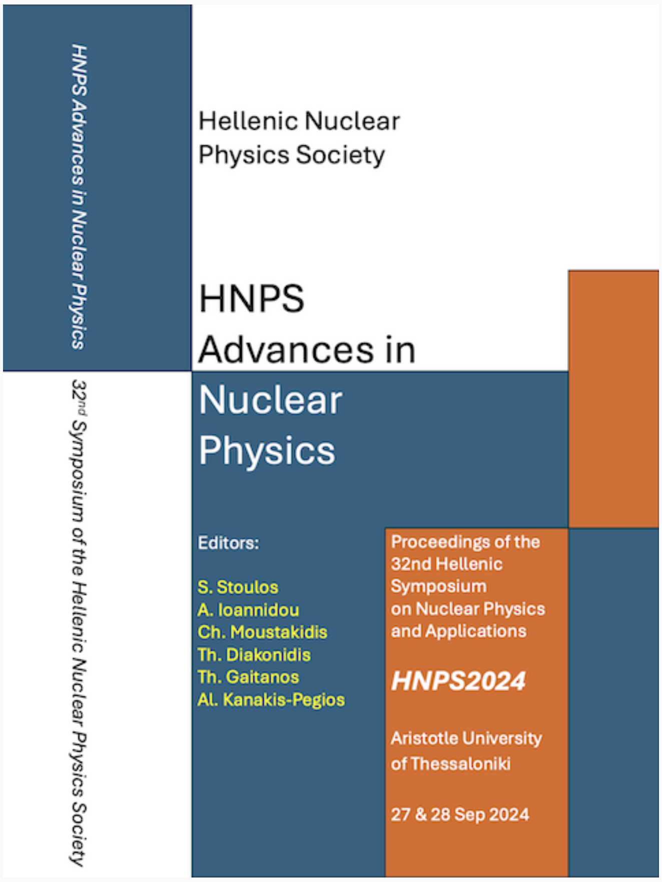
HNPS Advances in Nuclear Physics

Vol 31 (2025)

HNPS2024



Quarkyonic equation of state with momentum-dependent interaction and neutron star structure

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doi: 10.12681/hnpsanp.8159

Quarkyonic equation of state with momentum-dependent interaction and neutron star structure

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Abstract The structure and basic properties of dense nuclear matter still remain one of the open problems of physics. In particular, the composition of the matter that composes neutron stars is under theoretical and experimental investigation. Among the theories that have been proposed, apart from the classical one where the composition is dominated by hadrons, the existence or coexistence of free quark matter is a dominant guess. An approach towards this solution is the phenomenological view according to which the existence of quarkyonic matter plays a dominant role in the construction of the equation of state (EOS). In this paper we propose a phenomenological model for quarkyonic matter, borrowed from corresponding applications in hadronic models, where the interaction in the quarkyonic matter depends not only on the position but also on the momentum of the quarkynions. This consideration, as we demonstrate, can have a dramatic consequence on the shape of the EOS and thus on the properties of neutron stars.

Keywords Quarkyonic matter, neutron stars, equation of state, sound velocity

INTRODUCTION

One of the fundamental problems of physics remains the composition of dense nuclear matter as well as its basic properties both at zero and at finite temperature [1–4]. In particular, the equation of state of neutron star matter is the key quantity to study these objects. In this effort, a key problem that often arises is the inability of the EOSs to predict maximum masses for neutron stars that are compatible with recent observations (well above two solar masses) without simultaneously violating the sound speed causality.

