Momentum dependence of in-medium potentials: A solution to the hyperon puzzle in neutron stars

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Abstract   Neutron stars offer a great opportunity to study highly compressed hadronic matter experimentally and theoretically. However, the so-called hyperon-puzzle arises at neutron star densities. The hyperon coexistence with other particles in compressed matter softens the equation of state and many widely accepted models fail to reproduce precise observations of large neutron star masses. Here, we propose a mechanism to retain the stiffness of the high-density state with hyperons by considering the explicit momentum dependence of their in-medium potentials. Our approach modifies conventional strangeness threshold conditions and generates threshold effects on hyperons in high-density matter. We demonstrate these effects within the nonlinear derivative model, which incorporates baryon momentum-dependent fields based on empirical and microscopic studies. It turns out that even soft momentum-dependent strangeness fields do prohibit their populations in neutron star matter. The generic momentum dependence of strangeness potentials, as modeled by the nonlinear derivative approach, is crucial for resolving the long-standing hyperon-puzzle in neutron stars.

Keywords   neutron stars, hyperon puzzle, momentum dependent potentials

INTRODUCTION

Last decades observations of neutron stars (NS) of mass above $2M_\odot$ [1-4] brought the necessity of a review on the models that describe nuclear matter (NM) in the interior of these compact stars. In high density matter it is energetically allowed for hyperons (baryons with strangeness content) to be produced and live along with the nucleons. The problem arises from the fact that various nuclear matter approaches cannot predict large NS masses when they include hyperons [5,6], and this is what we call "the hyperon puzzle". The importance of solving the hyperon puzzle is not only an astrophysical issue, but it may help us gain a better understanding of elementary particles' interactions.

Describing NM in a NS is not a trivial problem. Inasmuch as various microscopic approaches predict an explicit momentum dependence (MD) of in-medium hyperon potentials and taking into consideration the success of Non-Linear Derivative (NLD) model in the description of NM systems [7-9,12], we take this model that incorporates the particle's MD and extend it to β-equilibrated matter with strangeness degrees of freedom. The MD of hyperon potentials in the NLD model change their threshold conditions [13].

THE NLD MODEL

The NLD model [7-9,12] is based on the conventional Quantum Hadrodynamics (QHD) formalism and is performed in a Relativistic Mean Field (RMF) context. Its Lagrangian density is the sum of the free Lagrangians for the baryons and exchange mesons, and the interaction Lagrangian:

$$\mathcal{L}_{NLD} = \sum_b \mathcal{L}_b + \sum_m \mathcal{L}_m + \sum_m \mathcal{L}_{int}^m \tag{1}$$

where $b = p, n, hyperons\ (Y)$, $m = \sigma, \omega, \rho$.

The crucial difference is that the interaction Lagrangian incorporates non-linear higher-order

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derivative operators D_b acting on the baryon spinor fields:

$$\mathcal{L}_{int}^m = \sum_b \frac{g_{mb}}{2} \left[\overline{\Psi}_b \overleftarrow{D}_b \Gamma_m \Psi_b \phi_m + \phi_m \overline{\Psi}_b \Gamma_b \overrightarrow{D}_b \Psi_b \right] \tag{2}$$

The operators D_b regulate the MD of all in-medium baryon potentials [7]. We choose them to be of a generic monopole-like form: $D_b(p) = \frac{\Lambda_1{}^2}{\Lambda_2{}^2 + p^2}$, where Λ_1 & Λ_2 are cut-off parameters.

Following the generalized Euler-Lagrange formalism, we end up to the baryons Dirac equations:

$$\left[\gamma_\mu \left(p^m - V_b^\mu(p) \right) - m_b^*(p) \right] u_b(p) = 0 \tag{3}$$

with effective baryon mass: $m_b^*(p) = m_b - S_b(p)$,

vector & scalar self-energies: $V_b^\mu(p) = g_{\omega b}\omega^\mu D_b^\omega(p) + \tau_{3b} g_{\rho b}\rho^b D_b^\rho(p)$ & $S_b(p) = g_{\sigma b}\sigma D_b^\sigma(p)$,

and to the meson-field equations:

