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Quarkyonic matter and applications in neutron stars

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Abstract   We consider a quarkyonic matter model in which, for large values of density and Fermi energies, we assume particles as quasiparticles where the low Fermi momentum states are occupied by quarks and the higher states in phase space are occupied by baryons. This idea has the aim to explain with a consistent way the phase transition in the interior of a neutron star or a hybrid star and to provide an equation of state which can produce neutron stars with masses greater than $2M_{sun}$ without violating causality. At extremely high density, quarkyonic matter is inferred from the properties of QCD when N_c is large and is a qualitative way to describe the particles in the momentum space. This state of matter can provide the sound velocity as a non-monotonic function of the baryon density which for higher values of density tends asymptotically to the conformal limit $1/\sqrt{3}$.

Keywords   Quarkyonic matter, neutron stars, phase transition, equation of state, sound velocity

INTRODUCTION

Recent gravitational wave, radio and X-ray detection from neutron stars and merging binary neutron star systems gave us some interesting insights about the structure of dense matter. The observation of neutron stars (NS) with masses greater than two-Solar masses shows that the pressure inside the inner core of a neutron star should be large. Also, gravitational wave detection placed an upper limit in radii of a NS, to not exceed the value of R=13.5 km [1,2]. So, we seek an equation of state to provide the sound velocity that increases rapidly as a function of the density. If we consider asymptotic freedom for large energy values, we expect to have a phase transition inside the inner core of a NS, from hadrons to quarks. The quarkyonic matter model is a theory candidate to describe with a consistent way this transition from hadronic to quark matter. It is not based on the QCD Lagrangian but it implements some features of QCD when N_c is large [3-5]. In this quarkyonic model, we consider that quarks form a Fermi sphere in the momentum space from zero momentum up to a fermi momentum k_{FQ} and baryons occupy higher momentum states and form a thin Fermi shell with a width of Δ. This idea can be illustrated in Fig. 1 [1].
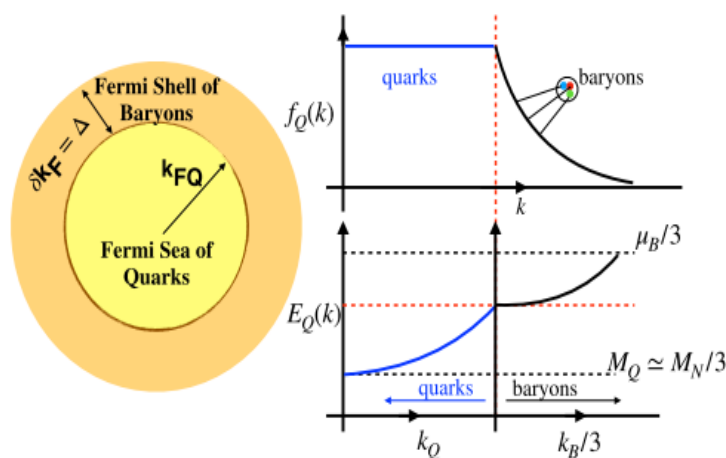

Figure 1. *The phase space in the quarkyonic model*

In the left-hand side of Fig. 1 we can see schematically the occupation of the states in momentum space and in the right panel we can see the energy and distribution functions, for quarks and baryons respectively, as a function of momentum. We assume that Chiral symmetry remains broken in our model so quark masses are obtained as $m_Q = m_N/N_c$, where N_c is the number of colors. For the thickness of the Fermi shell where nucleons reside, we impose the following relation [1,2],

$$\Delta = \frac{\Lambda_{Qyc}^3}{\hbar^3 c^3 k_{F_N}^2} + \kappa_{Qyc} \frac{\Lambda_{Qyc}}{\hbar c N_c^2} \tag{1}$$

where Λ_{Qyc} and κ_{Qyc} are parameters with values $\Lambda_{Qyc} \approx \Lambda_{QCD} \approx$ 200-300 MeV and $\kappa_{Qyc} \approx$ 0.2-0.3. The number density for quarks and nucleons will be

$$n_Q = \frac{g_s N_C}{2\pi^2} \sum_{Q=u,d} \int_0^{k_{F_Q}} k^2 dk \tag{2}$$

$$n_Q = \frac{g_s N_C}{2\pi^2} \sum_{N=p,n} \int_{k_{F_N}-\Delta}^{k_{F_N}} k^2 dk \tag{3}$$

