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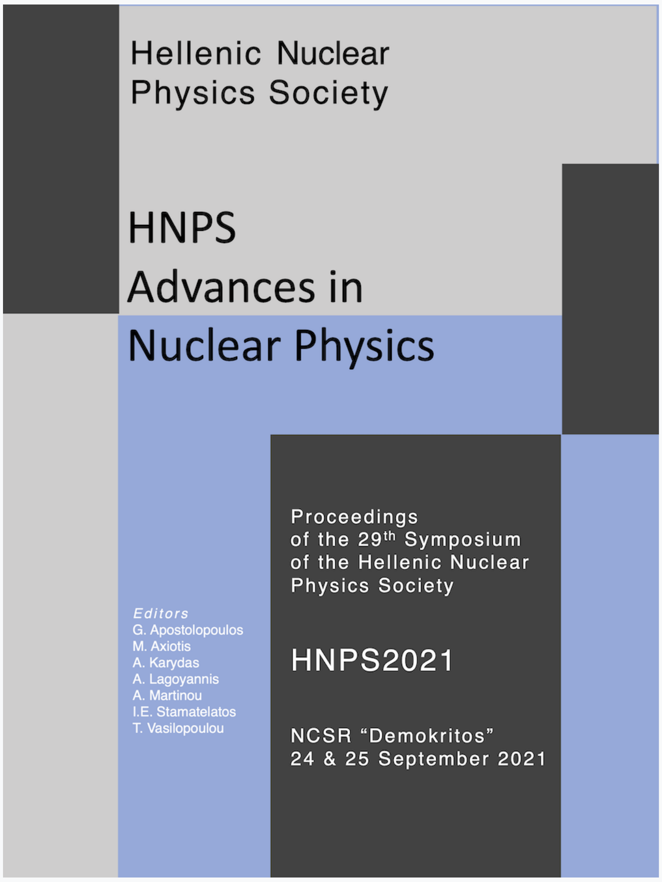

Validation of Empirical Formulas for Spallation Residue Production in 0.3A-1.5A GeV 56Fe+p Reactions

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Validation of Empirical Formulas for Spallation Residue Production in 0.3A-1.5A GeV ^{56}Fe+p Reactions

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Abstract The production of spallation residues in ^{56}Fe+p bombardments is described with empirical parametric formulas often used in cosmic-ray astrophysics, activation studies and isotope production for medical applications. Experimental observables including mass, charge and isotopic distributions are compared with calculations using two versions of the formulas of Rudstam and Silberberg-Tsao and the SPACS formula. For reference, a comparison is made with the predictions of a two-stage reaction model. Deviation factors obtained in these approaches are reported.

Keywords Spallation reactions, empirical formulas, two-stage model, isotope production

INTRODUCTION

Spallation reactions occurring in high-energy hadron-nucleus collisions involve the emission of several nucleons and produce a variety of isotopes. These reactions provide the ground for the development and testing of nuclear reaction models in the intermediate-energy regime. In applications, spallation of medium-weight targets is important in cosmic-ray studies, activation of materials in space and radiation safety [1,2].

Practical applications require knowledge of accurate cross sections of spallation products for various targets and bombarding energies. Experimental cross sections from many reaction systems are not always easily obtainable. The commonly used two-stage description is based on the Monte-Carlo method for the simulation of the intra-nuclear cascade and subsequent statistical evaporation of the excited pre-fragments. This method produces accurate results, but it is time-consuming, and may require a detailed tuning of the reaction parameters. For this reason, high-energy simulations often rely to semi-empirical cross-section formulas. The form of these formulas is suggested by the dominant physics processes and contain parameters whose values are obtained in a fitting procedure using available experimental cross section data.

In the present paper, we compare experimental spallation residue cross sections in ^{56}Fe+p reactions at 0.3A-1.5AGeV [3] with the results of four semi-empirical formulas and a two-stage model. We report the deviation factors obtained by each procedure.

EMPIRICAL FORMULAS AND THE TWO-STAGE MODEL

The empirical formulas we consider express the cross section for isotope production as a product of a mass yield and a charge dispersion factor. In the earliest formula developed by Rudstam [4], the cross section of a residue (A, Z) produced in the bombardment of a target (A_t, Z_t) with hadrons of energy E is given by

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$$\sigma(A,Z) = \sigma_0 exp(-P|A - A_t|)exp\left(-R\left|Z - Z_{prob}\right|^{\nu}\right)$$

Here, σ_0 is proportional to the total inelastic cross section. The second factor describes the product mass distribution, which depends strongly on the intra-nuclear cascade stage. The third factor describes the distribution of fragment charge around the most probable value Z_{prob}. The parameters P and R were determined with a least-squares fit using a large set of experimental cross sections. Two values of the exponent ν were considered. The version with $\nu = 3/2$ is referred to as the CDMD formula. The value of $\nu = 2$, corresponds to a Gaussian fit and results in the CDMD-G formula.

