1.4}$, μ_0 (in MeV), p_0 (in MeV fm^{-3}) and n_c/n_0 correspond to the various equations of state. The quark matter characteristics used are: a_4 = 0.7, Δ=0 MeV, $B_{eff}^{1/4}$ = 160 MeV.

K_0	L	η	R_{Mmax}	M_{max}	$R_{1.4}$	$\Lambda_{1.4}$	μ_0	p_0	n_c/n_0
220	40	70.607	10.5532	1.72952	11.9253	276.427	1115.71	47.1477	2.488
224	48	80.2128	10.5756	1.72503	11.999	284.358	1102.61	41.0373	2.3487
228	56	89.4205	10.602	1.72162	12.0709	291.481	1092.49	36.4622	2.2310
232	64	98.3141	10.6319	1.71897	12.1455	299.001	1084.5	32.9374	2.13
236	72	106.953	10.6654	1.71687	12.2246	307.424	1078.03	30.1431	2.0419
240	80	115.38	10.7043	1.71524	12.3118	317.98	1072.73	27.8873	1.9644
244	88	123.628	10.7474	1.71397	12.409	330.708	1068.29	26.0236	1.8952
248	96	131.724	10.7971	1.71304	12.5207	346.594	1064.54	24.4721	1.8332
252	104	139.687	10.8549	1.7124	12.6511	366.644	1061.35	23.1591	1.7772
256	108	147.534	10.9231	1.71202	12.8064	392.253	1058.57	22.0307	1.726
260	112	155.279	11.0063	1.71195	12.9981	426.323	1056.17	21.0603	1.6792

Table 2. The incompressibility K_0 (in MeV), the slope parameter L (in MeV), the parameter η (in MeV), R_{Mmax} (in Km), M_{max} (in $M_\odot$), $R_{1.4}$ (in Km), $\Lambda_{1.4}$, μ_0 (in MeV), p_0 (in MeV fm^{-3}) and n_c/n_0 correspond to the various equations of state. The quark matter characteristics used are: a_4 = 0.7, Δ=50 MeV, $B_{eff}^{1/4}$ = 160 MeV.

K_0	L	η	R_{Mmax}	M_{max}	$R_{1.4}$	$\Lambda_{1.4}$	μ_0	p_0	n_c/n_0
220	40	70.607	10.0639	1.74766	10.7861	108.739	984.88	5.78213	1.3424
224	48	80.2128	10.0737	1.7473	10.8047	108.286	983.135	5.17583	1.2577
228	56	89.4205	10.0845	1.74703	10.8248	107.852	981.792	4.71153	1.1836
232	64	98.3141	10.097	1.74682	10.8487	107.657	980.738	4.3482	1.1179
236	72	106.953	10.112	1.74668	10.8796	107.815	979.903	4.06146	1.0594
240	80	115.38	10.1296	1.74658	10.9061	108.352	979.227	3.82941	1.0065
244	88	123.628	10.151	1.74653	10.9469	109.379	978.682	3.64301	0.9585
248	96	131.724	10.1763	1.74651	10.9944	110.893	978.226	3.48718	0.9140
252	104	139.687	10.2071	1.74652	11.0521	113.052	977.85	3.35892	0.8727
256	108	147.534	10.2451	1.74658	11.1234	115.991	977.537	3.25207	0.8341
260	112	155.279	10.2926	1.74666	11.2135	119.946	977.264	3.15892	0.7972

On the other hand, an increase in Δ from 0 to 50 MeV, only leads to a small increase in the values of the maximum mass of the star, M_{max}. Apparently, the existence of quark matter in hybrid stars plays a decisive role for the mass M and radius R of the star, leading to significantly smaller values, as compared to the case of neutron stars with no quark matter. Actually, as can be seen, for both cases of quark matter, the maximum mass for hybrid stars (HS) obeys the inequality $M_{max}^{HS} \leq 1.74766 M_{Sun}$,when, for neutron stars (NS) with the same hadron model, it was found [3] that $M_{max}^{NS} \leq 2.442 M_{Sun}$. Also, the onset of quark matter for lower values of critical pressure P_0 with the increase of Δ, leads to significantly smaller values for $\Lambda_{1.4}$. Finally, another important aspect is the higher values for the hadron matter critical densities n_c , for the case Δ=0 MeV, with a maximum value of hadron density equal to 2.488 times the saturation density n_0.

CONCLUSIONS

So far, in the present study, we have studied the effects of the symmetry energy on some of hybrid stars' properties. For the specific hadron and quark matter models that we use, we find that a change in the parameter η (which encloses the combined effect of the incompressibility K_0 and the slope parameter L) has a stronger effect in $\Lambda_{1.4}$, the critical pressure P_0 , and the critical hadron number density n_c , as compared to its milder effect to other astrophysical observables of the hybrid star, R_{Mmax}, M_{max}, $R_{1.4}$.

Comparing the two quark matter cases that we present in this work, which differ only in the diquark pairing gap Δ by 50 MeV, we have found that an increase of Δ from 0 to 50 MeV, lowers significantly the critical pressure values P_0 for various cases of η. This leads to important differences between the two cases of hybrid stars, for quantities such as $R_{1.4}$ and n_c/n_0.

Our future goals include among others, study of extra cases of quark parameters and an effort to connect hybrid star properties with finite nuclei properties.

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