The energy density will be in the form

$$\epsilon_Q = \frac{g_s N_C}{2\pi^2} \sum_{Q=u,d} \int_0^{k_{F_Q}} k^2 \sqrt{(hck)^2 + m_Q^2 c^4} dk \tag{4}$$

$$\epsilon_N = \frac{g_s}{2\pi^2} \sum_{N=p,n} \int_{k_{F_N}-\Delta}^{k_{F_N}} k^2 \sqrt{(hck)^2 + m_Q^2 c^4} dk + V(n_N) \tag{5}$$

for quarks and for nucleons respectively. Initially, we consider that quarks are non-interacting and nucleons interact via a potential that depends on the number density. The chemical potentials and pressure are obtained from the thermodynamic relations,

$$\mu_i = \frac{\partial \epsilon_i}{\partial n_i} \tag{6}$$

$$P = -\epsilon + \sum_i \mu_i n_i \tag{7}$$

THE NDU QUARQYONIC MODEL

We start the study of quarkyonic matter with a simple model that consists of neutrons, up and down quarks. We ignore protons because all insights show that the proton fraction is very small inside a neutron star. We assume that the Fermi momentum for down quarks is $k_{Fd} = (k_{Fn} - \Delta)/3$ and $k_{Fd} = 2^{1/3}k_{Fu}$ to impose charge neutrality. Also, we set the number of colors and the spin degeneracy to be equal to $N_c = 3$ and $g_s = 2$. The number density for quarks and neutrons will become,

$$n_Q = n_u + n_d = \frac{1}{\pi^2}\left(k_{F_u}^3 + k_{F_d}^3\right) \tag{8}$$

$$n_n = \frac{1}{3\pi^2}\left(k_{F_n}^3 - \left(k_{F_n} - \Delta\right)^3\right) \tag{9}$$

so the total baryon density will be,

$$n_B = n_n + \frac{(n_u + n_d)}{3} = \frac{1}{3\pi^2}\left(k_{F_n}^3 - \left(k_{F_n} - \Delta\right)^3 + k_{F_u}^3 + k_{F_d}^3\right) \tag{10}$$

The total energy density of quarkyonic matter is

$$\epsilon_{tot} = \epsilon_n + \epsilon_Q = \frac{g_s}{2\pi^2}\int_{k_{F_N}-\Delta}^{k_{F_N}} k^2\sqrt{(hck)^2 + m_n^2c^4}\,dk + V(n_n)$$
$$+ \sum_{i=u,d}\frac{g_s N_c}{2\pi^2}\int_0^{k_{F_i}} k^2\sqrt{(hck)^2 + m_Q^2c^4}\,dk \tag{11}$$

where we assume a potential energy which depends only on the neutron number density. The potential energy for this model is in the form,

$$V(n_n) = \alpha n_n\left(\frac{n_n}{n_0}\right) + b n_n\left(\frac{n_n}{n_0}\right)^2 \tag{12}$$

where n_0=0.16 fm^{-3} is the saturation density and for the coefficients we set α=-28.8 MeV and b=10 MeV. We also set Λ_{Qyc} = 380 MeV and κ_{Qyc} = 0.3 and we extract the equation of state and the sound velocity.

The sound velocity can be derived from the following relation,

$$\frac{c_s^2}{c^2} = \frac{\partial P}{\partial \epsilon} \tag{13}$$

After these calculations we extracted the quarkyonic matter equation of state for this simple potential and the sound velocity. We plot the Pressure versus the total energy density and the sound velocity as a function of the baryon density. We did so for a pure neutron model (consists only of neutrons) to compare it with the quarkyonic one and the results are illustrated in the following figures.
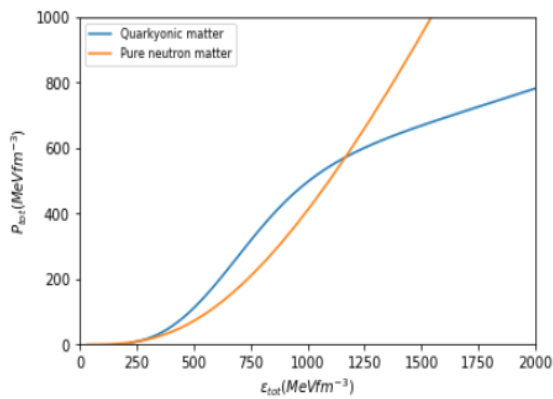

