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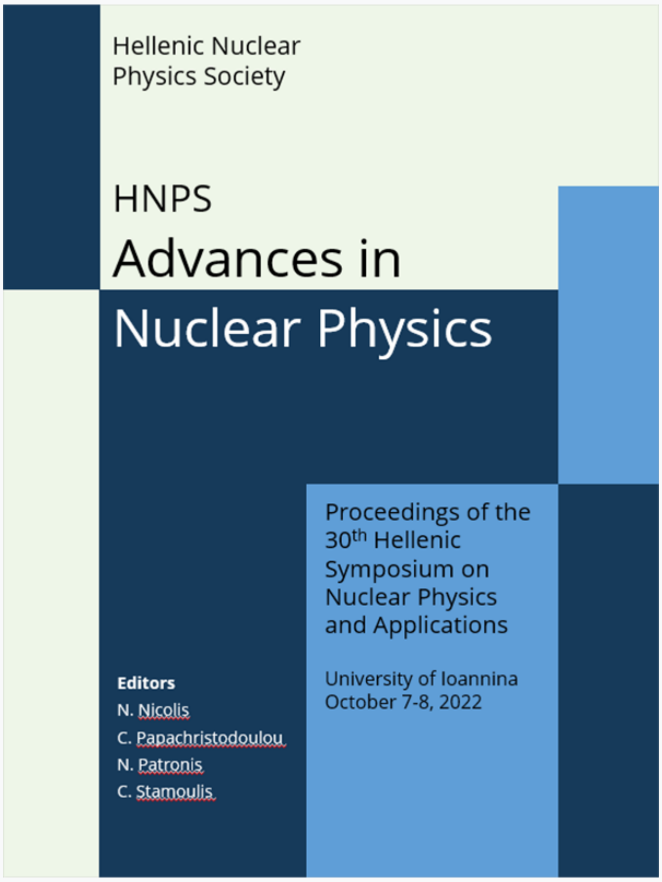

Universal relations and finite temperature neutron stars

Pavlos Laskos-Patkos, Polychronis S. Koliogiannis, Alkiviadis Kanakis-Pegios, Charalampos C. Moustakidis

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Universal relations and finite temperature neutron stars

P. Laskos-Patkos*, P.S. Koliogiannis, A. Kanakis-Pegios, Ch. C. Moustakidis

Department of Theoretical Physics, Aristotle University of Thessaloniki, Greece

Abstract In the past few years, a lot of studies have been devoted to the discovery of universal relations (equation of state independent relations). The significance of such expressions can be understood if we consider that they offer the opportunity for testing general relativity in a way that is independent of the nuclear equation of state and they also allow us to impose constraints on the structure of neutron stars. The aim of this work is twofold. Firstly, we wish to clarify if hot equations of state are able to reproduce established universal relations. Secondly, we investigate a possible universal connection between the binding energy and the dimensionless tidal deformability of a neutron star. These two bulk properties are associated with two very important candidates for multimessenger signals, binary neutron star mergers and supernova explosions. We find that the predictions of hot equations of state do not agree with the predictions of accepted universal relations. Subsequently, the use of universal relations, when thermal effects are present, may be erroneous. Additionally, we find that, for moderate neutron star masses, the binding energy and the dimensionless tidal deformability of a neutron star satisfy a universal relation. The latter allows us to impose constraints on the binding energy of 1.4 M_{sun} neutron star, using information from the analysis of the GW170817 event. Finally, we are able to present a universal relation between the compactness, the binding energy and the dimensionless tidal deformability, which is independent of the employed equation of state for zero and finite temperature.

Keywords hot nuclear matter, neutron stars, universal relations

INTRODUCTION

Old isolated neutron stars are considered to be cold. That means that the thermal energy of the system is very small compared to the Fermi energy due to degeneracy. However, there are many astrophysical phenomena, concerning neutron stars, where thermal effects may play an important role. For example, protoneutron stars (remnants of supernova explosions) or binary neutron star (BNS) merger remnants may be very hot, reaching temperatures of 30-100 MeV [1]. Additionally, the exact temperature range for neutron stars during the inspiral phase of a BNS merger is not well-constrained [2-4]. In any case, the new era of multimessenger astronomy demands precise computations concerning dense nuclear matter at finite temperature.

In the past years, several studies investigated the existence of universal relations regarding properly scaled neutron star properties [5-8]. Such empirical expressions are of most importance as they allow us to impose constraints on neutron star structure and they provide the opportunity for testing general theory of relativity in an EOS independent way [5, 9-10]. It is therefore crucial to establish whether finite temperature EOSs are compatible with established and widely used universal relations (which were derived using cold EOSs). Previous studies that have employed finite temperature EOSs in order to investigate the origin of empirical relations, suggest that universality is about insensitivity to the EOS and not to the thermodynamic conditions [11-14].

