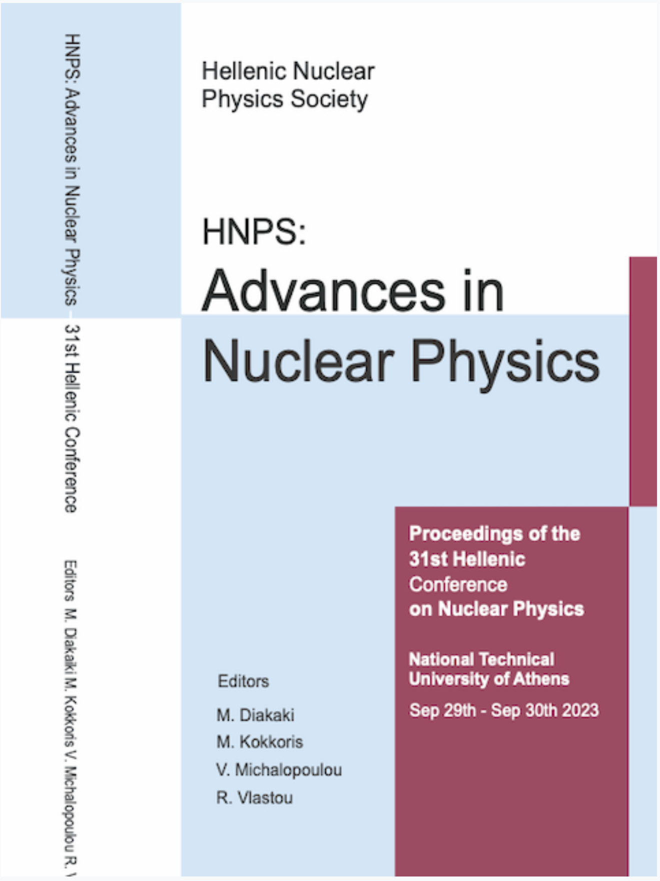
HNPS Advances in Nuclear Physics

Vol 30 (2024)

HNPS2023



Signatures of hadron-quark phase transition through the r-mode instability in twin stars

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doi: 10.12681/hnpsanp.6258

Signatures of hadron-quark phase transition through the *r*-mode instability in twin stars

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Abstract The observation of two compact stars with equal mass but different radius (twin stars) would be a strong indication of hadron-quark phase transition in dense nuclear matter. In this work we examine the differences that appear in the *r*-mode instability windows and evolution of twin stars to investigate the possibility of identifying them through the future detection of gravitational waves. We find that two stars with an identical mass and fairly similar temperature and rotational frequency may behave differently with respect to the *r*-modes. Hence, the future possible detection of gravitational radiation due to unstable *r*-modes from a star appearing in the stable region of the frequency-temperature plane (which will be determined by the absence of gravitation wave emission from existing and future pulsar observations) would be a clear sign for the existence of twin stars. In addition, we examine the compatibility of current data from low mass x-ray binaries with the predictions from hybrid equations of state that support the existence of third stable branch of compact objects. We find that, depending on the energy density jump and the crust elasticity, hybrid equations of state may serve as a viable solution for the explanation of existing observations.

INTRODUCTION

Neutron stars are considered to be unique extraterrestrial laboratories for the study of dense nuclear matter [1]. One of the most interesting unresolved questions, regarding the physics of ultra-dense strongly interacting systems, concerns the relevant degrees of freedom in the interior of compact stars. In particular, compact stars could be purely hadronic (composed of neutrons, protons and hyperons) but the very dense environment may allow for the existence of exotic forms of matter such as deconfined quarks [2-3]. The latter case gives rise to the possible existence of strange quark stars, which are purely composed of strange quark matter, or hybrid stars, where a quark matter core is surrounded by an outer layer of hadronic matter.

The possible identification of hybrid stars is a rather difficult task, as the theoretically predicted value for their radii is quite similar to the one of purely hadronic stars [1,4]. Therefore, one would have to rely on signature predictions of hybrid equations of state to identify the existence of hybrid compact stars. Such a prediction is the existence of twin star solutions [5-7]. More precisely, if the density discontinuity due to a first order phase transition, between hadronic and quark matter, is wide enough then (depending on the speed of sound profile in quark matter) two stars with an identical mass but a different radius could potentially exist. Interestingly, verifying the existence of twin star configurations would be the smoking gun evidence of hadron-quark phase transition in compact stars.

