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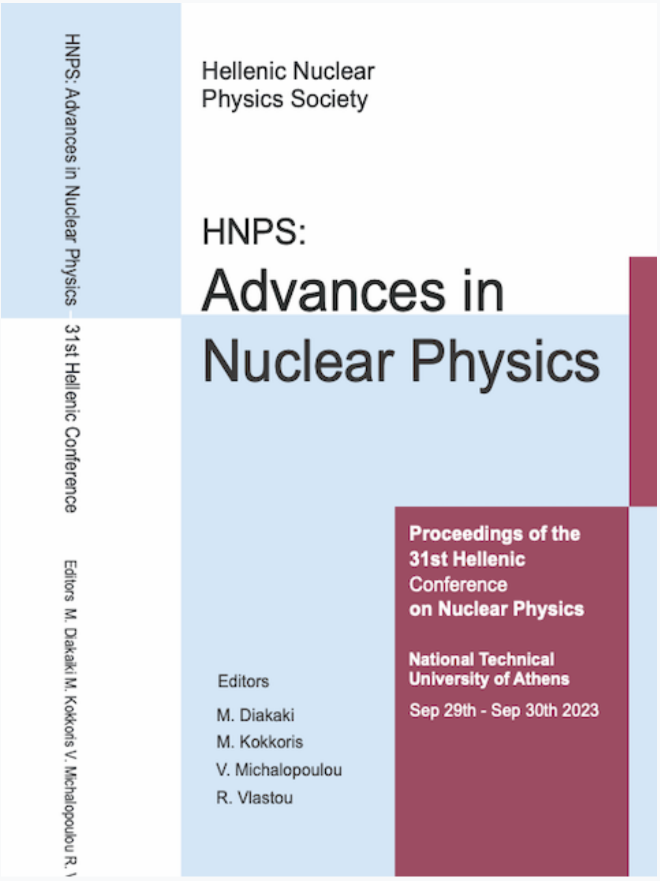

A study of the nuclear structure of even-even Te isotopes in the Interacting Boson Model framework

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A study of the nuclear structure of even-even Te isotopes in the Interacting Boson Model framework

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Abstract The even-even tellurium isotopes (Z=52) are part of a region beyond the closed proton shell at Z=50 and are generally considered to exhibit vibrational-like properties. These properties are investigated in the present work using the the Interacting Boson Model (IBM-1) framework to estimate the energy levels, transition quadrupole moments and the reduced transition probabilities B(E2) of the even-even $^{104-140}$Te isotopes. In addition, the deformation parameters β_2 and γ have been calculated providing a useful visualization of the nuclear shapes in these isotopes. The results have been compared to available adopted data and used as estimates for unavailable ones.

Keywords Interacting Boson Model, even-even Te isotopes, IBAR, deformation

INTRODUCTION

The Interacting Boson Model (version IBM-1), introduced by Arima and Iachello [1], is an algebraic framework for describing low-lying collective states of even-even nuclei far from closed shells, which are dominated only by valence nucleon excitations. According to the model, valence nucleons create pairs of protons or neutrons (ignoring their degrees of freedom) [2], which behave as bosons with total angular momentum and parity $J = 0^+$ and $J = 2^+$, called *s*-bosons and *d*-bosons, respectively. The total number of bosons, $N_B = N_\pi + N_\nu$, is given by the half number of valence nucleons or holes taken relative to the nearest closed shells (here Z=50 and N=50-82) where N_π and N_ν are the proton and neutron boson numbers, respectively.

For the calculations in the IBM framework the extended consistent Q formalism (ECQF) with the Hamiltonian [3],

$$H(\zeta,\chi) = c\left[(1-\zeta)\hat{n}_d - \frac{\zeta}{4N_B}\hat{Q}^\chi \cdot \hat{Q}^\chi\right] \tag{1}$$

has been used, where $\hat{n}_d = d^\dagger \cdot \tilde{d}$ is the quadrupole boson creation operator, $\hat{Q}^\chi = \left(d^\dagger s + s^\dagger \cdot \tilde{d}\right) + \chi(d^\dagger \cdot \tilde{d})^{(2)}$ is the quadrupole operator and c is a scaling factor. The E2 transition operator is defined in terms of the same quadrupole operator:

$$T(E2) = \mathrm{e_B}\hat{Q}^\chi, \tag{2}$$

where $\mathrm{e_B}$ represents the boson effective charge in (eb).

The two parameters, ζ and χ, involved in the Hamiltonian vary within the range $\zeta \in [0,1]$ and $\chi \in [-\sqrt{7/2}, +\sqrt{7/2}]$ and describe the parameter space of this Hamiltonian, represented schematically by a triangle known as the IBM symmetry triangle or the so-called Casten Triangle [3] with three dynamical symmetries located in the triangle vertices. The symmetry limits correspond to vibrational nuclei with a spherical form U(5), to an axially symmetric prolate rotor SU(3) and to a rotor with a flat potential O(6). There are also the critical point symmetries E(5) [4] and X(5) [5] describing the shape phase transitions in atomic nuclei, related to the transition from U(5) (vibrational) to O(6) (γ-unstable) nuclei, and to the transition from U(5) to SU(3) (rotational) nuclei respectively [6].

