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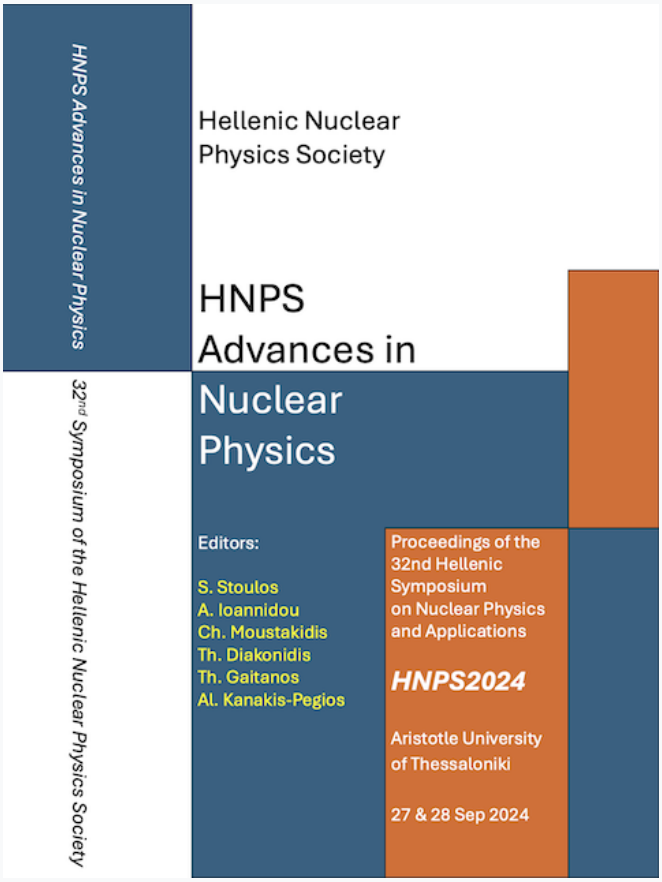

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Pavlos Laskos-Patkos, Georgios A. Lalazissis, Sibo Wang, Jie Meng, Peter Ring, Charalambos C. Moustakidis

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Maximally stiff equations of state and the structure of hybrid stars

P. Laskos-Patkos[1,*], G.A. Lalazissis[1], Sibo Wang[2], Jie Meng[3], Peter Ring[4], Ch.C. Moustakidis[1]

[1] *Department of Theoretical Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece*
[2] *Department of Physics and Chongqing Key Laboratory for Strongly Coupled Physics, Chongqing University, Chongqing 401331, China*
[3] *State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*
[4] *Department of Physics, Technische Universität München, D-85747 Garching, Germany*

Abstract The simultaneous reconciliation of the 1σ estimations on the mass (*M*) and radius (*R*) of the PSR J0030+0451 pulsar and the HESS J1731-347 remnant requires a region in the *M*-*R* plane where the slope *dM/dR* is positive. The latter reflects that the equation of state (EOS) should stiffen as the density increases, which requires a sufficiently large value for the speed of sound in dense matter. However, based on different theoretical assumptions, the sound velocity manifests an upper bound leading to a constraint on how stiff the resulting EOS can be. In the present contribution, we present our recent work on the possible simultaneous explanation of the aforementioned measurements in the context of hybrid EOSs, by describing the high-density phase as maximally incompressible (considering different suggestions for the upper speed of sound bound). A relevant discussion on possible constraints on the properties of first order transitions is also included.

INTRODUCTION

Compact stars are considered to be exceptional extraterrestrial laboratories for the physics of strongly interacting systems [1]. Given the fact that different nuclear models predict distinct values for bulk stellar properties, one may utilize precise astronomical measurements to impose constraints on the nuclear equation of state (EOS).

The first simultaneous mass and radius measurement for an isolated (not in a binary system) neutron star came by the NICER mission. In particular, in 2019, Riley *et al.* [2], and independently Miller *et al.* [3], suggested that the PSR J0030+0451 has a mass M~1.3-1.4 M_{sun} and a radius R~13 km. Since then, different theoretical studies have attempted to provide some insight on the properties of dense nuclear matter by exploiting the aforementioned estimation. One important result included the possible elimination of strong phase transitions below ~1.7n_0, which would allow for the existence of twin stars with significantly different radii at low masses [4].