An improvement of the Rudstam formula was reported by Silberberg and Tsao (S-T) [5]. This formula involves the product of three additional factors, namely, a nuclear structure factor depending on the density of states of the product nucleus, an enhancement factor for the light-mass evaporation products and a pairing factor which accounts for the enhancement of even Z residue cross sections. An update has been reported in Ref. [6] and denoted hereafter as S-T-B.

The EPAX formula is based on a parametrization of measured production cross sections in high-energy fragmentation of medium to heavy mass projectiles and targets. It is valid in the so-called "limiting fragmentation" regime. The version EPAX2.1 is known to provide a good quality of agreement for neutron-poor fragments.

The SPACS formula [7] resulted after a detailed examination of each term. The isobaric distributions are calculated with a functional form borrowed from EPAX, taking account of the bombarding energy dependence. The influence of closed shells and the even-odd staggering in the residue yields are explicitly introduced.

In a Monte-Carlo two-stage description, we model the reaction with the INC stage code ISABEL [8] coupled with the sequential binary decay code MECO [9]. Information on the calculation for the reactions considered in the present paper is reported in Ref. [10].

COMPARISONS WITH THE EXPERIMENTAL DATA

The symbols in Figs. 1-4 show experimental cross sections of ^{56}Fe + p reactions at the indicated bombarding energies. Columns 1 and 2 show the mass and charge distributions. Columns 3 and 4 show the values of <A>/Z and H.W.F.M. (σ_Z) of the isotopic distributions as a function of Z. In Fig.1, the dashed black lines show the calculated distributions with Rudstam's CDMD-G formula. The agreement with the experimental data improves with increasing bombarding energy. The red solid lines show the calculation with the CDMD formula. CDMD provides a slightly better description of the data than CDMD-G.

In Fig. 2, the results of calculations with the Silberberg and Chao (S-T) formula are shown with the dashed lines. The solid red lines show the results of the calculation with the updated formula by Silberberg, Tsao and Barghouty (S-T-B). These formulas improve the description of the A-, Z- and <A>/Z distributions with exceptions for the lowest-Z residues. The σ_Z-data are overestimated below Z=15-20 by both formulas at all bombarding energies. The S-T-B formula provides a better description of the data than the S-T formula.

