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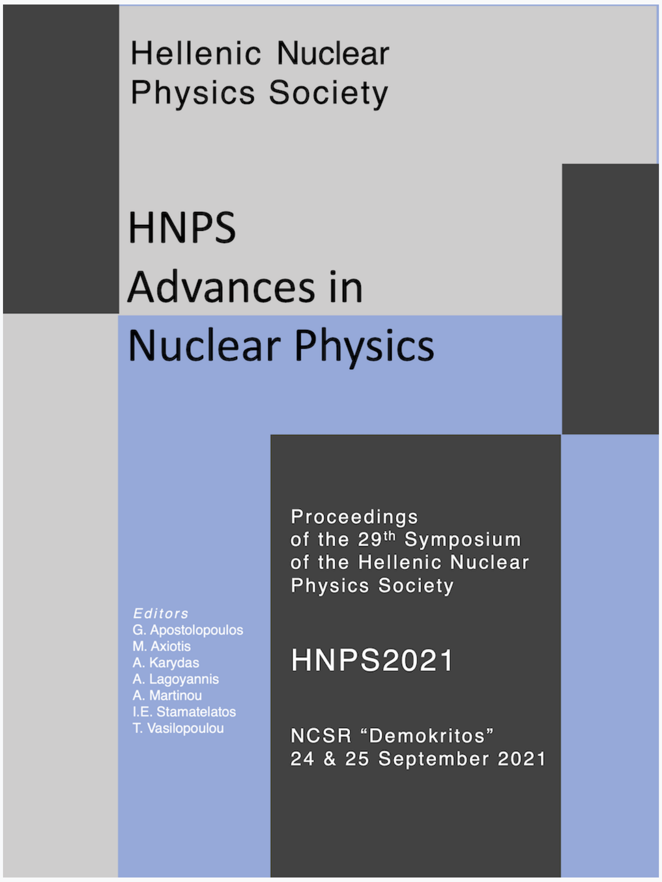

The s and d bosons into the Shell Model SU(3) wave functions

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Abstract The Shell Model SU(3) symmetry was discovered by Elliott in 1958 and since then has been the algebraic realization of the Nuclear Shell Model. It is considered to be a fermionic nuclear model, since it tackles the occupancies of the orbitals by nucleons. Elliott proved that the SU(3) symmetry appears in the LS (spin-orbit) coupling scheme of the nucleons. On the other side the Interacting Boson Model was proposed by Arima and Iachello in 1975 and it is a boson model. The bosons in the Interacting Boson Model are being derived by nucleon pairs. But the mapping (Otsuka-Arima-Iachello mapping) of the nucleon pairs into bosons is functional only in the jj coupling scheme and so it is applicable only in the U(5) and O(6) limits of the Interacting Boson Model. But what is the origin of the s and d bosons in the SU(3) limit of the Interacting Boson Model? Hereby I demonstrate that the s and d bosons are present into the spatial Shell Model SU(3) wave functions and that they derive from pairs of harmonic oscillator quanta.

Keywords Elliott SU(3), Shell Model, Interacting Boson Model

INTRODUCTION

The Shell Model SU(3) symmetry [1-4] is the algebraic realization of the Nuclear Shell Model and describes adequately the deformed nuclei. Elliott proved that if one uses the Shell Model single-particle orbitals in the cartesian coordinate system [3,5], s/he shall result to have the familiar rotational spectrum, *i.e.*, the nuclear states are organized in bands labeled by the K quantum number and possess the angular momentum L as a good quantum number.

The “Shell Model SU(3) symmetry” nowadays is called “Elliott SU(3) symmetry”, but I shall avoid this name, because a) Elliott never called his model “Elliott SU(3)” in his articles, b) some people today are misguided by the name and believe that the “Elliott SU(3) symmetry” is something different from the “Nuclear Shell Model” [6]. The Shell Model SU(3) symmetry is originally applied in valence nuclear shells, which exceed among the three dimensional harmonic oscillator nucleon magic numbers 2, 8, 20, 40, 70, 112,... Such magic numbers are usually present in light nuclei or in nuclei with shape coexistence [7]. Thus an extension of the model had to be discovered for the medium mass and heavy nuclei, which possess the spin-orbit like magic numbers 6, 14, 28, 50, 82, 126...[5]. Our theory group in Demokritos (along with collaborators) has developed the proxy-SU(3) symmetry [5,8,9], which treats all the orbitals of the spin-orbit like shells except one. In the proxy-SU(3) scheme both the normal and the intruder parity orbitals of the spin-orbit like shell are treated together. The excluded orbital is the last to be occupied (by two protons or neutrons) and so its exclusion from the shell is not affecting the nuclear properties.

The Shell Model SU(3) symmetry is valid in the LS coupling scheme. This means that the many-nucleon Slater determinant, which is the nuclear wave function, is separated into two parts: a) the spatial part and b) the spin-isospin part. The one part is the conjugate of the other and thus the product of them (the Slater determinant) is totally antisymmetric in the permutation of two nucleons. The spatial part results in having the SU(3) symmetry of Elliott and it is a many-quanta wave function

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[10]. It has been proven that the short-range character of the nucleon-nucleon interaction demands that the spatial part is as symmetric as possible in the interchange of the quanta [10].

