

## HNPS Advances in Nuclear Physics

Vol 28 (2021)

HNPS2021



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*Arsenia Chorozydou, Theodoros Gaitanos*

doi: [10.12681/hnps.3574](https://doi.org/10.12681/hnps.3574)

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### To cite this article:

Chorozydou, A., & Gaitanos, T. (2022). The Non-Linear Derivative (NLD) model for the in-medium hyperons & antihyperons dynamics. *HNPS Advances in Nuclear Physics*, 28, 54–59. <https://doi.org/10.12681/hnps.3574>

# The Non-Linear Derivative (NLD) model for the in-medium hyperons & antihyperons dynamics

A. Chorozidou<sup>1,\*</sup>, T. Gaitanos<sup>1</sup>

<sup>1</sup> Department of Physics, Aristotle University of Thessaloniki

**Abstract** The in-medium properties of hyperons and antihyperons are studied with the Non-Linear Derivative (NLD) model and focus is made on the momentum dependence of strangeness optical potentials [1]. The NLD model is based on the Relativistic Mean Field (RMF) approximation to Relativistic Hadrodynamics (RHD) approach of nuclear systems, but it incorporates an explicit momentum dependence of mean-fields. The extension of the NLD model to the baryon and antibaryon octet is based on SU(6) and G-parity arguments. It is demonstrated that with a proper choice of momentum cut-offs, the  $\Lambda$  and  $\Sigma$  optical potentials are consistent with recent studies of the chiral effective field theory and  $\Xi$  optical potentials are consistent with Lattice-QCD calculations, over a wide momentum region. We also present NLD predictions for the in-medium momentum dependence of  $\bar{\Lambda}$ -,  $\bar{\Sigma}$ - and  $\bar{\Xi}$ - hyperons. This work is important for future experimental studies, like CBM, PANDA at FAIR and is relevant to nuclear astrophysics as well.

**Keywords** optical potential, hyperons

## INTRODUCTION

In an attempt to explain both astrophysical observations ( $2M_{\odot}$  Neutron Stars, that seems to exclude soft hadronic Equation of State (EoS) at high baryon densities) and results from Heavy-Ion Collisions (they lead to a softening of the high-density EoS) with a proper nuclear matter model, an inclusion of hyperons degrees of freedom is performed in the NLD model. The NLD model is based on a basic RMF Lagrangian formulation, but it incorporates higher order derivatives in the nucleon-nucleon (NN) interaction Lagrangians. In previous work [2], it has been demonstrated that this Ansatz corrects the high-momentum behaviour of the interaction, makes the EoS softer at densities just above saturation, but at the same time it reproduces the  $2M_{\odot}$  pulsars at densities far beyond saturation. Given that the NN interaction is fairly well known in contrast to the hyperon-nucleon (YN) interaction, in order to approach YN interaction, we take the previously well studied NLD model for nucleons [2] and extend it to the strangeness degrees of freedom, in the spirit of SU(6) and G-parity arguments. Having done that, we discuss the momentum dependence of the in-medium YN-potentials.

## THE NLD MODEL OF THE BARYON OCTET

The NLD Lagrangian is based on the conventional RHD and for the baryon octet it reads as

$$\mathcal{L} = \frac{1}{2} \sum_B [\bar{\Psi}_B \gamma_{\mu} i \vec{\partial}^{\mu} \Psi_B - \bar{\Psi}_B i \vec{\partial}^{\mu} \gamma_{\mu} \Psi_B] - \sum_B m_B \bar{\Psi}_B \Psi_B - \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \vec{\rho}_{\mu} \vec{\rho}^{\mu} - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu} - \frac{1}{2} m_{\delta}^2 \vec{\delta}^2 + \frac{1}{2} \partial_{\mu} \vec{\delta} \partial^{\mu} \vec{\delta} + \mathcal{L}_{int}^{\sigma} + \mathcal{L}_{int}^{\omega} + \mathcal{L}_{int}^{\rho} + \mathcal{L}_{int}^{\delta}.$$

The sum over B runs over the baryonic octet  $\Psi_B = (\Psi_N, \Psi_{\Lambda}, \Psi_{\Sigma}, \Psi_{\Xi})^T$  with  $\Psi_N = (\psi_p, \psi_n)^T$ ,  $\Psi_{\Lambda} = \psi_{\Lambda}$ ,  $\Psi_{\Sigma} = (\psi_{\Sigma^+}, \psi_{\Sigma^0}, \psi_{\Sigma^-})^T$ ,  $\Psi_{\Xi} = (\psi_{\Xi^0}, \psi_{\Xi^-})^T$  for the isospin-doublets  $\Psi_N$  and  $\Psi_{\Xi}$ , isospin-triplet  $\Psi_{\Sigma}$  and the neutral  $\Psi_{\Lambda}$ . The interactions between the nucleon fields are described by the exchange

\* Corresponding author: achoro@auth.gr

of meson fields. These are the scalar  $\sigma$  and vector  $\omega^\mu$  mesons in the isoscalar channel, as well as the scalar  $\vec{\delta}$  and vector  $\vec{\rho}^\mu$  mesons in the isovector channel. Their corresponding Lagrangian densities are of the Klein-Gordon and Proca types. The term  $U(\sigma) = \frac{1}{3}b\sigma^3 + \frac{1}{4}c\sigma^4$  contains the usual self-interactions of the  $\sigma$  meson. The field strength tensors are defined as  $F^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$ ,  $\vec{G}^{\mu\nu} = \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$  for the isoscalar and isovector fields, respectively. In the following we restrict to a minimal set of interaction degrees of freedom. In the iso-scalar sector, the  $\sigma$ - and  $\omega$ -fields are obviously considered. In the iso-vector channel, we keep the vector, isovector  $\rho$ -meson field and neglect the  $\delta$ -field.