$$m_\sigma{}^2 \sigma + \frac{\partial U}{\partial \sigma} = \sum_b g_{\sigma b} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \le p_{F_b}} d^3p \frac{m_b^*}{E_b^*} D_b^\sigma(p) \tag{4}$$

$$m_\omega{}^2 \omega = \sum_b g_{\omega b} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}| \le p_{F_b}} d^3p D_b^\omega(p) \tag{5}$$

where $U(\sigma)$ is the self-interaction sigma term and the baryon energy $E_b^*(p) = \sqrt{m^{*2}(p) + p^2}$ (p_{F_b}: the baryon Fermi momentum). The various parameters of the model are fixed by the empirical values at saturation. As for the hyperon sector, the couplings and the strangeness cut-offs are calculated by fitting our results to results from microscopic calculations based on the chiral effective field (χ-EFT) theory [10]. Emphasis should be laid on the MD of self-energies and of the source terms in meson-fields equations. As a result, a particle's in-medium energy will not be any more a monotonic function of momentum (this happens in conventional models, where self-energies are not functions of momentum)

$$E_b(p) = \sqrt{\left(m_b - S_b(p) \right)^2 + p^2} + V_b^0(p) \tag{6}$$

a feature that changes the conventional strangeness thresholds.

Taking into consideration the available experimental information, we focus on the single-strangeness sector.

The NLD model for NS

To apply all the previous approach to the NS matter, we assume that NS consist of protons, neutrons, electrons and Λ & Σ hyperons, and we employ β-equilibrium. This theoretical context is expressed by the following three conditions [11]

1. conservation of baryon density: $\rho_B = \sum_b \rho_b$
2. charge neutrality: $\sum_b q_b \rho_b - \rho_e = 0$ (q_b: the baryon charge)
3. β-equilibrium: $\mu_b = \mu_n - q_b \mu_e$ (μ: the chemical potentials for the various types of particles) with $\mu_b = \sqrt{p^2 + [m_b - S_b(p)]^2} + V_b^0(p)$ at $p = p_{F_b}$.

For the β-equilibrium condition the procedure is: at a given baryon density the $E_b(p)$ (Eq. 6) plot should cross the hyperon (Y) threshold $\mu_b = \mu_n - q_b \mu_e$ at p_{F_b}, that is to have a solution of $\mu_Y = \mu_n - q_b \mu_e = E_Y(p_{F_Y})$. That conventionally requires that at $p = 0$, $E_Y(0) < \mu_Y = \mu_n - q_b \mu_e$. If that holds, hyperon must be populated and in conventional models, where $E_b(p)$ is a monotonic function (no MD of self-energies etc.), this is a trivial procedure. Nevertheless, in NLD context, where the

hyperon energy is not a monotonic function of momentum, the picture is different. In Fig. 1(a-c) some relevant cases are shown, where the threshold for hyperon appearance is obviously completely changed.
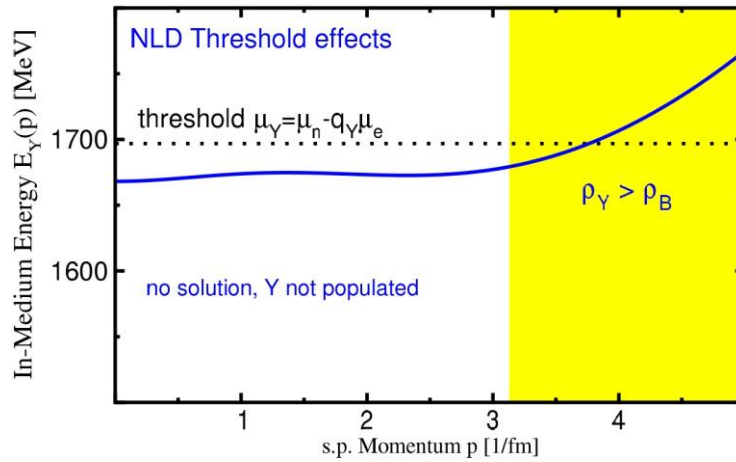