Figure 2. *The total pressure versus the energy density*
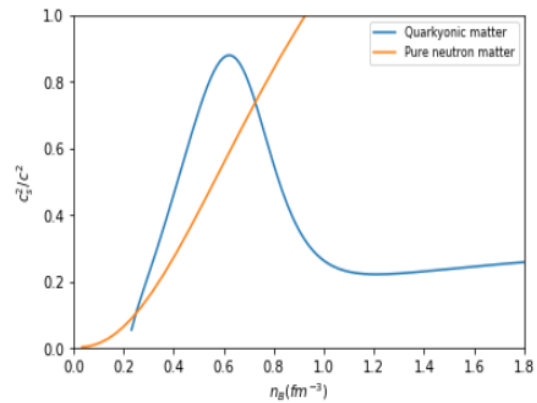

Figure 3. *The sound speed as a function of baryon density*

In Fig. 2 we can see the equation of state for quarkyonic matter (blue line) compared with a pure neutron model (orange line). In Fig. 3 we see the sound velocity for these two models (blue line corresponds to quarkyonic matter and the orange line to pure neutron matter).

THE NDU QUARKYONIC MODEL WITH MOMENTUM DEPENDENT INTERACTION

In the second model we assume the same quarkyonic state of matter including neutrons, up and down quarks but now we consider that both neutrons and quarks interact via a momentum dependent interaction (MDI) [6-8]. We introduce a potential in which the first terms depend on the baryon density and the last one has a momentum dependence and has been inserted to include finite range forces and short range repulsion. The potential energy we impose is in the form

$$V_{int}(n_n, k_{F_n}) = \frac{1}{3}An_0(1+x_0)u^2 + \frac{\frac{2}{3}Bn_0(1-x_3)u^{\sigma+1}}{1+\frac{2}{3}B'n_0(1-x_3)u^{\sigma-1}}$$
$$+ u\sum_{i=1,2}\frac{1}{5}[6C_i - 8Z_i]\mathfrak{I}_n^i \tag{14}$$

where $u = n_n/n_0$ and

$$\mathfrak{I}_n = \frac{2}{(2\pi)^3}\int d^3k\, g(n,\Lambda_i)f_\tau$$
$$= \frac{2}{(2\pi)^3}\int_{k_{F_n}-\Delta}^{k_{F_n}}4\pi\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}k^2 dk \tag{15}$$

is the finite range forces term for neutrons and

$$\mathfrak{I}_Q = \frac{2}{(2\pi)^3}\int d^3k\, g(n,\Lambda_i)f_\tau$$
$$= \frac{2}{(2\pi)^3}\int_0^{k_{F_Q}}4\pi\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1}k^2 dk \tag{16}$$

for quarks, respectively. k_{FQ} becomes k_{Fu} for up quarks and k_{Fd} for down quarks, respectively. The first two terms in Eqn. 14 are both momentum-independent. The first one corresponds to an attractive interaction, while the second term express a repulsive force and is dominant at high densities ($n>0.6$ fm^{-3}). The last one concerns to the momentum dependent part of the potential and corresponds to an attractive interaction. The total energy density now will be in the form,

$$\epsilon_{tot} = \epsilon_n + \epsilon_Q = \frac{g_s}{2\pi^2}\int_{k_{F_N}-\Delta}^{k_{F_N}}k^2\sqrt{(hck)^2+m_n^2c^4}dk + V_{int}(n_n,k_{F_n})$$
$$+ \sum_{i=u,d}\frac{g_s N_c}{2\pi^2}\int_0^{k_{F_i}}k^2\sqrt{(hck)^2+m_Q^2c^4}dk + V_{int}(n_i,k_{F_i}) \tag{17}$$

as in the previous model with the difference that in this case, we impose interactions also for quarks and the potential energy is given by Eqns. 14-16. For this potential we set up the parameters as: $\Lambda_1=1.5k_{F0}$, $\Lambda_2=3k_{Fo}$, A=-46.65 MeV, B=39.45 MeV, B'=0.3 MeV, σ=1.663 MeV, C_1=-83.84 MeV, C_2=23 MeV, χ_0=1.654 MeV, χ_3=-1.112 MeV, Z_1=3.81 MeV, Z_2=13.16 MeV, Λ_{Qyc}=220 MeV, κ_{Qyc}=0.3.