In a recent study, the authors reported a linear correlation between the dimensionless tidal deformability of a 1.4 M_{sun} neutron star ($\Lambda_{1.4}$) and the binding energy E_b of a specific mass configuration [15]. We estimate that this connection rises from the fact that tidal deformalibity and binding energy are associated via a universal relation. It is important to comment that the tidal deformability and the binding energy can be well-measured in the cases of BNS mergers and supernova explosions, respectively. Therefore, these two bulk neutron star properties are connected to two very

* Corresponding author: plaskos@physics.auth.gr

promising candidates for future multimessenger detection.

In this work, we examine the agreement of hot EOSs with known universal relations and we also investigate the possibility of a universal connection between the binding energy and the dimensionless tidal deformability of a neutron star.

BINDING ENERGY

Binding energy is defined as the energy gain from the assembly of N nucleons in order to form a stable star. This bulk neutron star property can be extracted from the observation of supernova neutrinos [16]. The mathematical expression for the binding energy is given by [15-16]

$$E_b = N m_b c^2 - M c^2, \tag{1}$$

where m_b stands for the average mass of nucleons, c is the speed of light and M is the gravitational mass of a star. There is a debate about which value of m_b is more appropriate. Several authors assume that it corresponds to the mass of free nucleons. In the present work, the average baryon mass is considered to be the mass of ^{56}Fe/56 = 930.412 MeV/c^2 [15-16].

TIDAL DEFORMABILITY

Gravitational waves emitted during the inspiral phase of a BNS merger are an important source for Earth based detectors [17-18]. Such observations may provide crucial information concering a bulk neutron star property known as tidal deformability λ [17-18]. This quanity is strongly dependent on the structure of a neutron star and thus can be used in order to constrain the nuclear equation of state.

Tidal deformability is defined as the proportionality coefficient between the induced quadrapole moment Q_{ij} of a star and the tidal gravitational field of its companion E_{ij} in a BNS system

$$Q_{ij} = -\lambda E_{ij}. \tag{2}$$

The calculation of the tidal deformability requires the evaluation of the second tidal love number k_2 and the radius R of the star. In particular,

$$\lambda = \frac{2}{3} k_2 \frac{R^5}{G}, \tag{3}$$

where G is the gravitational constant. For the determination of the second tidal love number one has to solve a non-linear differential equation and TOV equations in a self-consistent way [17-18].

At this point, it is convenient to define the dimensionless tidal deformability Λ as

$$\Lambda = \frac{2}{3} k_2 \frac{R^5 c^2}{GM} = \frac{2}{3} k_2 C^{-5}, \tag{4}$$

where C is the compactness of a star. It has to be noted that Λ follows a set of universal relations known as I-Love-Q and Love-C relations [5-6]. The Love-C relation connects Λ and C and it can be used in order to impose constraints on the radius of a neutron star by measuring its tidal deformability [9-10]. In the present work, Love-C relation is employed in order to test the behavior of hot EOSs concerning universal relations.

RESCALED ENTROPY

One of the universal relations employed in this work was found in Ref. [8]. Alexander et al. showed that if one scales the total entropy of a neutron star appropriately (rescaled entropy RE) there is universal relation with the compactness [8]. The rescaled entropy can be evaluated from the right hand side of the following equation

$$RE = \frac{T'S}{Mc^2} = \frac{4\pi}{Mc^2} \int_0^R (-g_{tt} g_{rr})^{\frac{1}{2}} (\varepsilon + p) r^2 dr\,, \tag{5}$$

where r is the radial distance from the center of the star, p and ε are the pressure and the energy density distributions and g_{tt}, g_{rr} correspond to the time and radial (diagonal) components of the metric tensor.

S is the total entropy and T' is a scaled form of temperature which remains a constant inside a cold star (for more details see Ref. [8]). Rescaled entropy exhibits an empirical relation with the compactness for both isotropic and anisotropic neutron stars. In the later case the universal trend tends approximately to the corresponding rescaled entropy value for a black hole [8].