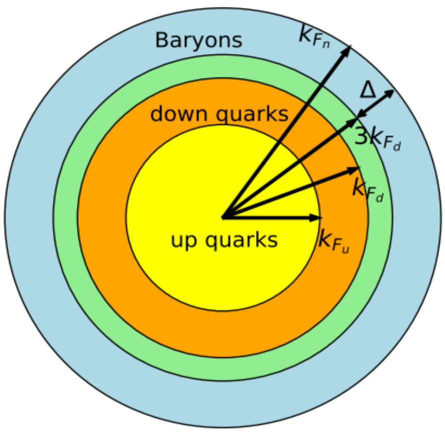

Figure 1. *The momentum space of the quarkyonic matter*

An interesting attempt in this direction is the consideration of a hybrid state of dense nuclear matter called quarkyonic matter. Following the analysis of Ref. [5,6] the basic assumption of quarkyonic matter is that at large Fermi energy, the degrees of freedom inside the Fermi sea may be treated as quarks, and confining forces remain important only near the Fermi surface where nucleons emerge through correlations between quarks. In this case one can consider that quarks, confinement at

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the Fermi surface, occupying a momentum shell of width $\Delta \simeq \Lambda_{QCD}$, produce triplets with spin 1/2, that are the baryons [5,7,8]. To better understand this idea, we illustrate schematically the momentum space in Fig. (1), where we indicate, with different colors, the low and high momentum states which are occupied by quarks and baryons, respectively.

PURE NEUTRON MATTER CASE

We start our calculations with a pure neutron model to compare it with the quarkyonic one. First of all, we have to compute the number density of neutrons which will be in the form,

$$n_n = \int_0^{k_{Fn}} \frac{d^3k}{(2\pi)^3} = \frac{g_s}{6\pi^2} k_{Fn}^3 \tag{1}$$

We obtain the energy density in the following form as,

$$\varepsilon_n = \frac{g_s}{2\pi^2} \int_0^{k_{Fn}} dk\, k^2 \sqrt{(\hbar ck)^2 + m_n^2 c^4} + V(k_{Fn}) \tag{2}$$

where the first term is the kinetic part and is obtained in the relativistic form and $V_{int}(k_{Fn})$ is the potential energy. After that, the chemical potential of neutrons will be given from the familiar thermodynamic relation,

$$\mu_n = \frac{\partial \varepsilon_n}{\partial n_n} \tag{3}$$

and the total pressure will be,

$$P = \mu_n n_n - \varepsilon_n \tag{4}$$

so to construct the equation of state.

As an initial effort, we assume that neutrons interact via a momentum dependent potential in the following form [1],

$$V(k_{Fn}) = \frac{1}{3} A n_s (1 + \chi_0) u^2 + \frac{\frac{2}{3} B n_s (1 - \chi_3) u^{\sigma+1}}{1 + \frac{2}{3} B'^{n_s(1-\chi_3)u^{\sigma-1}}} + u \sum_{i=1,2} \frac{1}{5} [6C_i - 8Z_i] J_n^i \tag{5}$$

where,

$$J_n^i = \frac{2}{(2\pi)^3} \int_0^{k_{Fn}} d^3\, k g(n_n, \Lambda_i) = \frac{2}{(2\pi)^3} \int_0^{k_{Fn}} dk\, 4\pi k^2 \left[1 + \frac{k}{\Lambda_i}\right]^{-1} \tag{6}$$

The parameterization we used for the momentum dependent potential, Eq. (5), is the following: $A=-46.65$, $B=39.45$, $B'=0.3$, $\sigma=1.663$, $C_1=-83.84$, $C_2=23$, $\chi_0=1.654$, $\chi_3=-1.112$, $Z_1=3.81$, $Z_2=13.16$, $\Lambda_1=1.5k_{Fn0}$, $\Lambda_2=3k_{Fn0}$, $u=n_n/n_0$, the saturation density $n_0=0.16$ fm^{-3} and k_{Fn0} is the neutron Fermi momentum at the saturation density.

The first two terms in Eq. (5) are both momentum independent and correspond to an attractive and a repulsive interaction respectively. The last term of the potential energy is momentum dependent, corresponds to an attractive interaction and expresses the finite range interaction forces.