After that, we calculate again the total pressure and the sound velocity as before and we extract the following diagrams (Figs. 4 and 5).

In Fig. 4 we can see the equation of state (pressure versus energy density) for quarkyonic matter (blue line) and for pure neutron matter (orange line), both for the momentum dependent interaction. In Fig. 5 we see the sound velocity as a function of the baryon density for the quarkyonic matter (blue line) and for pure neutron matter (orange line) both for the same momentum dependent interaction (MDI).
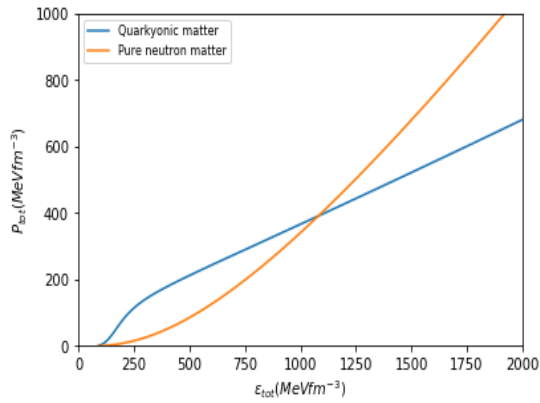
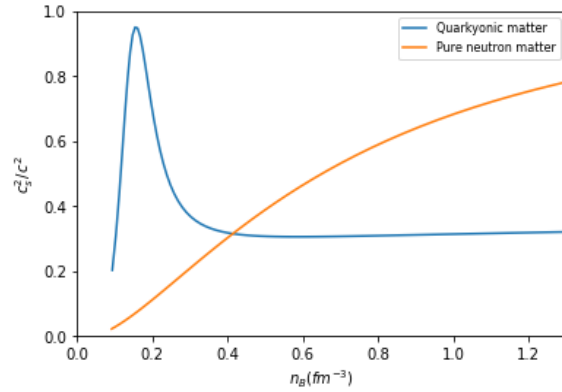**Figure 4**. *The total Pressure versus the energy density*

Figure 5. *The sound speed as a function of baryon density*

APPLICATIONS IN NEUTRON STARS -TOLMAN-OPPENHEIMER-VOLKOFF EQUATIONS

After the construction of equations of state for pure neutron and quarkyonic matter, we apply them in Einstein equations, for a static, spherically symmetric neutron star. This system of equations is the so called Tolman-Oppenheimer-Volkoff equations (TOV) which are in the following form,

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$
$$\frac{dP(r)}{dr} = \rho(r)c^2 \left(1 + \frac{P(r)}{\rho(r)c^2}\right)\frac{d\phi(r)}{dr} \tag{18}$$
$$\frac{d\phi(r)}{dr} = \frac{Gm(r)}{c^2 r^2}\left(1 + \frac{4P(r)r^3}{m(r)c^2}\right)\left(1 - \frac{2Gm(r)}{rc^2}\right)^{-1}$$

where G is the gravitational constant, c is the speed of light, P(r) is the total pressure, m(r) is the enclosed mass of the star, $\rho(r)$ is the total mass density, and $\varphi(r)$ is the gravitational field. We solve TOV equations numerically together with the equation of state and we get values for the mass and the radius of a neutron star for a central pressure. Then we repeat this process for several values of the central pressure, and we get the mass-radius diagram for each equation of state we constructed. In Figs. 6 and 7 we present our results.
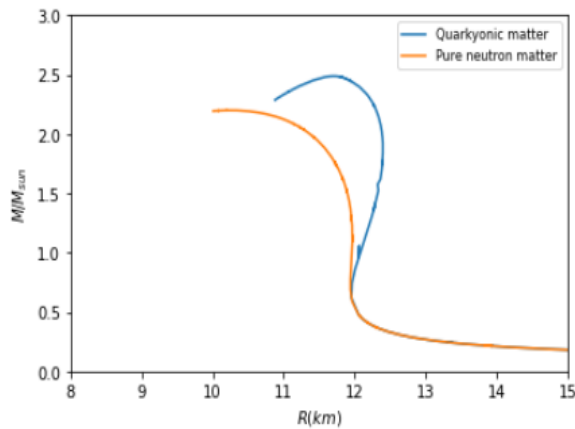