RESULTS AND DISCUSSION

In order to compare established universal relations with the results from hot EOSs, we employed a set of cold and hot EOSs. For information regarding the cold EOSs the reader is reffered to Refs. [19-21] and references therein. For the construction of finite temperature EOSs, we employed the MDI+APR1 model [1, 19]. The crust of cold and hot neutron stars was described via BPS [20] and Lattimer-Swesty [21] EOSs, respectively. In Figures 1(a), (b) one can find the mass-radius dependence for the EOSs employed in this work. As one can observe, the inclusion of temperature does not signifficanly affect the maximum mass of a neutron star. In contrast, the radius appears to be very sensitive to thermal effects [1, 24].

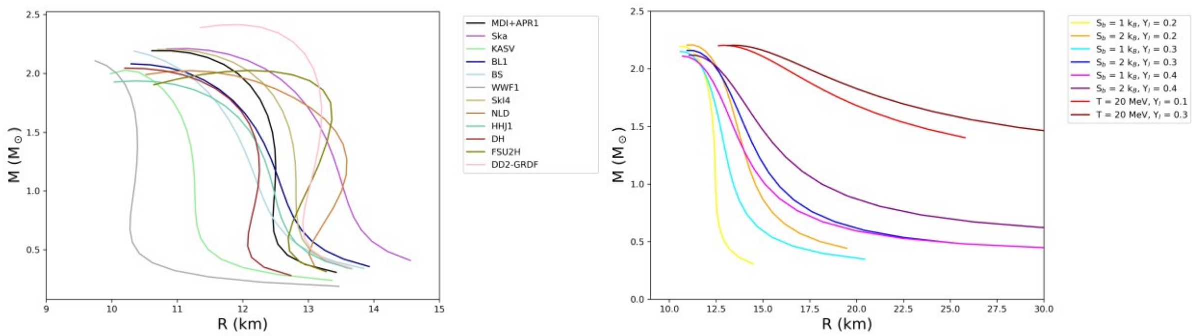

Figure 1. *Mass-Radius relations for (a) cold neutron stars, (b) hot neutron stars (MDI+APR1 EOS).*

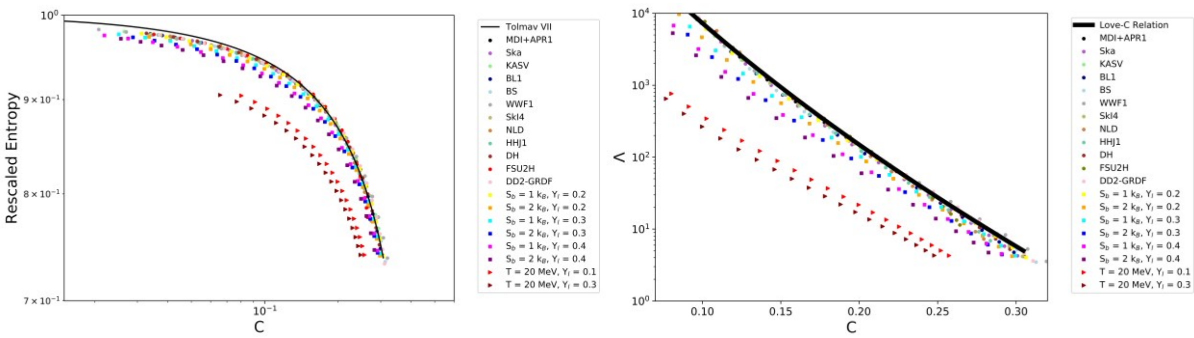

Figure 2. *(a) Relation between the rescaled entropy and the compactness [8] for several cold and hot EOSs. The solid line corresponds to the results for the Tolman VII analytical solution. (b) Relation between the dimensionless tidal deformability and the compactness. The solid line corresponds to Love-C universal relation found in Ref. [6]. In both panels the results denoted by circular, diamond and triangular points correspond to cold, isentropic and isothermal EOSs, respectively.*

Figure 2(a) depicts the dependence between the rescaled entropy and the compactness. The cold EOSs in fact follow an accurate universal relation. As one can observe, in the case of adiabatic (isentropic) EOSs, increasing the entropy per nucleon (S_b) leads to disagreement with the universal trend. Aditionally, the predictions from hot EOSs gradually diverge from the empirical relation as the lepton fraction (Y_l) increases. In the scenario of isothermal EOSs, the results exhibit large differences with the predictions of the universal relation.