We construct the equation of state for each model and we compute the sound velocity,

$$\frac{c_s}{c} = \sqrt{\frac{\partial P}{\partial \varepsilon}} \tag{7}$$

THE NDU (NEUTRONS, UP AND DOWN QUARKS) QUARQYONIC MODEL WITH MOMENTUM-DEPENDENT INTERACTION

To study quarkyonic matter and its effects in the neutron star properties, we start with a simple model in which we consider a neutron star that consists of only neutrons, up and down quarks, (NDU) model) [5, 9]. Also we assume that neutrons interact via a momentum dependent potential and quarks are asymptotically free. In this case, the equation for charge neutrality will take the simple form,

$$n_d = 2n_u \tag{8}$$

which in terms of the Fermi momentum becomes,

$$k_{Fd} = 2^{1/3} k_{Fu} \tag{9}$$

In this model we assume that the system is not in chemical equilibrium. Instead of this, we consider a simple relation for the Fermi momentum of down quarks and neutrons as in Ref. [5,10], $k_{Fd} = (k_{Fn} - \Delta)/3$, which derived by the basic assumption of the quarkyonic scenario, where Δ is the width of the momentum shell where baryons reside and we take it the form $\Delta = \Lambda_{Qyc}^3 / \hbar^3 c^3 k_{Fn}^2 + \kappa_{Qyc} \Lambda_{Qyc} / \hbar c N_c^2$ Ref. [5].

We set the parameters $\Lambda_{Qyc} \approx \Lambda_{QCD}$ and $\kappa_{Qyc} = 0.3$. Also we set the quark masses to be $m_u = m_d = m_Q = m_n/N_c$, the degeneracy of the spin and the number of colors for quarks to be $g_s = 2$ and $N_c = 3$ respectively. We compute the energy density, the number density and chemical potentials for quarks and neutrons respectively as before,

$$n_Q = \frac{g_S N_c}{2\pi^2} \sum_{i=u,d} \int_0^{k_{Fi}} k^2 \, dk \tag{10}$$

$$\varepsilon_Q = \frac{g_S N_c}{2\pi^2} \sum_{i=u,d} \int_0^{k_{Fi}} k^2 \sqrt{(\hbar c k)^2 + m_Q^2 c^4} dk \tag{11}$$

$$n_n = \frac{g_S}{2\pi^2} \int_{k_{Fn}-\Delta}^{k_{Fn}} k^2 \, dk \tag{12}$$

$$\varepsilon_Q = \frac{g_S}{2\pi^2} \sum_{i=u,d} \int_{k_{Fn}-\Delta}^{k_{Fn}} k^2 \sqrt{(\hbar c k)^2 + m_n^2 c^4} dk + V(n_n, k_{Fn}) \tag{13}$$

The interaction term for neutrons energy density will be in the form of Eq. (5), but in this case, the integration in the momentum space of the term in Eq. (6) is restricted in the momentum shell with the width equal to the parameter Δ.

The total baryon density and total energy density will be,

$$n_B = n_n + \frac{n_u + n_d}{3} = \frac{1}{3\pi^2} \left(k_{Fn}^3 - (k_{Fn} - \Delta)^3 + k_{Fu}^3 + k_{Fd}^3 \right) \tag{14}$$

and the total energy density will be,

$$\varepsilon_{tot} = \varepsilon_n + \varepsilon_Q \tag{15}$$

The chemical potential for each species of matter will be in the form,

$$\mu_i = \frac{\partial \varepsilon_{tot}}{\partial n_i} \tag{16}$$

and the total pressure will be,

$$P = -\varepsilon_{tot} + \sum_{i=n,u,d} \mu_i \, n_i \tag{17}$$

where index i express neutrons, up and down quarks respectively. After that, we compute the sound velocity from relation (7).

RESULTS AND DISCUSSION

The first result we extracted is the speed of sound for each model and for various values of the microscopic parameters. The reason is to see if the equations of state we provided are causal. We investigate several values of the transition density (n_{tr}) as well as for the parameter Λ_{Qyc}. We present these results in Fig. 2.

