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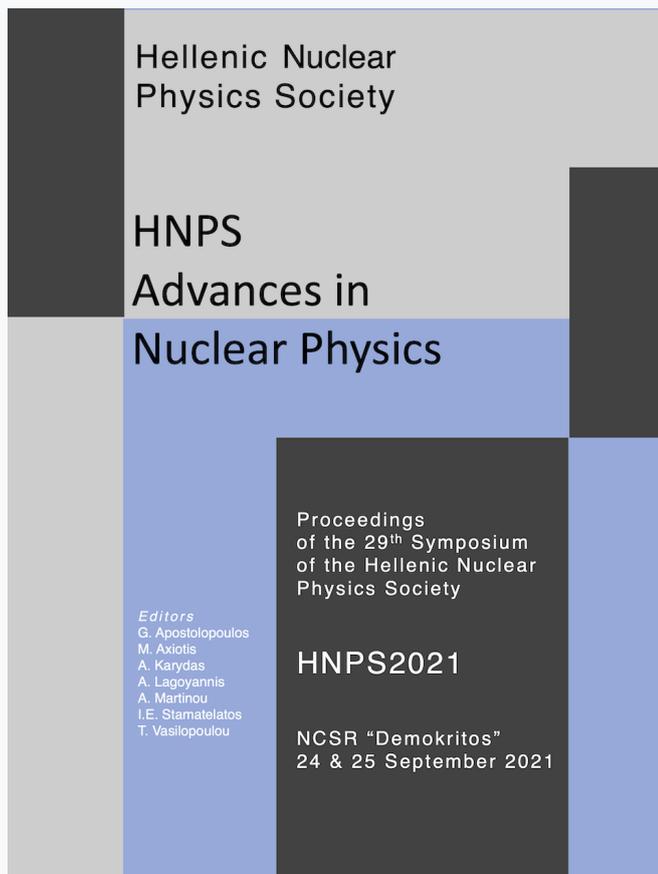
Unitary limit in heavy nuclei

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Abstract   The unitary limit refers to a scattering problem at infinite scattering length where a scattering state becomes bound. Such a limit is experimentally accessible in systems of cold atoms at the vicinity of Feshbach resonances. In nuclear physics, a physical manifestation of the unitary limit is of interest both from the experimental challenge to measure such a limit in nuclei and from the theoretical aspects that accompany that limit such as conformal symmetry, a quantum critical point and the BCS-BEC crossover. In this talk the application of a symmetry-based approach to the unitary limit in collective states of heavy, even-even nuclei is presented that is performed by means of the Interacting Boson Model of nuclear structure in conjunction with the Feshbach formalism of nuclear reactions. The results of this application start from the determination of what is to be measured in the experiment for the examination of the unitary limit in collective nuclear states. That is the fluctuations of the cross-section of the A+2n compound nucleus. The primary theoretical result concerns the representations of conformal symmetry in A+2n compound nuclei via the fluctuations of cross sections.

Keywords   Two neutrons, compound nuclei, Interacting Boson Model, conformal symmetry

INTRODUCTION

The unitary limit has sparked significant interest in nuclear physics that has been devoted mainly to light nuclei so far, see for instance [1]. A challenge is to determine the observable, which reflects the tuning of the scattering length in nuclear physics. The recent introduction of the unitary limit in heavy nuclei [2] points that a fluctuation of the cross-section of an A+2n (two neutrons) compound nucleus tunes the scattering length. A brief presentation of the unitary limit in heavy nuclei is presented in this short contribution. Some consequences are briefly discussed for the manifestation of conformal symmetry in nuclear physics.

DETAILS

A standard introduction to a scattering case with two spherical waves [3] starts from the entire scattering wavefunction

$$\Psi_0(r) = \frac{\exp(-ikr)}{r} - S\frac{\exp(ikr)}{r}, \quad S = \exp(2i\delta_0) = \exp(-2ika). \tag{1}$$

For reasons that will become clear later, a spherical incident wave is used, i.e., a wave whose distance from the scattering center is not constrained to one dimension but has spherical symmetry. The left-hand side of (1) shows the spherical incident wave $\exp(-ikr)/r$, and the outgoing $\exp(ikr)/r$ is affected by the factor S, which is the scattering matrix element and reflects the scattering amplitude. S is expressed in terms of the exponential of the phase shift δ_0. The scattering length a is defined for very low energies by the condition $\delta_0 = -ka$ and shows the intercept of the scattering wavefunction for the r axis.