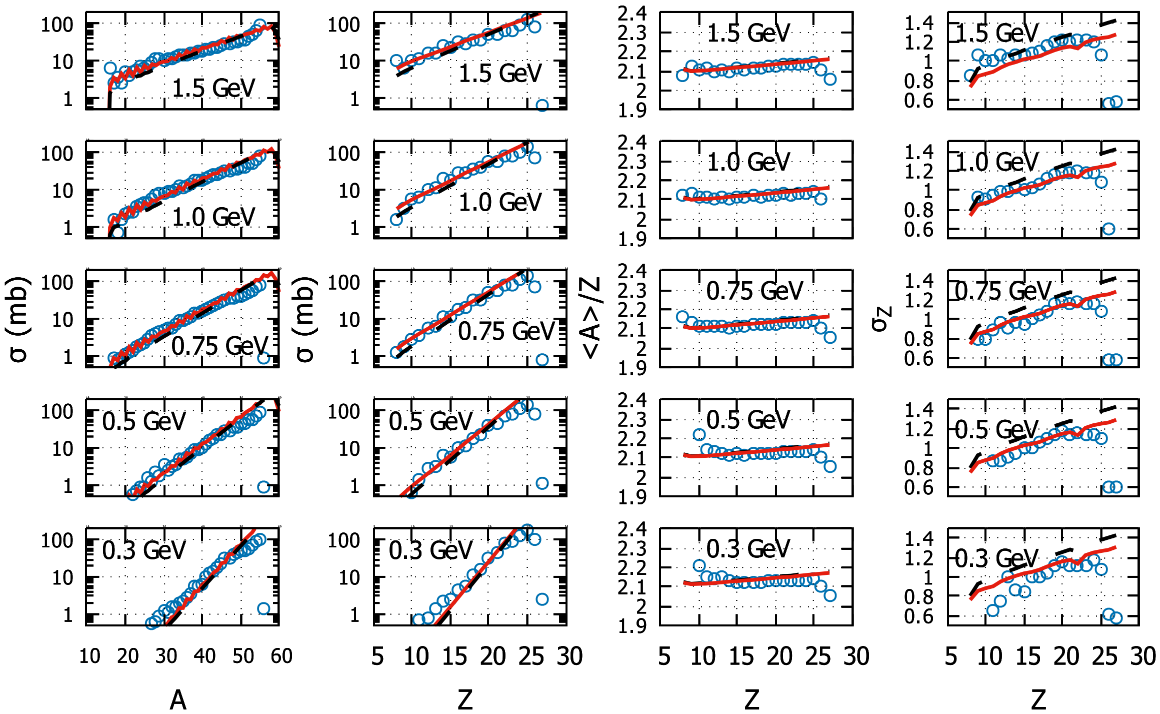

Fig. 1. *Mass and charge distributions, <A>/Z and σZ of the isotopic distributions as a function of Z at the indicated bombarding energies per nucleon. Experimental data (symbols) are compared with the CDMD-G (dashed lines) and CDMD (solid lines) formulas, respectively.*

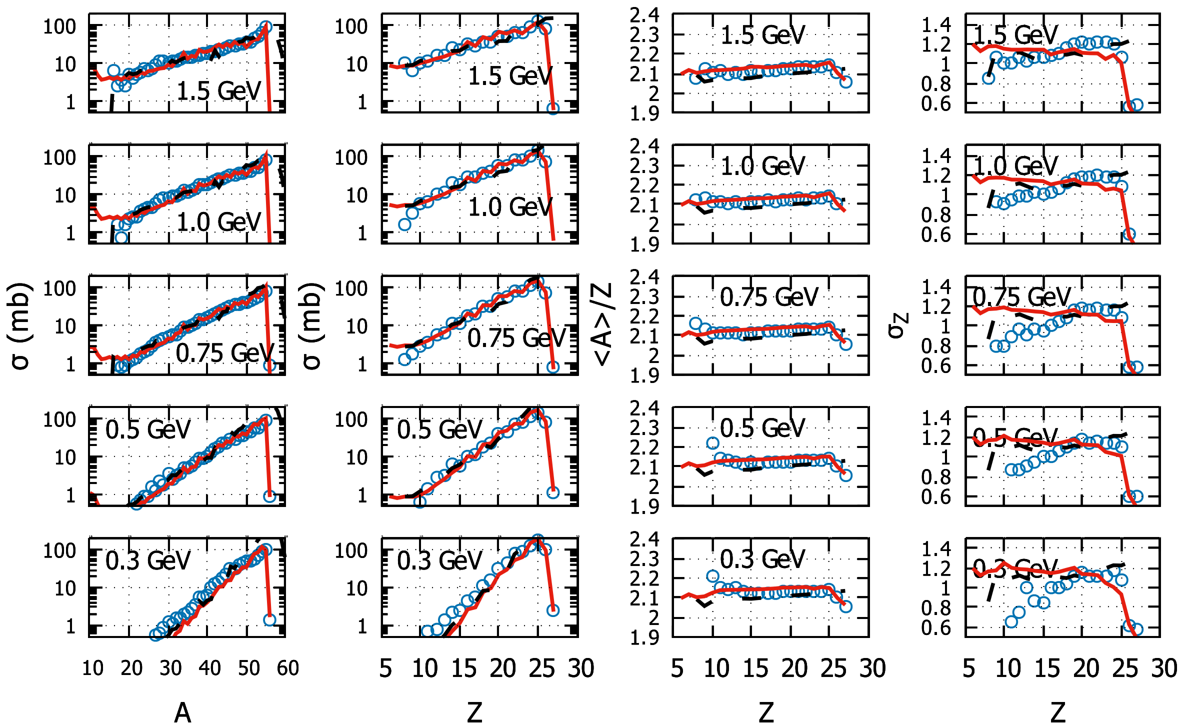

Fig. 2. *Experimental data described in the caption of Fig. 1 are compared with the results of S-T (dashed lines) and S-T-B (solid lines) formulas, respectively.*