In the case of isentropic EOSs, we can theoretically prove (using the generalized condition for thermal equilibrium in general relativity [24-25]) that the rescaled entropy is tighly connected to the binding energy of a neutron star. Specifically, for constant entropy per nucleon, we can show that

$$RE = \left(1 + \frac{E_b}{Mc^2}\right) e^{\Phi(R)}, \tag{6}$$

where $e^{\Phi(R)} = \sqrt{1-2C}$ is the star's redshift [24]. Therefore, we expect that the evaluation of the right hand side of Equation (6), for cold and hot isentropic EOSs, will produce the same results as in Figure 2(a). The latter leads to the conclusion that there is an approximate universal relation between the binding energy (divided by the mass) and the compactness of a neutron star. A similar expression has been suggested by Lattimer and Prakash [16]. We conclude that the relations found in Refs [8, 16] are essentially the same. It is notable, that the right hand side of Equation (6), provides a properly scaled version of the binding energy (rescaled binding energy) which is expected to satisfy a set of universal relations (at least for cold EOSs) [24].

In Figure 2(b) one sees the universal relation between the dimensionless tidal deformability and the compactness (Love-C relation) found by Masselli et al. [6] and its comparison with the results derived using hot EOSs. As we have mentioned, this relation has been previously employed in order to produce contraints on the radius of a neutron star [9-10]. The empirical relation is satisfied only in the case where $S_b = 1$ and $Y_l = 0.2$. This reveals the need for narrowing down the range of possible temperatures for a neutron star during the inspiral phase of BNS merger [4, 24].

Moving on to the second part of this study, we are going to investigate the existence of an approximate universal relation that involves the binding energy and the dimensionless tidal deformability. Figure 3(a) shows the results for the dependence between E_b/M and Λ for cold and hot EOSs. As one can observe, cold EOSs follow a specific trend which is quite accurate, at least for moderate compact star masses (error less than 10%). Once again, the inclusion of temperature in our calculations leads to large differences from the empirical relation. For the data from cold EOSs we provide a fit of the following form

$$\frac{E_b}{Mc^2} = \sum_{k=0}^{3} \alpha_k \, (\ln \Lambda)^{-k}, \tag{7}$$

where $\alpha_0 = -0.08399$, $\alpha_1 = 1.52078$, $\alpha_2 = -2.91006$ and $\alpha_3 = 1.85495$.

We need to comment that the observed neutron star masses usually lie in the range [1 M_{sun}, M_{max}]. Specifically, the lightest neutron star observed (up to the time that this reserach was conducted) had a mass of 1.174 ± 0.004 M_{sun} [26]. In the afforementioned range, we can fit a simple linear expression to the data from cold EOSs

$$\frac{E_b}{Mc^2} = b_0 + b_1 \ln \Lambda, \tag{8}$$

where $b_0 = 0.22350$ and $b_1 = -0.02017$. Figure 3(b) depicts the relation between the binding energy and the dimensionless tidal deformability in the range [1 M_{sun}, M_{max}]. The shaded area corresponds to constraints on the tidal deformability of a 1.4 M_{sun} from the GW170817 merger event [9]. These constraints allow us to impose bounds on the binding energy of 1.4 M_{sun} neutron star (which can be found in Table 1) [24].

Table 1. *Constraints on the binding energy of a 1.4 M_{sun} neutron star using Equation (8) and the analysis of the GW170817 event. The 1σ error for the fit is included.*

Λ	E_b/Mc^2	$E_b(10^{53}\ erg)$
70	$0.1378 \pm 3.1 \times 10^{-3}$	3.4489 ± 0.0793
580	$0.0952 \pm 3.9 \times 10^{-3}$	2.3814 ± 0.0977

At this point, we are going to examine the dependence between the rescaled binding energy and the dimensionless tidal deformability. In Figure 4(a) one can observe this relation for cold and hot EOSs. Wave found that cold EOSs satisfy a very accurate universal relation (max error for stable configurations is less than 0.6%). Surprisingly, hot isentropic equations of state respect the aforementioned universality (with the same accuracy). It is noteworthy that isothermal EOSs exhibit large differences from the empirical trend but this is due to the fact that Equation (6) does not hold for constant temperature neutron stars [24]. For the data from cold and hot isentropic EOSs, we provide a fit of the following form

$$\left(1+\frac{E_b}{Mc^2}\right)\sqrt{1-2C}=\sum_{k=0}^{3} c_k\,(\ln\, \Lambda)^k, \tag{9}$$

where $c_0 = 0.66656$, $c_1 = 0.05855$, $c_2 = -0.00402$ and $c_3 = 0.00010$. Figure 4(b) depicts the same dependence but only for 1.4 M_{sun} neutron stars and shows possible constraints on the rescaled binding energy (using the GW170817 event [9]).