In nuclear physics and in the physics of cold atoms, there are scattering cases where the element of the scattering matrix S is affected by a fluctuation that represents the formation of an intermediate state. In nuclear physics, the intermediate state represents a compound nucleus while in cold atoms the intermediate state represents a diatomic molecule. In both cases, these intermediate states states

exemplify a resonance of the Feshbach formalism, the so-called Feshbach resonance [4]. In those cases, the scattering wavefunction takes the form

$$\Psi_0(r) = \frac{\exp(-ikr)}{r} - \left(1 - \frac{i\Gamma_m}{E - E_m + i\Gamma_m/2}\right)\exp(2i\delta_0)\frac{\exp(ikr)}{r}. \qquad (2)$$

In the language of nuclear physics, Eq (2) gives the scattering matrix element $S' = S\, S_R$, with $S_R = 1 - i\Gamma_m/(E - E_m + i\Gamma_m/2)$ a fluctuating part [5] that fluctuates with energy and contains resonances of energies E_m and widths Γ_m. In general, in nuclear reactions, states of compound nuclei are represented by such a fluctuation in the total cross section after the latter has being averaged in a certain energy range. These average values and their fluctuations are the fundamentals of statistical models for the scattering matrix, see for instance [6]. For the purposes of the examination of the unitary limit in nuclear physics, one observes that $S' = S\, S_R = \exp(-2ik(a + a'))$, i.e that the fluctuating part of the scattering matrix defines the effective scattering length

$$a_{eff} = a + a', \quad a' = \frac{1}{2k}\tan^{-1}\left[\frac{\Gamma_m(E - E_m)}{(E - E_m)^2 + \Gamma_m^2/4}\right], \qquad (3)$$

which is of the same form with the effective scattering length of the Feshbach formalism in systems of cold atoms [4]. One now recalls that cross-sections are computed by the absolute magnitude of the elements of the scattering matrix and a fluctuation of the scattering matrix is imprinted on the cross-section of the reaction under study [5,6]. In other words, one observes that in nuclear physics the tuning of the scattering length is observable via the fluctuation of the cross-section [2] i.e that the scattering length maximizes itself at that fluctuation which represents the resonance corresponding to the intermediate state. That observation points out to what should be measured for the examination of the unitary limit in compound nuclei – the fluctuation of the cross-section which is presented in the next section.

On the other hand, in experiments with cold atoms, the effective scattering length is directly proportional to the applied magnetic field responsible for their trapping [4]. The scattering length is tuned to infinity when the applied magnetic field takes that value which raises the energy between the scattered atoms (open channel) to the energy of a bound molecular state (closed channel) – the formation of the diatomic molecule. In that case, a resonance occurs in a scattering wavefunction with a fluctuating part of the form (2) [4]. This open-closed channel crossing defines the Feshbach resonance, which permits the experimental observation of the unitary limit in systems of cold atoms. Such an open-closed channel crossing is prescribed in [2] for the collective states of heavy even-even compound nuclei in the context of the Interacting Boson Model [7] and a very brief narration of some parts of this prescription follows below.

The unitary limit at the vicinity of Feshbach resonances in systems of cold atoms is incorporated into a symmetry-based approach [8] on the Schrodinger equation that describes the trapping of cold atoms into a harmonic oscillator potential in hyperspherical coordinates. One proves [2] that this equation obeys the O(6) symmetry for trapping two atoms. Furthermore, based on a group theoretical analysis [2], this equation is algebraically compared with the equation of the O(6) limit for the Interacting Boson Model [7]. The result is the definition of a scattering problem of two incident neutrons (2n) onto the collective state of a heavy, even-even target nucleus. The scattering wavefunction now reads

$$\Psi_0(r) = \frac{\exp(-ik_r r)}{r^{5/2}} - \left(1 - \frac{i\Gamma_m}{E - E_m + i\Gamma_m/2}\right)\exp(2i\delta_0)\frac{\exp(ik_r r)}{r^{5/2}}, \qquad (4)$$

which is of the same form with (2). However there are substantial differences. In general, a spherical wave in d dimensions is accompanied by the radial factor $1/(r^{(d-1)/2})$ and in $d = 3$ the radial factor is $1/r$. In (4) the radial factor is $1/(r^{5/2})$ and reflects the generalization of the scattering problem in the simultaneous scattering of two particles or a pair of particles. The wavenumber k_r symbolizes the relative momentum between the incident (outgoing) neutron pair and the collective state of the target nucleus. The expression of k_r in terms of the momenta of each incident neutron is given in [2]. The Schrodinger equation of the scattering lives now in $d = 6$ dimensions and the partial wave analysis for two particles is determined by the O(6) symmetry which produces the cross-section

$$\sigma = \frac{(4\pi)^3}{k_r^2 + 1/a_r^2(k_r)}. \tag{5}$$

Note here the cubic power of the solid angle factor $(4\pi)^3$ which signifies the pair in contrast with the cross-section of a single neutron which would contain only the first power of 4π. The quantity $a_r(k_r)$ is a generalized scattering length defined by and subjected to the effective range expansion

$$k_r \cot \delta_0 = \frac{1}{a_r(k_r)}, \quad a_r(k_r) = \frac{1}{a_r} - \frac{1}{2} k_r^2 r^* + \cdots \tag{6}.$$

r^* is introduced as an effective range for the pair-collective state interaction and is experimentally determinable by the width of the pair-collective state resonance [2].