Motivated by several studies [8-10] examining the differences in the bulk properties of twin stars, in this contribution we present the deviations that may appear in their *r*-mode instability windows (for more details see the original work of Ref. [11]), as such an approach could potentially allow for their future distinction.

EQUATION OF STATE

To derive hybrid equations of state that predict twin star solutions we employed the well-known constant speed of sound parametrization [1,12]. In particular, under Maxwell construction, the energy

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density reads

$$\varepsilon(p) = \begin{cases} \varepsilon_h(p_{tr}), & p \leqslant p_{tr} \\ \varepsilon_h(p_{tr}) + \Delta\varepsilon + \left(\dfrac{c_s}{c}\right)^{-2}(p - p_{tr}), & p > p_{tr} \end{cases} \tag{1}$$

where c_s, $\Delta\varepsilon$ and p_{tr} stand for the speed of sound, the energy density discontinuity and the transition pressure, respectively. In addition, the first line of Eq. (1) corresponds to the equation of state of hadronic matter, while the second line stands for the quark phase. The speed of sound is selected to be equal to the speed of light (c) in order to construct the maximally stiff equation of state for quark matter.

Note that a first order phase transition is not sufficient for the appearance of twin star solutions. According to Seidov's calculations [1,13], a distinct stable hybrid branch may only appear if the energy density jump exceeds a critical value which is given by

$$\Delta\varepsilon_{cr} = \frac{1}{2}\varepsilon_{tr} + \frac{3}{2}p_{tr}. \tag{2}$$

Therefore, if $\Delta\varepsilon > \Delta\varepsilon_{cr}$, then two separate stable branches may appear on the mass-radius plane.

For the description of the low-density hadronic phase, we considered two widely employed equations of state, namely the DD2-GRDF (from now on DD2 for simplicity) and NL3 models [14-15]. The outer layers of the star are modeled through the well-established equation of state of Baym *et al.* [16].

R-MODE INSTABILITY

The study of compact star oscillations and instabilities has garnered a lot of attention in the past two decades, as such dynamic processes may lead to the emission of detectable gravitational radiation [17-18]. In principle, neutron stars may suffer a number of different instabilities which come in different flavors, but they have a general feature in common, they can be directly associated with unstable modes of oscillation. The present work is focused on the so-called *r*-mode instability [19]. Specifically, the *r*-modes are oscillations that manifest in rotating stars and their restoring force is the Coriolis force. Interestingly, the gravitational radiation-driven instability of these modes has been proposed as a possible explanation for the fact that compact stars do not spin up to their theoretically allow limit, known as Kepler frequency.

The *r*-mode instability will only occur if the angular velocity (Ω) of a star exceeds a critical value (Ω_c). In order to evaluate the critical spin frequency ($f_c = \Omega_c/2\pi$) one needs to equate the critical timescales of the processes that drive and damp the instability. In particular [20-21],

$$\frac{1}{\tau_{gr}(\Omega,T)} + \frac{1}{\tau_{el}(\Omega,T,S)} + \frac{1}{\tau_{bv}(\Omega,T)} + \frac{1}{\tau_{sv}(\Omega,T)} = 0, \tag{3}$$

where τ_{gr}, τ_{el}, τ_{sv} and τ_{bv} stand for the critical timescales of the gravitational radiation, the viscous boundary layer, the shear and the bulk viscosity mechanisms, respectively. In principle, gravitational radiation tends to make the *r*-modes unstable, while the later three mechanisms dissipate the instability [20-21]. For more details concerning the evaluation of the aforementioned timescales the reader is referred to Ref. [11]. As one can observe, the aforementioned timescales depend on the angular velocity, the temperature (T) and the crust elasticity/slippage factor (S). Therefore, for a given value of S, one can solve Eq. (3) to derive the critical spin frequency as a function of temperature. The latter curve is known as *r*-mode instability window and evidently, if the relevant properties of star form a point that lies above the instability window, then the star would be *r*-mode unstable and would emit gravitational waves (and spin-down).

It has been shown that the *r*-mode instability is quite sensitive to the radius of a star [21]. Therefore, if twin stars do exist their instability windows would be different due to their different radius. In addition, the critical timescales, mentioned before, depend on the composition of stellar matter [4,22]. Considering that twin configurations are composed of different particles (the one is hadronic, while the other hybrid), the mechanisms that block the growth of the *r*-mode instability are going to be different in their interior. For the latter two reasons we studied how the parameters of the phase transition may affect the deviations that manifest in the *r*-mode instability windows of twin stars.