Figure 1a. *Strangeness threshold effects in NLD model.*
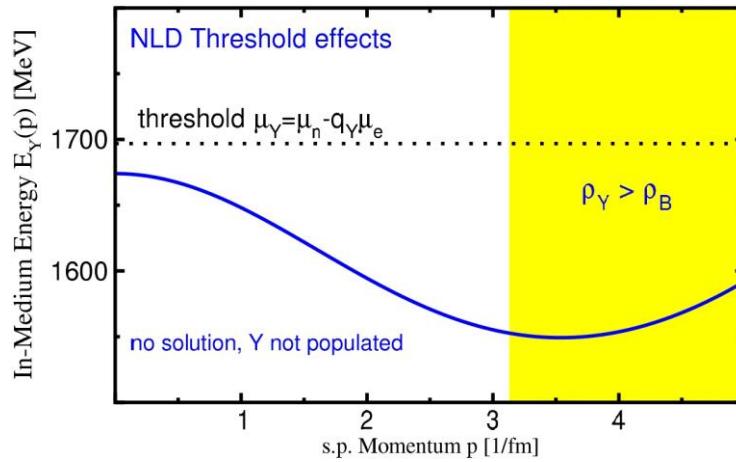*Case I: threshold satisfied but there is no solution (yellow area is forbidden).*

Figure 1b. *Strangeness threshold effects in NLD model.*
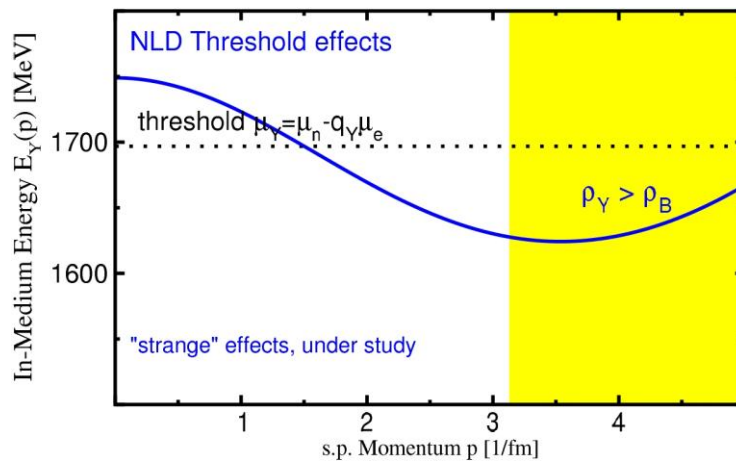*Case II: threshold satisfied but there is no solution.*

Figure 1c. *Strangeness threshold effects in NLD model.*
Case III: threshold is not satisfied, but a solution shows up (interpretation under study).

The NLD results

A key quantity that helps us to comprehend the strangeness threshold conditions in β-equilibrium is the Schrödinger-equivalent optical potential (its real part)

$$U_{opt}^b = -S_b + \frac{E_b}{m_b} V_b + \frac{1}{2m_b}\left(S_b{}^2 - V_b{}^2\right).$$

It describes the hadronic mean-field felt by the baryon b with momentum $p = |\vec{p}|$ relative to the hadronic matter at rest at a given baryon density ρ_B. The results we take for Λ and Σ^- hyperons are shown in Fig. 2. As ρ_B increases, the Λ potential exhibits the expected repulsive character for all momentum values, but the MD becomes softer.
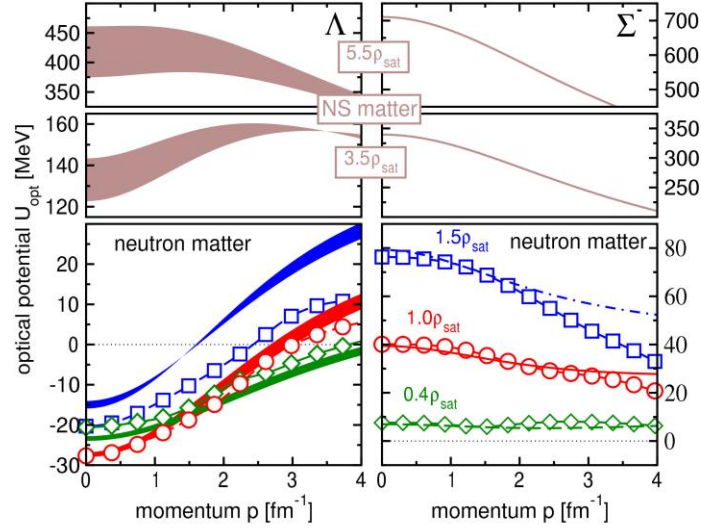