The Interacting Boson Model [11] treats the s and d bosons for the construction of the nuclear spectrum. The s, d bosons are spherical tensor operators of degree 0, 2 respectively. In the Otsuka-Arima-Iachello mapping [12] the nucleons are coupled into pairs with total angular momentum 0 and 2. Such pairs are mapped into the s and d bosons. But this kind of mapping is valid only in the jj coupling scheme, in which the angular momentum $\boldsymbol{l}$ is coupled with the spin $\boldsymbol{s}$ of the nucleon to create the total angular momentum $\boldsymbol{j}=\boldsymbol{l}+\boldsymbol{s}$. Afterwards the total angular momenta of all the nucleons are coupled to deliver the nuclear angular momentum J. This kind of coupling is suitable for the O(5) symmetry and so it is applicable only in the O(6) and U(5) limits of the Interacting Boson Model, which possess the O(5) as a sub-algebra [13]. The question is, what is the origin of the s and d bosons of the Interacting Boson Model in the majority of nuclei, which is deformed and thus described by the SU(3) symmetry and the LS coupling scheme?

THE SHELL MODEL SU(3) WAVE FUNCTIONS

Harvey gave an example of the many quanta SU(3) wave functions [11]. It is crucial to understand that the SU(3) wave functions contain only the spatial part of the full nuclear wave function and so the vacuum is the state of no quanta $|0\rangle$. The many quanta SU(3) wave functions are represented by irreducible representations (irreps) of the type (λ,μ), in which the $\lambda+\mu$ quanta are symmetric upon their interchange [10]. Thus a (2,0) irrep represents two quanta in the z axis, which are symmetric upon their interchange. Similarly, an irrep (4,0) has 4 symmetric quanta in the z axis etc.

The harmonic oscillator quanta are being created by the creation operators of the harmonic oscillator:

$$a_k^\dagger=\sqrt{\frac{m\omega}{2\,\hbar}}k-\frac{i}{\sqrt{2m\omega\hbar}}p_k,\quad (1)$$

where m is the mass of the nucleon, ω is the frequency of the harmonic oscillator, $k=x,y,z$ is the position of the particle in the cartesian coordinate system and p_k is the relevant momentum. These operators act on the spatial vacuum $|0\rangle$ to create the cartesian quanta. We may use the above operators to create a spherical tensor of degree 1:

$$u_{-1}^\dagger=\frac{a_x^\dagger-ia_y^\dagger}{\sqrt{2}},\qquad u_0^\dagger=a_z^\dagger,\qquad u_1^\dagger=-\frac{a_x^\dagger+ia_y^\dagger}{\sqrt{2}}.\quad (2)$$

These operators when acting on the vacuum create a harmonic oscillator quantum with angular momentum 1 and projection $m=-1,0,1$ respectively.

The harmonic oscillator quanta can be coupled into symmetric pairs. For instance the spatial wave function of the SU(3) irrep (2,0) is the:

$$\varphi_{zz}=\varphi_z(i_1)\varphi_z(i_2)\quad (3)$$

or

$$\varphi_{zz}=a_z^\dagger(i_1)a_z^\dagger(i_2)|0\rangle,\quad (4)$$

where the i_1,i_2 ennumerate the valence particles and obtain the values 1, 2, 3,..., A_{val}, with $A_{val}=Z_{val}+N_{val}$ being the number of the valence protons and neutrons. As Harvey wrote the i_1,i_2 may obtain the same value, if the quanta derive from the same nucleon [11].

The spherical harmonic oscillator quanta can be coupled into symmetric pairs too. This is achieved by the coupling of the spherical tensors:

$$(F_M^L)^\dagger(i_1,i_2) = \sum_{m,m'} (1m1m'|LM) u_m{}^\dagger(i_1)\, u_{m'}{}^\dagger(i_2), \quad (5)$$

where the $(1m1m'|LM)$ stands for the Clebsch-Gordan coefficient. The symmetric pairs of the spherical harmonic oscillator quanta result to spherical tensors of degree $L = 0, 2$. These spherical tensor operators can be labeled as:

$$(F_0^0)^\dagger(i_1,i_2) = s^\dagger(i_1,i_2), \qquad (F_M^2)^\dagger(i_1,i_2) = d_M^\dagger(i_1,i_2). \quad (6)$$

Using a Clebsch-Gordan calculator and the relations (2) we get for instance that:

$$s^\dagger(i_1,i_2) = -\frac{1}{\sqrt{3}}\left(a_x^\dagger(i_1)a_x^\dagger(i_2) + a_y^\dagger(i_1)a_y^\dagger(i_2) + a_z^\dagger(i_1)a_z^\dagger(i_2)\right), \quad (7)$$

and

$$d_0^\dagger(i_1,i_2) = -\frac{1}{\sqrt{6}}\left(a_x^\dagger(i_1)a_x^\dagger(i_2) + a_y^\dagger(i_1)a_y^\dagger(i_2)\right) + \sqrt{\frac{2}{3}}\,a_z^\dagger(i_1)a_z^\dagger(i_2). \quad (8)$$