In 2022, the analysis of Doroshenko *et al.* [5] on the mass and radius of the central compact object in the HESS J1731-347 remnant provided rather puzzling values. In particular, the authors suggested that M ~0.7M_{sun} and R ~ 10 km, which are significantly lower compared to values previously inferred from the study of millisecond pulsars. As a consequence, a lot of studies have been devoted to examining the implications of such a strangely light compact object. In that direction, several different assumptions for the nature of ultra-dense matter have been adopted. Those included the absolute stability of strange quark matter [6-8], the appearance of first-order phase transitions [9-10] and the possible presence of dark matter [9]. Of course, the explanation of the HESS J1731-347 properties with the consideration of ordinary nuclear matter has not been ruled out [11-13], but the low radius indicates a soft EOS at low density, characterized by a low symmetry slope.

Interestingly, as indicated in the study of Kubis *et al.* [13], the simultaneous explanation of HESS J1731-347 and PSR J0030+0451 would require a "Z-like" shape for the mass-radius dependence of neutron stars. This is of utmost importance, as it indicates that the nuclear EOS should encounter

* Corresponding author: plaskos@physics.auth.gr

significant stiffening as the density increases. Notably, the stiffening of an EOS is directly controlled by the speed of sound (c_s). Given that there is an upper bound on the speed of sound, then there is also a bound on how stiff an EOS can be. Therefore, it is particularly interesting to examine how the consideration of distinct, theoretically motivated speed of sound constraints may impact the potential simultaneous explanation of the aforementioned measurements.

In the present conference contribution, we first discuss our recent results on the possible reconciliation of HESS J1731-347 and PSR J0030+0451, by imposing the speed of sound bounds related to causality and conformality. In particular, according to the special theory of relativity the sound velocity does not exceed the speed of light. However, there have been suggestions that the upper limit on the speed of sound could be significantly lower [14]. According to Bedaque and Steiner [15], there are several arguments indicating that $c_s^2 < c^2/3$ in the interior of compact stars. Secondly, we conduct a relevant discussion on the possible existence of twin star configurations in light of the considered astronomical constraints. For more details and additional results, we refer to the original work [16].

CONSTANT SPEED OF SOUND PARAMETRIZATION

For the construction of hybrid EOSs with a maximally stiff high-density phase, one has to rely on the widely employed constant speed of sound parametrization. In the latter framework, and under the assumption of a large surface tension between the two phases (Maxwell construction), the energy density will be given as [1]

$$\varepsilon(p) = \begin{cases} \varepsilon_h(p_{tr}), & p \leqslant p_{tr} \\ \varepsilon_h(p_{tr}) + \Delta\varepsilon + \left(\frac{c_s}{c}\right)^{-2}(p - p_{tr}), & p > p_{tr} \end{cases} \tag{1}$$

In Eq. (1) the first line stands for the EOS of hadronic matter, while the second line indicates the maximally stiff description of the high-density phase. In the present contribution, we considered the speed of sound bounds related to special theory of relativity and conformality. Therefore, we constructed two families of hybrid EOSs with the: a) causality limit $c_s \leq c$ and b) conformal limit $c_s \leq c/\sqrt{3}$.

The consideration of a highly sophisticated nuclear model is of utmost importance as the constraints that we aim to derive on the properties of phase transitions will be subject to uncertainties related to the description of the low-density phase. Therefore, for hadronic matter we anchored our analysis on a state-of-the-art relativistic, microscopic model. In particular, we relied on a recently derived self-consistent relativistic Brueckner-Hartree-Fock (RBHF) calculation in full Dirac space (which means that no approximations were made for the neglection of negative energy states in the single-particle spectrum) [17-18]. The realistic nuclear potential employed was BonnA. The properties of neutron stars produced by the employed hadronic model can be found in Ref. [18]. For more details regarding the application of the RBHF theory for the study of compact star properties the reader is referred to Refs. [19-24].

RESULTS AND DISCUSSION

Our main goal was the extraction of constraints on the properties of first-order phase transitions. First, we constructed a large set of hybrid EOSs by varying the values of the transition density and energy density jump in the ranges [0.16,0.40] fm^{-3} and [0,350] MeV fm^{-3}, respectively. Then, we solved the well-known Tolman-Oppenheimer-Volkoff equations to derive the stellar properties predicted by each of the considered EOSs. Finally, we tested which of the hybrid models satisfied the constraints related to PSR J0030+0451 and HESS J1731-347 at the 1σ level. Of course, any reasonable model should be in accordance with the existence of massive compact stars. Therefore, we also examined whether the derived EOSs satisfy the conservative $2M_{sun}$ maximum mass constraint.