Figure 6. *Mass-Radius diagram for QM and PNM*
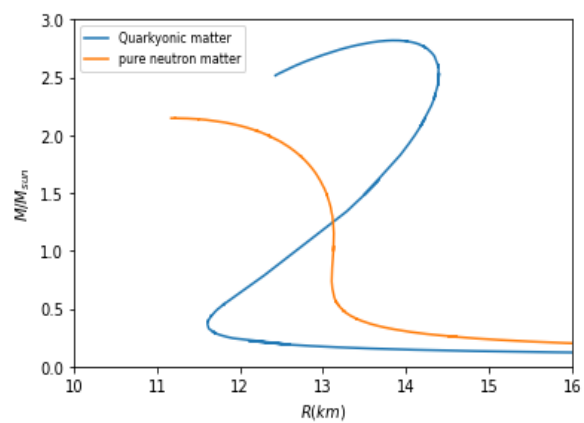

Figure 7. *Mass-Radius diagrams. QM versus PNM for the MDI model*

In Fig. 6 we see the mass-radius diagram for the quarkyonic matter (QM) (blue line) versus the pure neutron matter (PNM) (orange line) for the density dependent potential (Eqn.12). In the Fig. 7 we see the mass-radius diagram for the quarkyonic matter (blue line) and for the pure neutron matter (orange line) interacting via a momentum dependent potential (Eqn. 14).

RESULTS AND DISCUSSION

After this initial effort we can note some interesting features of quarkyonic matter. First of all, quarkyonic matter provides the sound speed as a non-monotonic function of the baryon density. We obtain a rapid increase up to a maximum value, without exceeding the speed of light, after it decreases and eventually it is reaching asymptotically the value $1/\sqrt{3}$ which is the conformal limit. In the first model the transition from neutrons to quarks occurs around a baryon density $n_B=0.24$ fm^{-3} and the sound speed has a maximum value about $0.9c$. In the momentum dependent quarkyonic model the phase transition occurs around a baryon density $n_B=0.2$ fm^{-3} and the sound speed reaches a maximum value about $0.96c$. Quarkyonic matter can predict more massive neutron stars (about 2.5-2.8 Solar masses), without violating the causality in contrast with pure hadronic matter models which violate causality and can't provide neutron stars with masses greater than $2.2M_{sun}$. Also, we can see from M-R diagrams that this state of matter predicts slightly greater values for the radius of a neutron star for the same masses. If we compare the quarkyonic matter model with the potential given by Eqn.12 and the quarkyonic matter model interacting via the momentum dependent interaction, we can see that in the second one, quarks appear at lower densities and also this equation of state gives greater masses for a neutron star. It is important to note that for the momentum dependent interaction model, the sound speed does not exceed the speed of light neither in the pure neutron matter case. This will be a very interesting feature for the study of nuclear matter and condensed matter physics, as in the study of the structure of a neutron star.

CONCLUSIONS

We study quarkyonic matter, a hybrid state of matter in which we consider that quarks are deconfined for low momentum states and form a fermi sphere in the momentum space. For higher values of momentum, quarks are confined into baryons and form a fermi shell which surrounds the quarks fermi sphere. We believe that this state of matter exists at densities close to that of nuclear matter and at high fermi energies. We construct equations of state for a simple model which includes only neutrons, up and down quarks for two different potentials and we compare them with pure neutron matter equations of state. We extract some interesting results which shows the advantages of this theory. This theory is very recent and there are a lot of open questions about it, but also has many perspectives and applications in several theoretical physics fields. In future work we have to extend our model to include protons and electrons to impose β-equilibrium and to apply quarkyonic matter in finite temperature neutron stars. Also, we must investigate if there is any fundamental theory which can provide this state of matter [9-11]. We expect that future gravitational wave detections from binary neutron star systems will lead us to constrain further some of the microscopic parameters of our model, so that to test and to improve our equations of state.

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