This means that the symmetric state of Eq. (4) can be written as:

$$\varphi_{zz} = a_z^\dagger(i_1)a_z^\dagger(i_2)|0\rangle = \frac{1}{\sqrt{3}}\left(-s^\dagger(i_1,i_2) + \sqrt{2}d_0^\dagger(i_1,i_2)\right)|0\rangle. \quad (9)$$

The $s^\dagger(i_1,i_2)$ operator creates the ground state 0^+ with L=K=M=0, while the $d_0^\dagger(i_1,i_2)$ operator creates the first excited state 2^+ with L=2, M=K=0. The probability for the ground state is $\frac{1}{3}$, while the probability for the excited state is $\frac{2}{3}$. The same probabilities are derived if one uses the L-projection technique (see Table 2A of Ref. [14]) for the SU(3) irrep (2,0).

The $-s^\dagger(i_1,i_2)$ operator in Eq. (9) is positive due to the negative sign in Eq. (7). If the $s^\dagger, d_M^\dagger$ operators in Eq. (9) created the s and d bosons of the Interacting Boson Model, then the state of Eq. (9) coincides with the coherent states of Ginocchio and Kirson [15]:

$$\frac{1}{\sqrt{1+\beta^2}}\left(s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{\beta\sin\gamma}{\sqrt{2}}\left(d_2^\dagger + d_{-2}^\dagger\right)\right)|0\rangle, \quad (10)$$

for $\beta = \sqrt{2}, \gamma = 0^\circ$, where β, γ are the deformation variables of the Bohr-Mottelson Model. In this scenario the wave function of Eq. (9) is a state of the SU(3) limit of the Interacting Boson Model (since $\beta = \sqrt{2}$) and represents a prolate nuclear shape (since $\gamma = 0^\circ$).

The important point is that in the spatial Shell Model SU(3) wave functions the 3 states $|n_z = 1\rangle, |n_x = 1\rangle, |n_y = 1\rangle$ (where n_z, n_x, n_y are the numbers of the harmonic oscillator quanta in the three cartesian directions) are being occupied by harmonic oscillator quanta, which are bosons. Thus an infinite number of quanta may occupy a single state and so an infinite number of symmetric pairs of quanta may occur in a Shell Model SU(3) state.

CONCLUSIONS

The Shell Model SU(3) symmetry is valid in the LS coupling scheme. In this scheme the many-particle wave function is separated into a spatial many-quanta part, which possesses the SU(3) symmetry and into a spin-isospin part. The many-quanta wave function of the SU(3) symmetry may possess an infinite number of quanta in a cartesian direction, since the quanta are bosons. The quanta are coupled into symmetric pairs. These pairs can be expressed into spherical tensor operators of degree 0 or 2, thus they are very similar with the s and d bosons of the Interacting Boson Model. Furthermore, the transformation of the cartesian pairs of quanta into the spherical pairs of quanta gives the correct probabilities for the nuclear states with good L and K (the ones which are being derived by the L-projection technique [14]). In addition the Shell Model states with symmetric pairs of quanta coincide with the coherent states of Ginocchio and Kirson for the SU(3) limit of the Interacting Boson Model. Do all these facts support that the $s^{\dagger}, d_M^{\dagger}$ operators of Eq. (6) are the s, d bosons of the Interacting Boson Model? Can we interpret the s, d bosons of the Interacting Boson Model as symmetric pairs of quanta?

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References

[1] J. P. Elliott, Proc. Roy. Soc. Ser. A, 245, p. 128 (1958)
[2] J. P. Elliott, Proc. Roy. Soc. Ser. A , 245, p. 562 (1958)
[3] J. P. Elliott, M. Harvey, Proc. Roy. Soc. Ser. A, 272, p. 557 (1963)
[4] J. P. Elliott and C. E. Wilsdon, Proc. Roy. Soc. Ser. A, 302, p. 509 (1968)
[5] Andriana Martinou et al., EPJA, 56, 239 (2020)
[6] M. Goeppert-Mayer, Phys. Rev., 74, p. 235 (1948)
[7] Andriana Martinou et al., EPJA, 57, 84 (2021)
[8] Dennis Bonatsos et al., PRC, 95, 064325 (2017)
[9] Dennis Bonatsos et al., PRC, 95, 064326 (2017)
[10] Andriana Martinou et al., EPJA, 57, 83 (2021)
[11] M. Harvey, Advances in Nuclear Physics vol 1, Plenum Press, New York (1968).
[12] T. Otsuka et al., Phys. Lett. B, 76, p. 139, (1978)
[13] F. Iachello and A. Arima (1987). *The Interacting Boson Model*. Cambridge University Press
[14] J. D. Vergados, Nucl. Phys. A, 111, p. 681 (1968)
[15] J. N. Ginocchio and M. W. Kirson, Nucl. Phys. A, 350, p. 31 (1980)