In Fig. 1 we depicted the parameter values for which different astronomical measurements are reconciled. The left panel includes the results that were derived by considering the special relativity sound velocity constraint, while the right panel stands for the case where the conformal limit was imposed. The blue region shows the parametrizations that yield results compactible with the existence of PSR J0030+0451. The green regime illustrates the values for which the HESS J171-347 constraint is satisfied. Last, the orange area indicates the overlap between the blue and the green regions, suggesting that both of the aforementioned constraints are met. Note that all of the depicted parametrizations are compatible with the existence of compact stars that exceed the two solar masses.

As one can observe in Fig. 1a, the simultaneous reconciliation of both PSR J0030+0451 and HESS J1731-347 constraints requires a transition density below ~0.28 fm^{-3} and an energy density jump lower than ~160 MeV fm^{-3}. Notably, the derived upper bound on $\Delta\varepsilon$ is quite general. This is fairly simple to understand as, for a given transition density, the largest possible energy density jump is imposed by the lowest estimation on the radius of PSR J0030+0451. Since any softer description of the high-density phase (compared to maximal stiff one employed here) would produce compact stars with lower radii (for the same stellar mass), one may deduce that the explanation of the PSR J0030+0451 constraints for larger density jump values is impossible (see Ref. [16] for more details). On the other hand, the upper bound on the transition density is not as general since it depends on the lower energy density jump values which yield results compatible to HESS J1731-347. However, this lower bound is dictated by the higher radius estimation for HESS J1731-347. Hence, the use of a softer EOS (compared to the maximally stiff one) could potentially allow for EOSs that are able to produce low radius values even under the consideration of a smaller energy density jump. However, one needs to keep in mind that reducing the considered speed of sound may significantly impact the resulting maximum mass prediction.

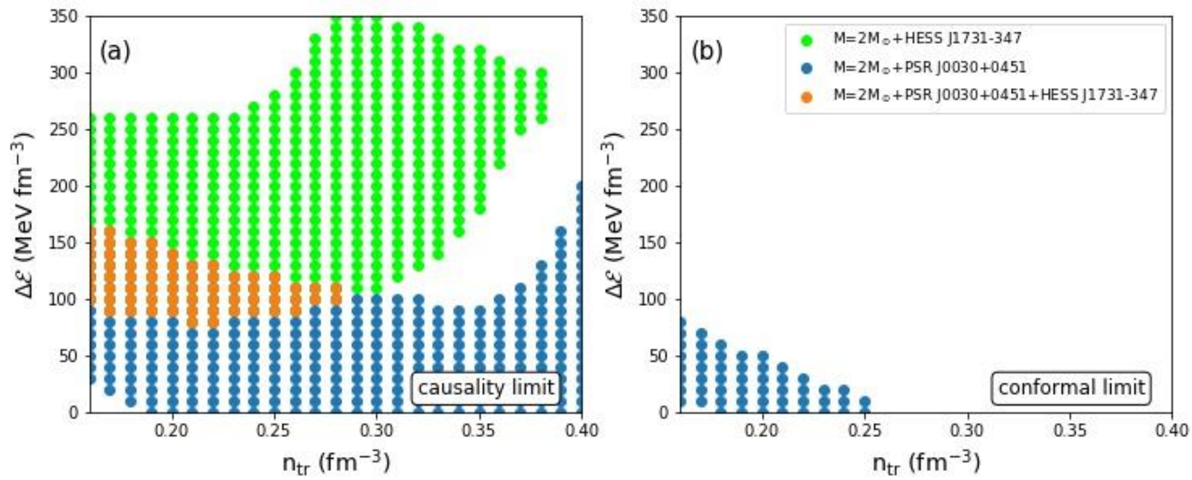

Figure 1. *$\Delta\varepsilon$-n_{tr} values that are compatible to the considered observations. The blue (green) dots indicate the parameter values that are compatible to PSR J0030+0451 (HESS J1731-347). The orange dots denote the region where both PSR J0030+0451 and HESS J1731-347 are satisfied. All of the depicted parameter sets are compatible to the $2M_{sun}$ maximum mass constraint. Panel (a) stands for the case where the maximal speed of sound is considered to be the speed of light, while in panel (b) the conformal limit is imposed. Taken from Ref. [16].*

As is evident from Fig. 1b, the conformal constraint is inconsistent with the existence of the central compact object in the HESS J1731-347 supernova remnant. That is related to the fact that, for the given realistic hadronic model, the reconciliation of the HESS J1731-347 constraint requires a nonzero value for the energy density discontinuity. However, the latter has a significant impact on the resulting

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maximum mass prediction. As a consequence, the derived EOSs do not align with the $2M_{sun}$ constraint and, therefore, they are ruled out.