RESULTS AND DISCUSSION

To study the effects of the relevant parameters on the deviations between the *r*-mode instability windows of twin stars we constructed a set of hybrid equations of state. In Fig. 1 we depict the mass-radius dependance for the derived models. The shaded contours correspond to constraints derived from the GW170817 event [23]. Evidently, as the energy density jump increases there is a reduction on the corresponding maximum mass prediction. Hence, this places an upper limit on the considered value of $\Delta\varepsilon$ for a given value of the transition density. In addition, the larger the density discontinuity, the higher the difference between the radii of twin configurations. The latter is of particular importance, as the radius difference of twins is expected to play a major role concerning the differences in the resulting spin frequencies [21].

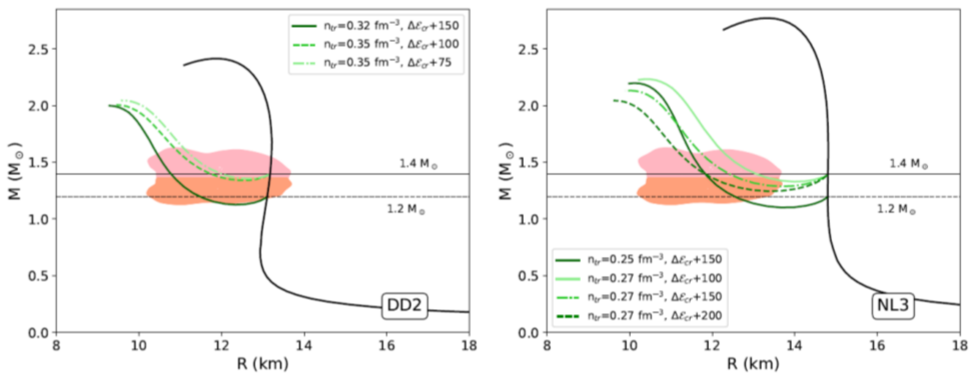

Figure 1. *Mass-radius diagrams for the constructed hybrid equations of state. The pink and orange contours correspond to constraints derived from the GW170817 event [23]. Each equation of state is identified by the transition density and the value of the energy density jump (the units of which are MeV fm^{-3} and are omitted in the legend for simplicity). Reproduced from Ref. [11], published by APS, 2023.*

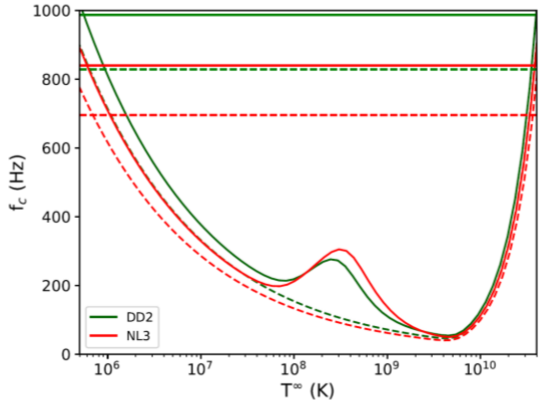

Figure 2. *The r-mode instability windows of 1.4 M_{sun} twin stars without the inclusion of damping due to a viscous boundary layer. The horizontal lines stand for the respective Kepler frequencies. With red (green) we denote the results when the NL3 (DD2) hadronic model is employed. The dashed (solid) line stands for the instability window of the hadronic (hybrid) twin. Reproduced from Ref. [11], published by APS, 2023.*

In Fig. 2 we display the r-mode instability windows of 1.4 M_{sun} twin stars, in a minimal scenario where the damping mechanism due to a rigid stellar crust is not included. Note that, on the x-axis instead of temperature (T) we considered the so-called redshifted temperature (as this is an observationally attainable quantity), which is given by $T\sqrt{1-2C}$ (C corresponds to the compactness of a star). As it is evident, for both of the considered low density models, there are certain differences that become apparent in the limiting spin frequencies of twin configurations. In particular, in the low temperature regime ($T^{\infty} < 10\ K$) the deviations are mainly due to the different radii values of the two stars [21]. For moderate temperature values ($10^8 K \leqslant T^{\infty} \leqslant 10^{10}\ K$) the dominant damping mechanism is the bulk viscosity of quark matter [22]. Consequently, the r-mode oscillations are damped more efficiently in the hybrid star. Finally, for large values of temperature ($T^{\infty} > 10^{10}\ K$) the bulk viscosity of hadronic matter becomes dominant, leading to similar predictions for both of the instability windows. Let us now consider the effects of a viscous boundary layer on the dissipation of the r-mode instability.