The above relations define the (2n)-collective state scattering problem, and their solutions are presented in detail in [2]. In that case, the IBM Hamiltonian of the O(6) limit [7] plays the role of the trapping potential for the incident neutrons. The Feshbach formalism applies here and the intermediate state of energy E_m and width Γ_m gives rise to the A+2n compound nucleus. The unitary limit manifests itself at the crossing energy between the open channel defined by the two incident neutrons (2n) onto the ground state of the target nucleus that is represented by N_b bosons with the closed channel of $N_b + 1$ bosons. This is equivalent with the capture of the two neutrons (2n) as an intermediate boson by the target nucleus. In general, a set of these resonances – for the formation of the intermediate state of the A+2n compound nucleus as an intermediate boson – is provided by the energies of the IBM states of the closed channels. When the energy of the open channel crosses with the energy of the closed channel of the IBM state, the generalized scattering length satisfies the resonance condition $1/a_r(k_r)=0$. This is the analog of the open-closed channel crossings in systems of cold atoms. For low k_r, such that $k_r^2 \to 0$, the only term that survives in the effective range expansion (6) is the energy independent scattering length $1/a_r$. In that case, the resonance condition for the $a_r(k_r)$ signifies the unitary limit. Therefore the unitary limit manifests itself in heavy nuclei at the formation (decay) of the A+2n compound nucleus at that particular situation where the incident (outgoing) 2n are captured (decays) as a boson.

That intermediate boson manifests itself by the fluctuation of the cross-section which is to be observed at the energy that separates the captured neutron pair from the target nucleus. It turns out that this energy is the two-neutron separation energy S_{2n} [2]. Therefore the observation of a fluctuation of the cross-section at the energy of S_{2n} above the ground state of the target nucleus signals an infinite scattering length in the sense of a resonance with the intermediate state of Eq (3). The width of such a fluctuation is determined for a particular type of coupling that proposes a two neutron transfer reaction in an exotic nucleus [2]. This is

$$\Gamma_m = b^2 \frac{4M}{\hbar^2} k_r, \tag{7}$$

where M symbolizes the neutron mass and b is proportional to the boson number of the closed channel state up to a spectroscopic factor.

In [2], cross-section rules for the incident (outgoing) neutron pair are derived for the A+2n compound nucleus. In particular, the rules of the reaction cross-section, of the elastic cross-section and of the compound-elastic cross section are derived for a neutron pair. In the simplest case, when the exit channel of the target nucleus remains the same with the entrance channel, the fluctuation of the total cross section is given by the compound-elastic cross section which for the A+2n compound nucleus reads

$$\sigma = \frac{(4\pi)^3}{k_r^{\ 2}} \frac{\Gamma_m^2}{(E-E_m)^2 + \Gamma_m^2/4}.$$

$$(8)$$

The examination of the unitary limit in a heavy even-even nucleus is therefore reflected on the examination of the cross-section of Eq (8) as a distribution around the resonance state with a peak centered at the two-neutron separation energy and width given by Eq (7). At the resonance, the pair-collective state scattering length is maximized and the unitary limit is achieved for a very low k_r. It deserves to be mentioned that the unitary limit in the (2n)-collective state scattering reflects a unitary pair-collective state interaction in the A+2n compound nucleus. Such an interaction is compensated at the reaction channels by the boundary condition

$$\frac{4\pi^3 a_r \hbar^2}{M} \delta(r) \to \lim_{r\to 0} \Psi_0(r) = \frac{C}{r^4} - \frac{1}{a_r^{\ 4}},$$

$$(9)$$

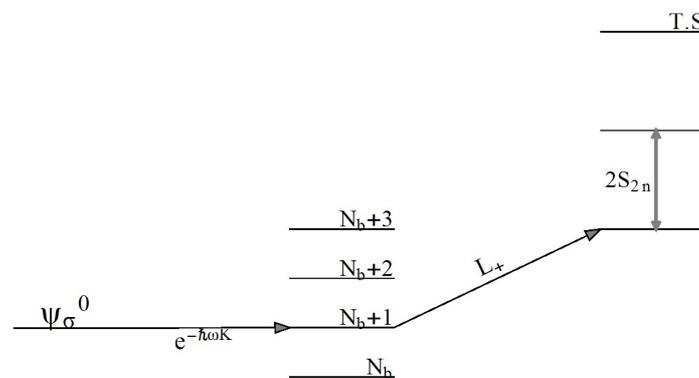where $\Psi_0(r)$ shows the entire pair-collective state scattering wavefunction.