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RESULTS AND DISCUSSION

Utilizing the software IBAR, developed by R.J. Casperson [7], precise IBM calculations were carried out for each boson number. The calculations spanned the parameter space of ζ and χ with a step size of 0.01 and contour plots depict the anticipated values of IBM-1 observables for each nucleus, illustrating variations for different configurations of ζ and χ as shown in Figure 1.

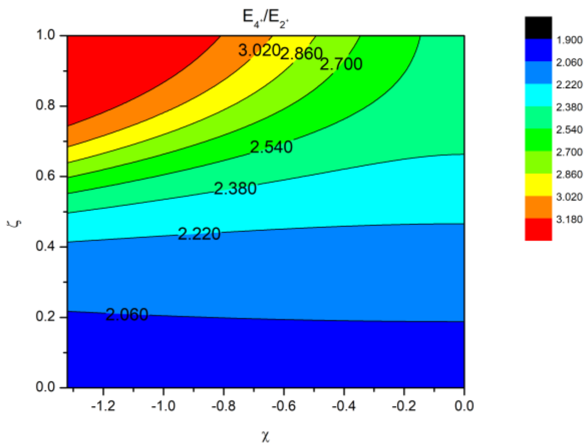

Figure 1. IBM calculated $R_{4/2}$ energy ratio for ^{108}Te

Upon establishing the best pair of the ζ and χ parameters (or polar coordinates projected inside the IBM symmetry triangle) the IBM calculations can generate a wide array of observable quantities such as energy levels and reduced transition probabilities (Figures 3 and 4). Furthermore, the nuclear shape can be described adequately by the deformation parameters β_2 and γ expressing the axial asymmetry and the triaxial deformation of the nuclear shape, respectively and can be extracted from Refs. [8-10]:

$$\beta_2 = \left(\frac{4\pi}{3ZR_0^2}\right)\left[\frac{B(E2;0_1^+ \to 2_1^+)}{e^2}\right]^{\frac{1}{2}}, \qquad \sin(3\gamma) = \frac{3}{2\sqrt{2}}\sqrt{1-\left(\frac{E(2_\gamma^+)-E(2_1^+)}{E(2_\gamma^+)-E(2_1^+)}\right)^2}, \tag{3}$$

where the B(E2) is the reduced electric transition probability from the ground state 0^+ to the first 2^+ state and 2_γ^+ is the bandhead of the quasi-γ band. Table 1 shows IBM calculated values for the quadrupole parameters β_2 and γ, as well as the intrinsic quadrupole moment Q_0, which is related to the reduced electric transition probability B(E2) by:

$$Q_0 = \left[\frac{16\pi}{5}\frac{B(E2;0_1^+ \to 2_1^+)}{e^2}\right]^{\frac{1}{2}} (in\ b)\,. \tag{4}$$

Concerning a typical collective even-even nucleus, a few essential observables can aptly explain its structure [3] like the energy ratio $E(4_1^+)/E(2_1^+)$, which provides a good perspective on the (low-spin) structure of a typical collective even-even nucleus and is equal to 2.0 for spherical harmonic vibrator [11] (Fig. 3).