In Fig. 3 we depict the instability windows of 1.4 M_{sun} twin stars for different values of energy density jump and crust elasticity. We notice that an increase on the density discontinuity results into wider deviations between the instabilities windows of twin configurations. In particular, for the case of the NL3 hadronic model with $\Delta\varepsilon = \Delta\varepsilon_{cr} + 200$, the differences in limiting spin frequencies may reach values up to 400 *Hz*. The most profound effect due to the inclusion of a rigid crust is that the characteristic peak in the instability window of the hybrid twin flattens. The latter is explained by the fact that the aforementioned dissipative mechanism is strong and common for both twins, regardless of their composition.

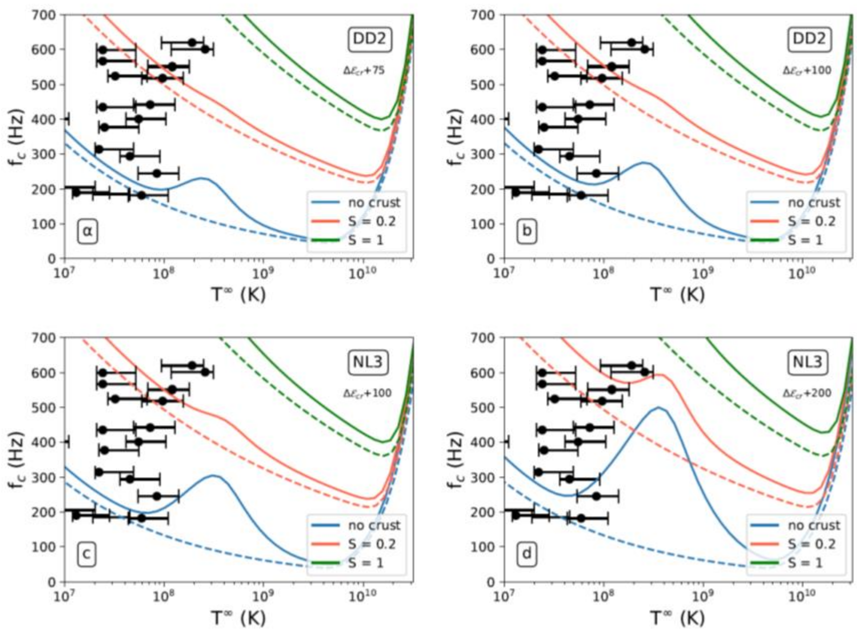

Figure 3. *The r-mode instability windows of 1.4 M_{sun} twin stars for different values of energy density jump and crust elasticity. Panels a), b) stand for the case where the DD2 model is employed, while panels c), d) correspond to the case where the NL3 model is used. The dotted points correspond to pulsar data [27]. Reproduced from Ref. [11], published by APS, 2023.*

We were able to illustrate that, depending on the value of $\Delta\varepsilon$, the values of the critical spin frequency of twin stars may be considerably different. The latter is of particular importance, as in the era of the gravitational wave astronomy we could potentially detect gravitational radiation due to unstable r-mode oscillations. In principle, if compact stars are purely hadronic then all neutron stars with equal mass should be described by a single instability window. The latter means that if a star with temperature T and angular velocity Ω_0 is stable with respect to the r-modes, then any other configuration with equal mass, similar temperature and $\Omega < \Omega_0$ should be stable as well. However, this is not the case if twin stars exist, since their instability windows deviate (schematically illustrated in Fig. 4). Consequently, the future possible detection of gravitational radiation from a star with lower angular velocity compared to the one of an r-mode stable star (with equal mass and similar temperature) would be a strong indication for the existence of twin star configurations.