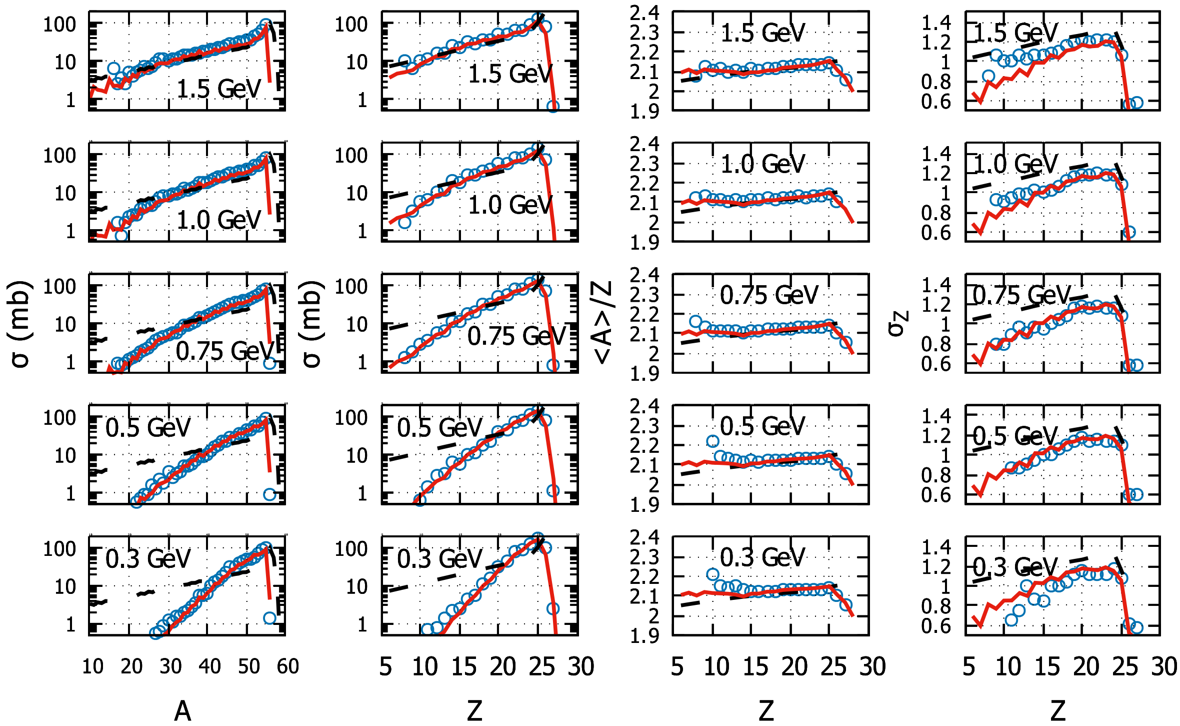

Fig. 3. *Experimental data described in the caption of Fig. 1 are compared with the results of EPAX2.1 (dashed lines) and SPACS (solid lines) formulas, respectively.*