Fig. 1. *The emergence of the tower of equally spaced states (T.S) from the unitary limit [1]. N_b is the boson number of the closed-channel state while ψ_σ^0 represents the state at the unitary limit. The repeated application of the SO(2,1) ladder operators on ψ_σ^0, build successively the members of the tower.*

DISCUSSION

The consequences of the unitary limit start from the Bardeen Cooper Schriefer-Bose Einstein Condensation (BCS-BEC) crossover, the operator-state correspondence of an underlying conformal field theory, and the hydrodynamics of zero viscosity [9]. So far, we have examined the consequences of the unitary limit concerning conformal symmetry. In the A+2n compound nucleus, the conformal group in one dimension is isomorphic to the SO(2,1) group that is defined by the generators

$$L_+ = -(d^\dagger d^\dagger + s^\dagger s^\dagger), \quad L_- = -(dd + ss), \quad L_0 = \frac{1}{2}\left(d^\dagger d + s^\dagger s + \frac{6}{2}\right). \qquad (9)$$

The s and d bosons here should be perceived as the elements of the U(6) algebra in their most general sense [7]. The generators $L_\pm$ create non-compact boosts on the Hamonic Oscillator of the O(6) limit with an harmonic oscillator length that is determined by the two-neutron separation energy [2]. That O(6) Hamiltonian coincides with the L_0 generator of Eqs (9). Moreover, the $L_\pm$ create and annihilate states with two bosons. A tower of equally spaced states emerges by the repeated application of the $L_\pm$ operators on the resonance state, as shown in Figure 1 (T.S). Conformal symmetry is represented on the mappings of the resonance state, the ψ_σ^0 in Figure 1, to IBM states of the closed channels. This mapping is equivalent to the statement that the formation of the neutron pair 2n as an intermediate boson at unitarity represents the primary state of the conformal algebra. Equivalently, this mapping reflects the one-dimensional conformal transformation on the boson number radius $\rho(t) = \rho/\lambda(t)$ and signals the capture of the incident neutron pair that rescales the boson number radius. One proves [2] that the invariant quantity (scaling dimension) of that mapping is the closed channel state's O(6) quantum number and coincides with the boson number in the lowest representation.

The repeated application of the operators to the primary state generates the tower of equally spaced states with the energy separation of two bosons. The primary operator of the conformal algebra that creates the primary state is the $s^\dagger + s$ and acts on the ground state of the target nucleus with N_b bosons. This operator is tentatively compared with a two-neutron transfer to a nucleus that however emits back the transferred pair in [2]. The amplitude of that process determines the fluctuation that represents the compound-elastic reaction with the rule of the cross section to be given in Eq (8). One expects that the tower of equally spaced states manifests itself as a regularity pattern of a whole sequence of fluctuations of the cross-section. How such a regularity pattern of a sequence of fluctuations is to be measured is not determined in [2]. However, in the tentative example of the two neutron transfer, that regularity means that either one changes the target nucleus by one boson and performs the two neutron scattering again for a series of isotopes or one varies the number of incident neutron pairs on the same target nucleus. In both cases, the sequence of the fluctuations of the cross sections should give one regular pattern.

CONCLUSIONS

Two are the main conclusions of the introduction of the unitary limit in heavy even-even nuclei. The first conclusion regards the experimental observable to examine the unitary limit in nuclear physics. That observable is the fluctuation of the cross-section as it tunes the scattering length. For example, in the A+2n compound nucleus, the experimental measurement of a compound-elastic cross-section at the energy of the two neutron separation energy from the ground state of the A nucleus with the width of Eq (7) signals the unitary limit.

The second conclusion is the proposition of the observable fact that emerges out of the conformal symmetry at the unitary limit in heavy nuclei. Such an observable fact is the regularity pattern of a sequence of fluctuations of the cross-section of determined energies and widths. That regularity pattern contrasts the usual random pattern of the fluctuations of the cross-sections in A+1n compound nuclei. It, therefore, constitutes the new observable fact that emerges out of the application of the unitary limit and conformal symmetry in heavy A+2n compound nuclei.

Acknowledgments

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