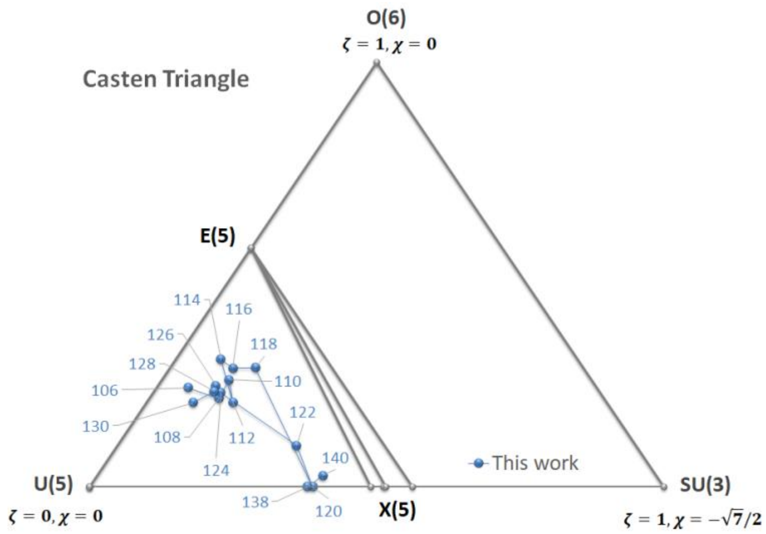

Figure 2. *Trajectories in the IBM symmetry triangle for* $^{106-140}Te$

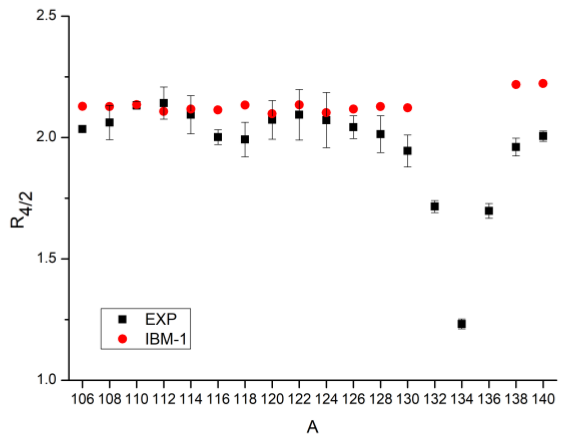

Figure 3. *Experimental [12] and IBM calculated energy ratios for* $^{106-140}Te$

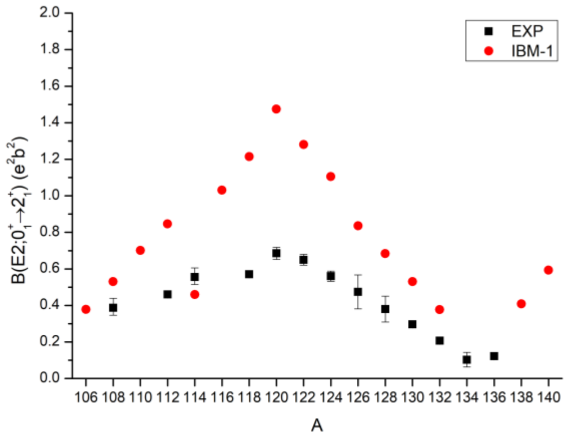

Figure 4. *Experimental [13] and IBM calculated for B(E2) values for* $^{106-140}Te$

The values of $R_{4/2}$ calculated using the IBM display a spherical anharmonic vibrational character for the Te isotopes. A maximum value of the calculated $B(E2; 0_1^+ \rightarrow 2_1^+)$ at ^{120}Te is obtained which is near the midshell and is in agreement with the available experimental data [12-13].

Table 1. IBM calculated values from Eqs. (3) and (4)

Isotope	*γ (°)*	β_2	Q_0 (b)	*Isotope*	*γ (°)*	β_2	Q_0 (b)
^{106}Te	27.16	0.154	1.949	^{122}Te	25.39	0.239	3.333
^{108}Te	26.98	0.180	2.310	^{124}Te	27.26	0.206	2.899
^{110}Te	26.91	0.204	2.655	^{126}Te	27.17	0.184	2.622
^{112}Te	27.05	0.221	2.916	^{128}Te	27.04	0.160	2.310
^{114}Te	27.22	0.242	3.219	^{130}Te	27.17	0.134	1.947
^{116}Te	27.14	0.259	3.495	^{138}Te	24.73	0.134	2.028
^{118}Te	26.67	0.282	3.851	^{140}Te	24.29	0.160	2.442
^{120}Te	25.13	0.260	3.589				

The electric quadrupole moment increases (see Table 1) with increasing neutron hole pairs with a maximum at ^{118}Te and if we assume a constant-charge distribution within the nucleus, the overall shape of the nucleus is primarily determined by its intrinsic quadrupole moment where for positive values a nuclei with prolate form occurs [14]. Deformed nuclei can be broadly classified as prolate, oblate, and triaxial, reflecting the nuclear dynamic symmetries inherent in the deformed Hamiltonian. This classification is based on the three principal axes of rotation within the ellipsoid. The symmetries governing nuclear shapes are evident on the β-γ plane, which comprises the set of intrinsic shape variables (β, γ) utilized in the Bohr Hamiltonian. All shapes are distinctly represented in the γ= (0°,60°) sector, considered as the representative one. Prolate nuclei align along the γ=0° axis, oblate nuclei along the γ=60° axis, and nuclei situated between γ=0° and γ=60° are characterized as triaxial. Numerous nuclei within the rare-earth region of the nuclear chart are recognized for their substantial deformation, frequently displaying quadrupole deformation parameter values β_2 greater than 0.2 for either the ground or low-lying states [15-17]. The U(5) limit corresponds to a particular set of parameters in the IBM, where the nucleus is characterized by a spherical shape and exhibits a vibrational or vibrational-like behavior as previously stated. The tellurium isotopes of this study are situated near the lower-left corner of the Interacting Boson Model (IBM) symmetry triangle exhibiting vibrational-like properties associated with the U(5) symmetry [1].

CONCLUSIONS

This study primarily focuses on the collective properties and evolutionary trends of the even-even Te isotopes as the neutron number rises. The inquiry into the collective behavior of even–even isotopes ranging from $^{106-140}$Te involves the computation of diverse observables, including energies and reduced transition probabilities. The calculated energy ratios and B(E2) obtained with the Interacting Boson Model (version IBM-1) show good agreement with the available experimental data, showing similar trends and confirming the effectiveness and utility of IBM in this region.

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