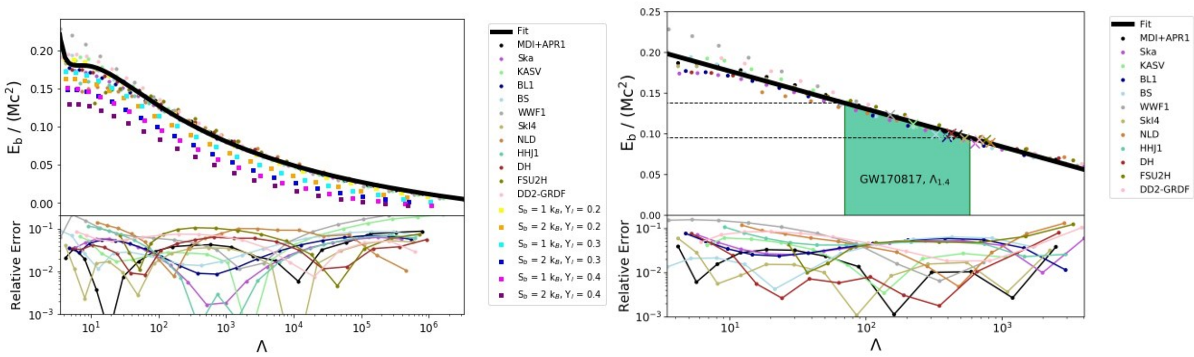

Figure 3. *(a) Relation between the binding energy and the dimensionless tidal deformability for cold and hot EOSs. The solid line corresponds to the fit of Equation (7). The results denoted by circular and diamond points correspond to cold and isentropic EOSs, respectively. In the bottom panel one can find the relative error for the fit of Equation (7). (b) Relation between the binding energy and the dimensionless tidal deformability for cold EOSs and neutron star masses in the range [1 M_{sun},M_{max}]. The solid line corresponds to the fit of Equation (8). The cross points correspond to 1.4 M_{sun} neutron star configurations. The shaded area corresponds to the constraints on $\Lambda_{1.4}$ from the GW170817 event [9]. In the bottom panel one can find the relative error for the fit of Equation (8).*

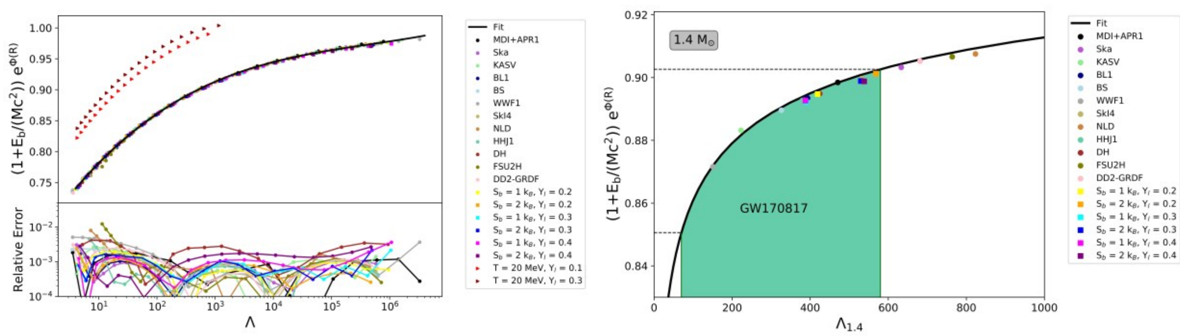

Figure 4. *(a) Relation between the rescaled binding energy and the dimensionless tidal deformability for cold and hot EOSs. The solid line corresponds to the fit of Equation (9). The results denoted by circular and diamond points correspond to cold and isentropic EOSs, respectively. In the bottom panel one can find the relative error for the fit of Equation (9). (b) Relation between the rescaled binding energy and the dimensionless tidal deformability for cold and isentropic EOSs and 1.4 M_{sun} neutron stars. The shaded area corresponds to the constraints on $\Lambda_{1.4}$ from the GW170817 event [9].*

CONCLUSION

Considering the importance of universal relations, we investigated their agreement with the predictions derived using finite temperature EOSs. We have found that hot EOSs do not always reproduce established empirical formulas (by employing the rescaled entropy-compactness and the Love-C relations). In addition, we examined the existence of a possible universal connection between the binding energy and the dimensionless tidal deformability. We have found that the dependence between E_b/M and Λ exhibits an approximate universal trend. The latter, provided the opportunity for constraining the binding energy of a 1.4 M_{sun} neutron star using data from the analysis of the GW170817 event. Additionaly, we discovered a new universal relation that involves the binding energy, the dimensionless tidal deformability and the compactness of a neutron star. This new empirical relation also holds for hot isentropic EOSs and its accuracy is remarkable as the maximum relative error is less than 0.6% for stable configurations.

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