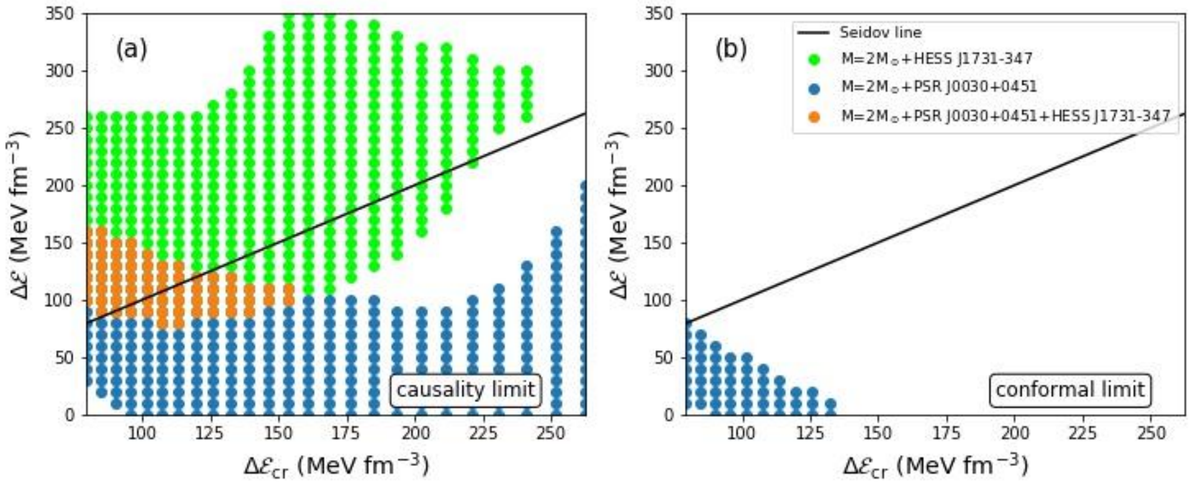

Figure 2. *The viable energy density jump values (found in Fig. 1) as a function of the corresponding critical energy density jump indicated by Seidov [25]. Panel (a) stands for the case the maximal speed of sound is considered to be the speed of light, while in panel (b) the conformal limit is imposed.*

It is particularly intersting to examine further implications of the simultaneous reconciliation of the aformentioned measurements. More precisely, we aimed to gain some insight on the possible existence of twin stars in view of the HESS J1731-347 and PSR J0030+0451 constraints. Therefore, in Figure 2 we plotted the viable (compatible to observation) energy density jump values as a function of the so-called critical energy density jump. This critical density discontinuity is given by the following expression [25]

$$\Delta\varepsilon_{cr} = \frac{1}{2}\varepsilon_{tr} + \frac{3}{2}p_{tr} \qquad (2)$$

and it sets an approximate benchmark for the lowest $\Delta\varepsilon$ value needed for the appearance of a descending branch in the mass-radius diagram (appearance of twin star configurations). In addition, the plot includes a straight line indicating the values at which $\Delta\varepsilon = \Delta\varepsilon_{cr}$ (marked as Seidov line). Note that, as PSR J0030+0451 sets the upper bound on $\Delta\varepsilon$ it also determines the possible existence of twins. We observed that, in the case where $c_s = c$, about half of the parameter values of the constructed configurations are characterized by an energy density jump that exceeds the aforementioned critical value. In addition, the lower the transition density, the wider the range of energy density jumps which are above the critical line. However, it should be clearly stated that transition densities close to nuclear saturation are not realistic as nuclei are known to be the stable form of matter at that density region. In the case where the sound velocity was bound by the conformal constraint none of the configurations that were compatible to PSR J0030+0451 were found to potentially support twin star solutions.