Figure 2. *Optical potential of Λ and Σ^- hyperons as a function of their momentum for various densities of nuclear matter (pure neutron matter). Lines & bands for NLD model, while squares circles and diamonds for χ-EFT model. Our results are adjusted to χ-EFT results only at $\rho=\rho_{sat}$.*

The non-trivial dependence of the hyperon potentials on both the ρ_B & the momentum is manifested in the in-medium energies in Fig.3. For Λ hyperons, the stiff MD of the in medium energy

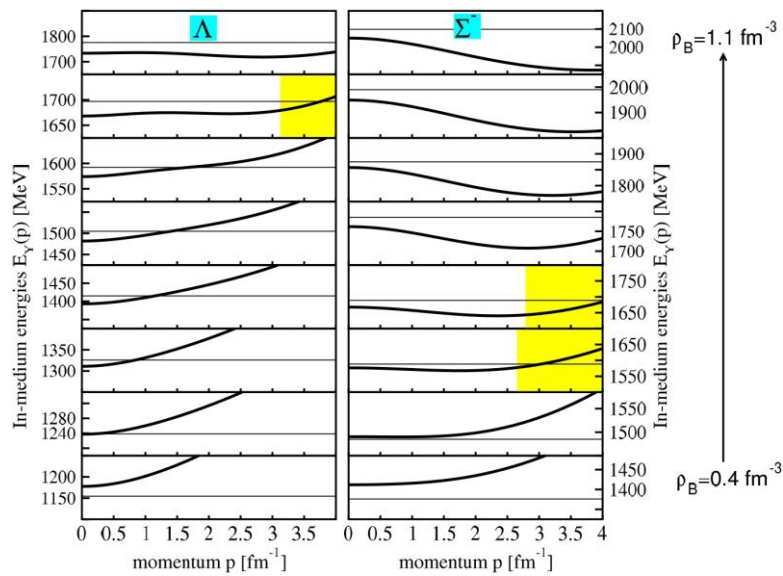**Figure 3.** *NLD hyperons threshold effects at relevant NS densities ($\rho_b = 0.4 fm^{-3}$ up to $\rho_b = 1.1 fm^{-3}$ in steps of $0.1 fm^{-3}$). The thin lines correspond to the threshold $\mu_n - q_Y \mu_e$. The shaded areas are forbidden regions: $p_Y > p_F$ at a given ρ_B.*

$E_Y(p)$ at the low-density area allow the threshold to be crossed at momenta below the Fermi-momentum of the given baryon density, and thus they can be populated. For Σ^- hyperons, the repulsive behaviour of the chemical potential does not permit their population in the same area. But as the density increases, the picture is different. For Σ^- hyperons, as the U_{opt} MD becomes soft, the $E_Y(p)$ does not exceed the corresponding threshold at momenta below the Fermi-momentum of that ρ_B, even if the threshold is fulfilled at vanishing momentum, meaning that they cannot be populated. For Λ hyperons, the more attractive nature along with the stiff MD of the $E_Y(p)$ let them to be populated, but as the density goes even higher, their production is prevented (in some cases the $E_Y(p)$ does surpass the threshold line, but only at momenta higher than the allowed maximum value of the Fermi-momentum, so the hyperons production is not possible there-yellow areas in Figs. 1 & 3).