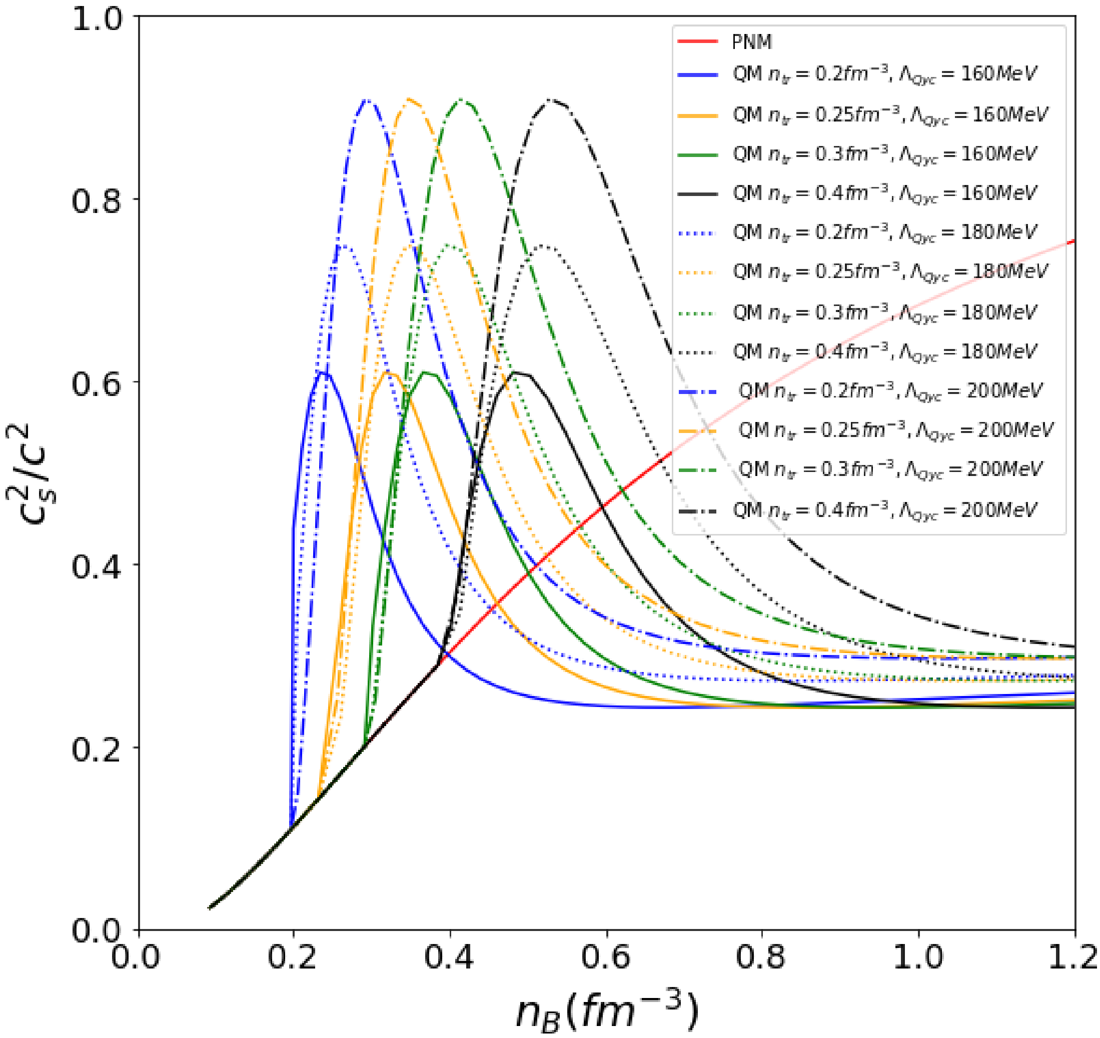

Figure 2. *The sound velocity for the quarkyonic matter (QM) interacting via the momentum-dependent interaction (upper figure) as a function of baryon density for ntr = 0.2, 0.25, 0.3, 0.4fm−3 (blue, yellow, green and black lines respectively) and for ΛQyc = 160, 180, 200 MeV (solid, dotted and dashed-dotted lines respectively). The solid red line corresponds to the pure neutron matter model (PNM).*

It is important to note that for the different values of transition density, the pick in the speed of sound as a function of the baryon density is not affected at all. On the other hand, we can see that as the parameter Λ_{Qyc} increases, the maximum speed of sound also increases, leading to a violation of causality for values $\Lambda_{Qyc} > 210$ MeV.