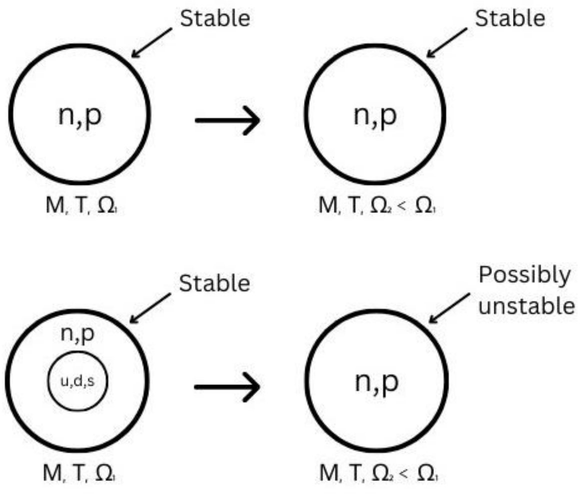

Figure 4. *Schematic illustration of a signature r-mode behavior associated with the existence of twin stars. Top: hadronic stars with equal mass and temperature but different angular velocity. Bottom: twin stars with equal temperature but different angular velocity.*

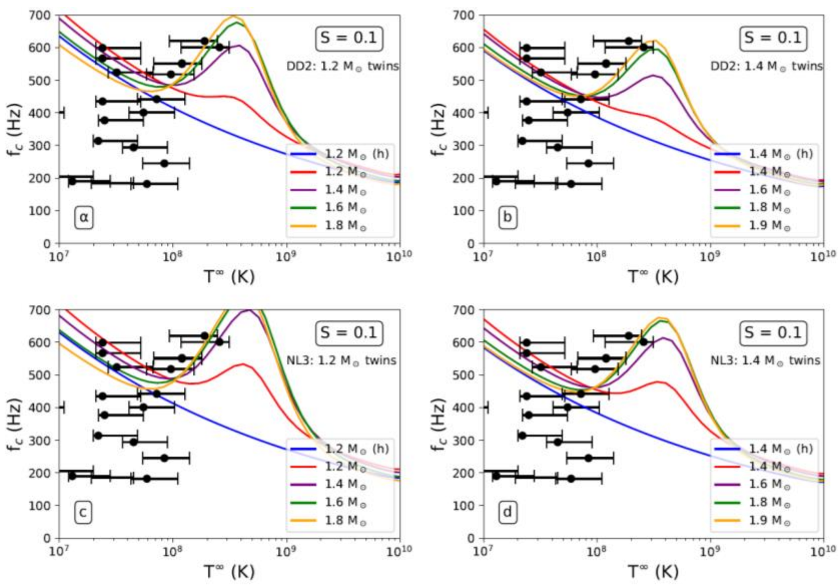

Figure 5. *The r-mode instability windows for different mass configurations. Panels a), b) stand for the case where the DD2 model is employed, while panels c), d) correspond to the case where the NL3 model is used. In all of depicted scenarios, the energy density jump is $\Delta\varepsilon_{cr}$+150 MeV fm^{-3} (except for panel b where we restrict to $\Delta\varepsilon_{cr}$+100 due to the 2M_{sun} constraint). The dotted points correspond to pulsar data [27]. Reproduced from Ref. [11], published by APS, 2023.*

Finally, it is important to comment that the search for gravitational radiation has just started [24-26]. However, no signal has been detected yet. That raises a question regarding the capability of hybrid equations of state to explain the *r*-mode stability of the observed compact stars. In Fig. 5 we depict the *r*-mode instability windows for different stellar mass configurations in comparison to current low mass x-ray binary data (crust elasticity is set to a conservative value of 0.1) [27]. In the case of equations of state that predict twin stars with $1.2M_{sun}$ (panels a and c), the damping mechanisms are sufficient to stabilize *r*-modes for moderately massive compact stars in the entire area covered by the observed pulsars. For later phase transitions (panels b and d) more massive compact star configurations are essential to ensure the *r*-mode stability of the observed stars.

CONCLUSIONS

In the current contribution we have presented a new approach that could potentially allow us to distinguish twin star configurations. In particular, we have shown that, depending on the strength of the phase transition, the *r*-mode instability windows of twin stars may be considerably different. As a consequence, the detection of gravitational radiation, due to unstable *r*-modes, from a star that lies on the stable region of the frequency-temperature plane (mapped by existing and future observation), would be a strong indication for the existence of twin pairs. Furthermore, we have demonstrated that the *r*-mode stability of the observed pulsars is compatible with equations of state that predict the existence of a third family of compact objects.

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