The results from the above analysis for the various particle fractions are shown in Fig. 4 [12] at the top left side, while for comparison purposes we have the corresponding results from a conventional relativistic mean field (RMF) model on the right side. The RMF model named NLρ has been chosen due to some similarities between the two models: a similar NS EoS for nucleons and comparable hyperon potentials at low momenta. At the bottom the EoS for both models are presented. Typical softening of the EoS when hyperons are included that happens in RMF models, is not manifested in NLD. This feature allows a prediction of maximum mass for a NS at around 2.05M$_\odot$ - a value consistent with observations that conventional RMF models cannot predict. The relation between the mass M and the radius R that can be calculated for the NLD model is shown in Fig. 5.
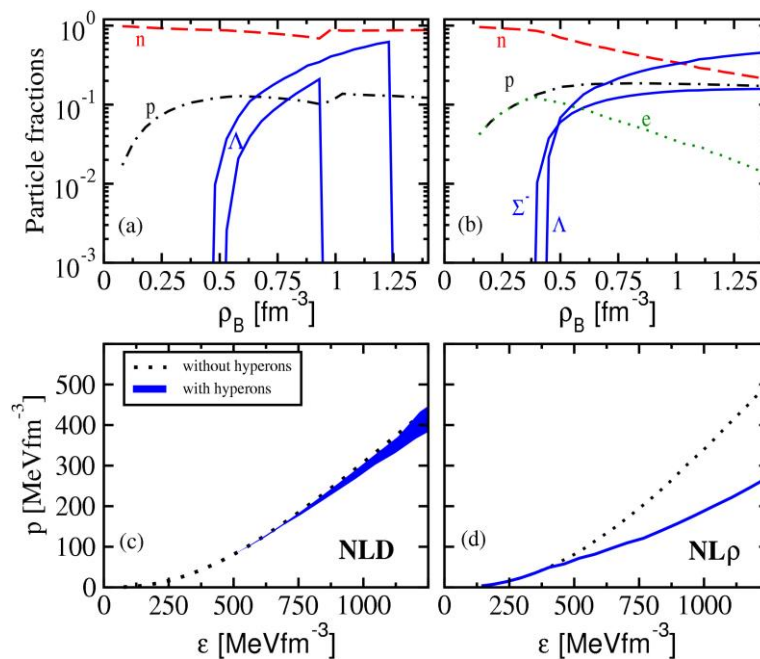

Figure 4. *Particle fractions (top) and EoS (bottom) from NLD calculations for NS matter with Λ & Σ hyperons (left) VS NLρ (a conventional RMF) model (right). Notice that in NLD model Σ hyperon is not populated and Λ hyperon may exist in a limited density region. The NLD EoS is not softened with hyperons presence, as it happens in conventional models.*
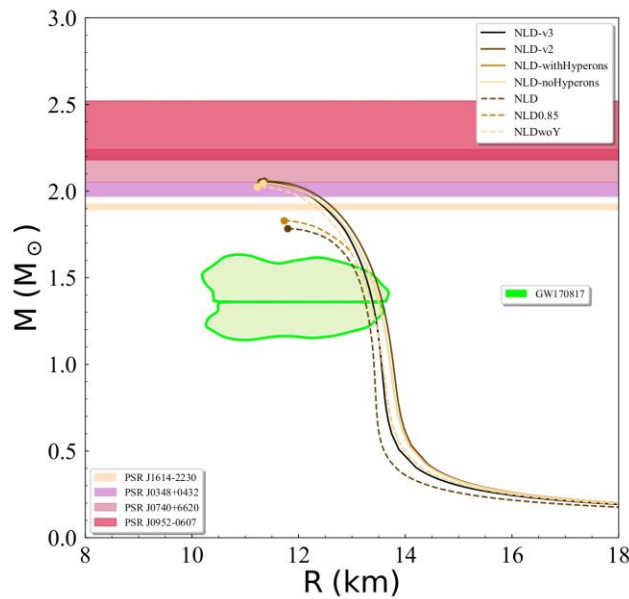
Figure 5. *M-R diagram from NLD calculations for NS matter including Λ & Σ hyperons (with A. Kanakis-Pegios & Ch. Moustakidis)*

CONCLUSIONS AND OUTLOOK

Our proposed solution to NS hyperon-puzzle, that is based on the in-medium strangeness MD, describes successfully the non-trivial features of empirical and microscopic baryon in-medium optical potentials and is applied appropriately to NS matter with hyperons. The hyperons thresholds are modified, and hyperons population is blocked even in some cases when the threshold condition is satisfied. Based on the MD of the microscopic χ-EFT calculations, the NLD model predicts NS matter with only low Λ-hyperon fractions in a restricted density region, resulting in a stiff NS EoS, that can predict masses up to $2.05 M_\odot$ for NS.

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