CONCLUSIONS

In the present contribution, we presented our recent results on the possible reconciliation of HESS J1731-347 and PSR J0030+0451 at the 1σ level. The main idea is that the simultaneous reconciliation of these two measurements requires stiffening of the EOS. Therefore, we used two different theoretically motivated speed of sound bounds to investigate how altering the maximal possible stiffness of the EOS may impact the explanation of the considered astronomical constraints. A key finding was that the energy density jump is constrained below ~160 MeV fm^{-3}. In addition, we compared the viable (compatible to observations) energy density jump values with the corresponding critical energy density jump for each of the constructed hybrid EOSs, in an attempt of gaining some insight about the possible occurrence of twin star solutions in view of HESS J1731-347 and PSR J0030+0451. We found that, for $c_s = c$, the existence of compact stars with identical mass and different radii is possible but it would require a rather early phase transition.

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References

[1] J. Schaffner-Bielich, *Compact Star Physics* (Cambridge University Press, Cambridge, England, 2020); doi: 10.1017/9781316848357
[2] T.E. Riley, et al., Astrophys. J. Lett. 887, L21 (2019); doi: 10.3847/2041-8213/ab481c
[3] M.C. Miller, et al., Astrophys. J. Lett. 887, L24 (2019); doi:10.3847/2041-8213/ab50c5
[4] J.-E. Christian and J. Schaffner-Bielich, Astrophys. J. Lett. 894, L8 (2020); doi: 10.3847/2041-8213/ab8af4
[5] V. Doroshenko, et al., Nat. Astron. 6, 1444 (2022); doi: 10.1038/s41550-022-01800-1
[6] F. Di Clemente, et al., Astrophys. J. 967, 159 (2024); doi: 10.3847/1538-4357/ad445b
[7] J.E. Horvath et al., Astron. Astrophys. 672, L11 (2023); doi: 10.1051/0004-6361/202345885
[8] P.T. Oikonomou and Ch.C. Moustakidis, Phys. Rev. D 108, 063010 (2023); doi: 10.1103/PhysRevD.108.063010
[9] V. Sagun, et al., Astrophys. J. 958, 49 (2023); doi: 10.3847/1538-4357/acfc9e
[10] P. Laskos-Patkos, et al., Phys. Rev. D 109, 063017 (2024); doi: 10.1103/PhysRevD.109.063017
[11] L. Brodie and A. Haber, Phys. Rev. C 108, 025806 (2023); doi: 10.1103/PhysRevC.108.025806
[12] J.J. Li and A. Sedrakian, Phys. Lett. B 844, 138062 (2023); doi: 10.1016/j.physletb.2023.138062
[13] S. Kubis, et al., Phys. Rev. C 108, 045803 (2023); doi: 10.1103/PhysRevC.108.045803
[14] J.M. Lattimer, *Astrophysics and Cosmology, in Proceedings of the 26th Solvay Conference on Physics, edited by R. Blandford, D. Gross, and A. Sevrin* (World Scientific, Singapore, 2014).
[15] P. Bedaque and A.W. Steiner, Phys. Rev. Lett. 114, 031103 (2015); doi: 10.1103/PhysRevLett.114.031103
[16] P. Laskos-Patkos, et al., Phys. Rev. C 111, 025801 (2025); doi: 10.1103/PhysRevC.111.025801
[17] S. Wang, et al., Phys. Rev. C 103, 054319 (2021); doi: 10.1103/PhysRevC.103.054319
[18] S. Wang, et al., Phys. Rev. C 106, L021305 (2022); doi: 10.1103/PhysRevC.106.L021305
[19] H. Tong, et al., Astrophys. J. 930, 137 (2022); doi: 10.3847/1538-4357/ac65fc
[20] S. Wang, et al., Phys. Rev. C 106, 045804 (2022); doi: 10.1103/PhysRevC.106.045804
[21] X. Qu, et al., Sci. China Phys. Mech. Astron. 66, 242011 (2023); doi: 10.1007/s11433-022-2048-3
[22] P. Qin, et al., Phys. Rev. D 107, 103009 (2023); doi: 10.1103/PhysRevD.107.103009
[23] D. Farrell and F. Weber, Astrophys. J. 969, 49 (2024); doi: 10.3847/1538-4357/ad47b8
[24] X. Qu, et al., Astrophys. J. 980, 3 (2025); doi: 10.3847/1538-4357/ada76b
[25] Z.F. Seidov, Sov. Astron. 15, 347 (1971).