After the construction of equations of state for the pure neutron case as well as for quarkyonic model, we solve Tolman-Oppenheimer-Volkoff (TOV) equations so to calculate mass, radius, tidal deformability and other bulk quantities of a neutron star.

This system of equations for a static, spherically symmetric neutron star has the following form,

$$\begin{gathered}\frac{dm(r)}{dr}=4\pi r^2\rho(r),\\ \frac{dP(r)}{dr}=\rho(r)c^2\left(1+\frac{P(r)}{\rho(r)c^2}\right)\frac{d\Phi(r)}{dr},\\ \frac{d\Phi(r)}{dr}=\frac{Gm(r)}{c}\left(1+\frac{4P(r)r^3}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}\end{gathered}\tag{18}$$

where P(r) is the total pressure, m(r) is the enclosed mass of the star and ρ(r) is the total mass density. We present the results for mass and radius relations in Fig. (3).

In Fig. 2 we can see the sound velocity as a function of the baryon density. As a general result, the sound speed increasing rapidly until a maximum value at densities 2-3 times greater the nuclear saturation density (0.16 fm^{-3}). After that, when quarkyonic matter appears, it dicreases and assymtotically it converges to the conformal limit ($1/\sqrt{3}$). So quarkyonic matter provides a stiff-soft equation of state and predicts massive neutron stars without violating the causality.

In Fig. 3, we present our results for the mass and radius for a cold neutron star. We assume for transition density the values n_{tr}=0.2, 0.25, 0.3, 0.4 fm^{-3} and for parameter Λ_{Qyc} we set Λ_{Qyc}=160, 180, 200 MeV. Also, we include some recent observational data from LIGO and HESS experiments, so to make a comparison with our results. Our goal is to make some constrains for the transition density and Λ_{Qyc}.

One can see in Fig. 3 that quarkyonic model for n_{tr}=0.3 and 0.4 fm^{-3}, as well as the pure neutron matter model, predict neutron stars with masses around 1.4 solar masses to have radius about 13-13.5 km, which is compatible with the observational data resulting from LIGO. If we set the transition density to be 0.3 fm^{-3} and below, our predictions are far away from the observational data.
Another interesting feature to notice is that for values of transition density near the saturation density and below, the masses and the radius provided are very large, and as we increase the transition density, the quarkyonic model tends to be equivalent to the pure neutron case.

CONCLUSIONS

After this initial effort we can note some interesting features of quarkyonic matter. First of all, quarkyonic matter provides the sound speed as a non-monotonic function of the baryon density, without exceeding the speed of light and it is reaching asymptotically the value $1/\sqrt{3}$ which is the conformal limit. This fact, along with the maximum neutron star masses predicted by quarkyonic equations of state, constitutes an important tool for explaining the properties of dense nuclear matter as well as the bulk properties of compact objects. Also, quarkyonic matter may bridge the gap between hadronic and quark matter and explain a possible phase transition between these two phases.

In this work we achive to constrain some microscopical parameters from recent gravitational wave data as long as from the speed of sound causality. The fisrt one is the transition density, which in our model can’t be lower than 0.3 fm^{-3}. Also the parameter Λ_{qyc} is constrained to be lower than 210 MeV.

In future work we have to extend our model to include protons and electrons to impose β-equilibrium and to apply quarkyonic matter in finite temperature neutron stars. Also we have to investigate if there is any fundamental theory which can provide this state of matter [2, 3, 10]. We expect that future gravitational wave observations from binary neutron star systems will give us information to constrain further some of the microscopic parameters of our model, so that to test and to improve our equations of state.