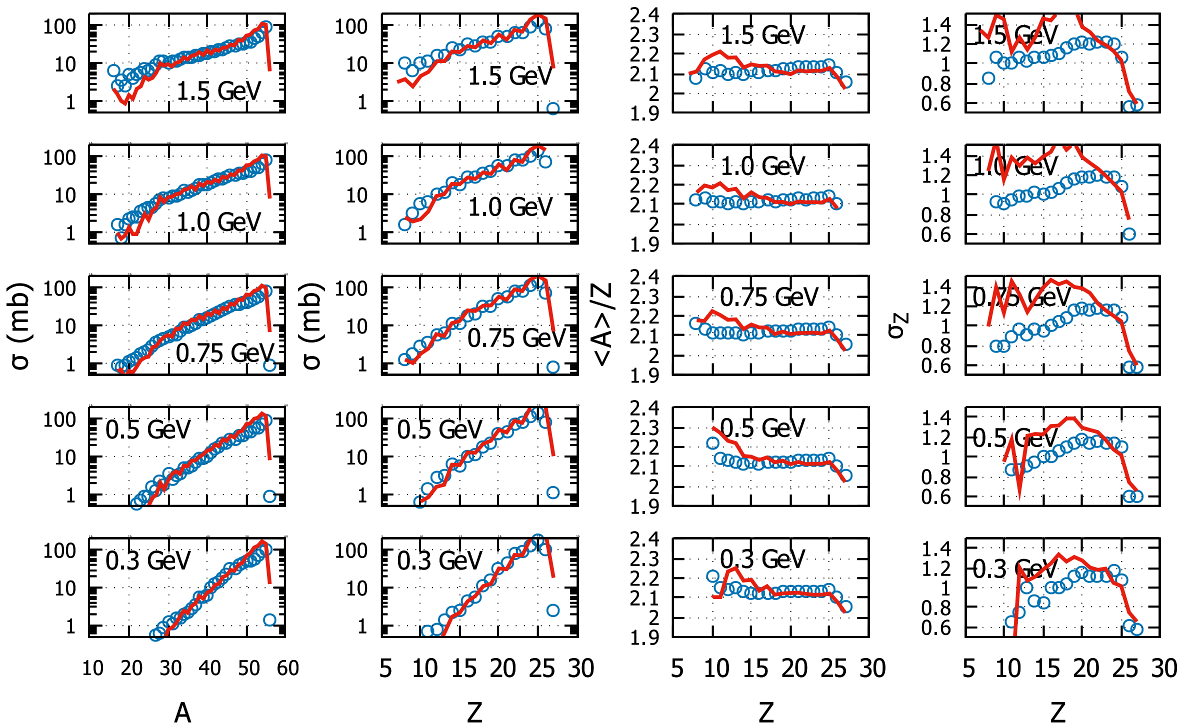

Fig. 4. *Experimental data described in the caption of Fig. 1 are compared with the results of a two-stage ISABEL-MECO reaction model calculation (solid lines).*

In Fig. 3, the results of the SPACS formula are shown with the solid lines. The agreement with all data at all bombarding energies is very good. The dashed lines show the results of the EPAX formula (version 2.1). This formula expresses the residue distributions corresponding to a full fragmentation scenario. Experimental data at the highest bombarding energy of $1500 MeV$ are consistent with this expectation.

The solid lines in Fig.4 show the results of the ISABEL-MECO calculation with parameters reported in Ref. [10]. For the mass and charge distributions, the agreement is very good at all bombarding energies. However, the experimental data for $< A >/Z$ and σ_Z are overpredicted below Z=15 and Z=20, respectively.

Quantitative comparisons of the above calculations with the experimental data were made with the F-test [11]. Table I gives the deviation factor F deduced from comparisons of the empirical formulas with the experimental A-, Z-, <A>/Z and σ_Z data at the indicated bombarding energies.

Table 1 *Deviation factor F of the formulas considered in the present work evaluated for the A and Z-Distributions (Z ≥ 6), <A>/Z and σ_Z at the indicated bombarding energies*

	E (MeV)	CDMD-G	CDMD	S-T	S-T-B	SPACS	EPAX 2.1	ISABEL-MECO
A-Distribution	300	3.513	3.281	3.094	2.347	1.598	5.755	2.467
	500	2.746	2.724	2.671	1.274	1.328	3.627	1.844
	750	2.362	2.316	2.872	1.235	1.271	2.977	1.589
	1000	2.660	2.601	2.581	1.130	1.451	2.871	1.869
	1500	2.474	2.419	2.285	1.266	1.479	2.490	1.957
Z-Distribution	300	6.027	5.663	5.184	2.172	1.588	5.449	2.755
	500	5.047	4.950	4.630	1.244	1.265	3.267	1.892
	750	4.353	4.225	4.063	1.282	1.203	2.469	1.755
	1000	1.554	1.460	1.543	1.383	1.201	1.949	1.453
	1500	3.872	3.689	1.461	1.201	1.296	1.566	2.011
$< A >/Z$	300	1.022	1.021	1.023	1.012	1.014	1.022	1.023
	500	1.022	1.020	1.024	1.013	1.014	1.021	1.024
	750	1.016	1.015	1.018	1.010	1.008	1.016	1.021
	1000	1.009	1.009	1.016	1.008	1.005	1.013	1.021
	1500	1.015	1.014	1.016	1.008	1.004	1.010	1.022
σ_Z	300	1.543	1.457	1.552	1.483	1.302	1.470	1.922
	500	1.421	1.340	1.399	1.303	1.172	1.320	1.736
	750	1.387	1.312	1.365	1.309	1.155	1.321	1.899
	1000	1.242	1.200	1.209	1.144	1.094	1.189	1.831
	1500	1.342	1.304	1.282	1.142	1.162	1.168	1.733

CONCLUSIONS

From Figs. 1-4 and Table I we may draw conclusions on the quality of agreement of our calculations with the experimental data. The empirical formulas improve in the order: CDMD-G, CDMD, S-T, S-T-B and SPACS. The EPAX formula provides its best description at the highest bombarding energy. The two-stage model with global statistical decay parameters describes well the experimental mass and charge distributions. Minor refinements could improve the description of the isotopic distributions.

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