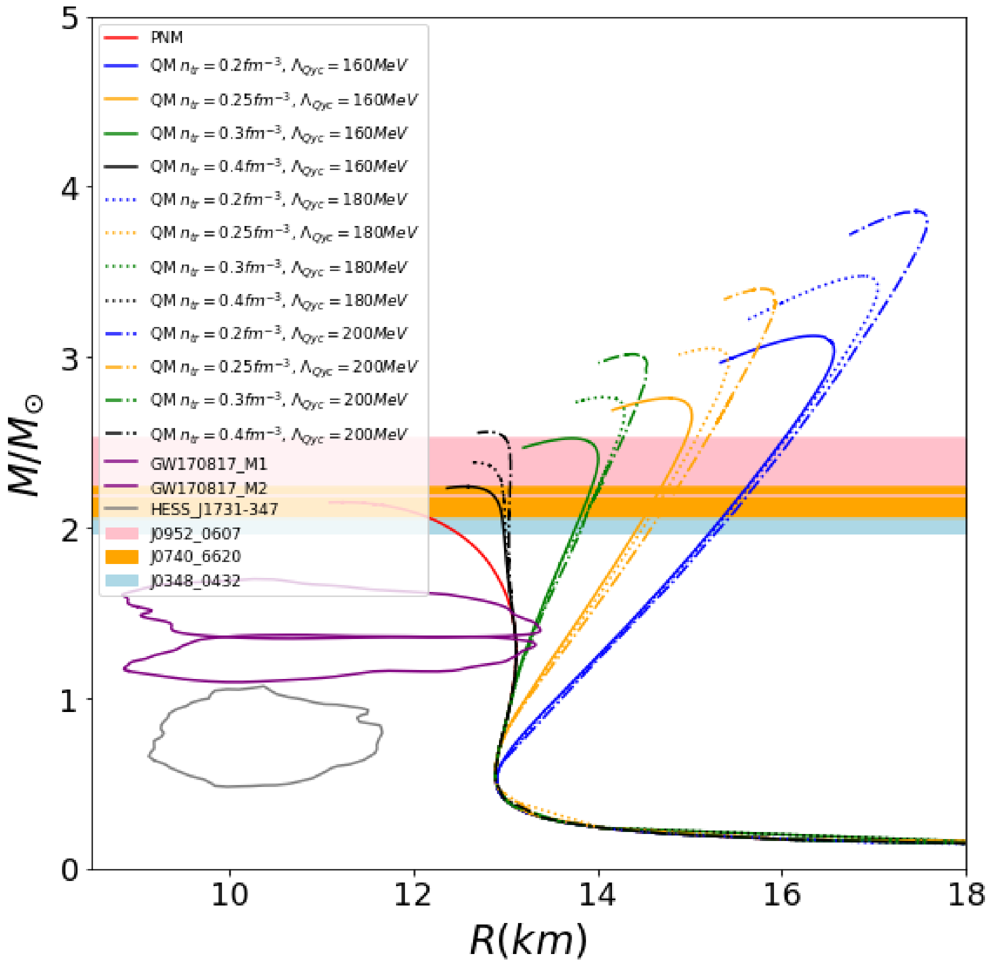

Figure 3. *M-R diagrams for the quarkyonic model (QM) for n_{tr}=0.2, 0.25, 0.3, 0.4 fm^{-3} (blue, yellow, green and black lines, respectively) and for Λ_{Qyc}=160, 180, and 200 MeV (solid, dotted and dashed-dotted lines, respectively). The solid red line corresponds to the pure neutron matter. The shaded regions correspond to possible constraints on the maximum mass from the observation of PSRJ0348+0432, PSR J0740+6620 and PSR J0952+0607 [11–15]. The purple lines correspond to data resulting from LIGO and the grey one corresponds to data from the HESS observation [16].*

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