The NLD interaction Lagrangians contain the conventional and simple combinations between the bilinear baryon-and linear meson-fields, however, they are extended by the inclusion of non-linear derivative operators  $\vec{\mathcal{D}}$ ,  $\overleftarrow{\mathcal{D}}$  for each baryon species  $B$ :

$$\begin{aligned}\mathcal{L}_{int}^\sigma &= \sum_B \frac{g_{\sigma B}}{2} [\bar{\Psi}_B \overleftarrow{\mathcal{D}}_B \Psi_B \sigma + \sigma \bar{\Psi}_B \vec{\mathcal{D}}_B \Psi_B] \\ \mathcal{L}_{int}^\omega &= - \sum_B \frac{g_{\omega B}}{2} [\bar{\Psi}_B \overleftarrow{\mathcal{D}}_B \gamma^\mu \Psi_B \omega_\mu + \omega_\mu \bar{\Psi}_B \gamma^\mu \vec{\mathcal{D}}_B \Psi_B] \\ \mathcal{L}_{int}^\rho &= - \sum_B \frac{g_{\rho B}}{2} [\bar{\Psi}_B \overleftarrow{\mathcal{D}}_B \gamma^\mu \vec{\tau} \Psi_B \vec{\rho}_\mu + \vec{\rho}_\mu \bar{\Psi}_B \vec{\tau} \gamma^\mu \vec{\mathcal{D}}_B \Psi_B]\end{aligned}$$

for the isoscalar-scalar, isoscalar-vector and isovector-vector vertices, respectively. The arrows on the non-linear operator  $\mathcal{D}$  indicate the direction of their action. The only difference with respect to the conventional RHD interaction Lagrangian is the presence of additional operator functions  $\vec{\mathcal{D}}_B$ ,  $\overleftarrow{\mathcal{D}}_B$ . These operator functions will regulate the high momentum component of hyperons. These operator functions (or regulators)  $\vec{\mathcal{D}}_B$ ,  $\overleftarrow{\mathcal{D}}_B$  are hermitian and generic functions of partial derivative operator. That is,  $\vec{\mathcal{D}}_B := \mathcal{D}(\vec{\xi}_B)$  and  $\overleftarrow{\mathcal{D}}_B := \mathcal{D}(\vec{\xi}_B)$  with the operator arguments  $\vec{\xi}_B = -\zeta_B^\alpha i\vec{\partial}_\alpha$  &  $\vec{\xi}_B = i\vec{\partial}_\alpha \zeta_B^\alpha$ . The four vector  $\zeta_B^\mu = \frac{v^\mu}{\Lambda_B}$  contains the cut-off  $\Lambda_B$  and  $v^\mu$  is an auxiliary vector. They are assumed to act on the baryon spinors  $\Psi_B$  and  $\bar{\Psi}_B$  by a formal Taylor expansion with respect to the operator argument. The functional form of the regulators is constructed such that in the limit  $\vec{\Lambda}_B \rightarrow \infty$  the original RHD Lagrangians are recovered, that is,  $\vec{\mathcal{D}}_B = \overleftarrow{\mathcal{D}}_B^\dagger \rightarrow 1$ .

From the generalised Euler-Lagrange formalism, we obtain the equations of motion for the degrees of freedom in the NLD model. The meson field equations of motion read

$$\begin{aligned}\partial_\alpha \partial^\alpha \sigma + m_\sigma^2 \sigma + \frac{\partial U}{\partial \sigma} &= \frac{1}{2} \sum_B g_{\sigma B} [\bar{\Psi}_B \overleftarrow{\mathcal{D}}_B \Psi_B + \bar{\Psi}_B \vec{\mathcal{D}}_B \Psi_B] \\ \partial_\mu F^{\mu\nu} + m_\omega^2 \omega^\nu &= \frac{1}{2} \sum_B g_{\omega B} [\bar{\Psi}_B \overleftarrow{\mathcal{D}}_B \gamma^\nu \Psi_B + \bar{\Psi}_B \gamma^\nu \vec{\mathcal{D}}_B \Psi_B] \\ \partial_\mu G^{\mu\nu} + m_\rho^2 \vec{\rho}^\nu &= \frac{1}{2} \sum_B g_{\rho B} [\bar{\Psi}_B \overleftarrow{\mathcal{D}}_B \gamma^\nu \vec{\tau} \Psi_B + \bar{\Psi}_B \vec{\tau} \gamma^\nu \vec{\mathcal{D}}_B \Psi_B]\end{aligned}$$

for the isoscalar-scalar, isoscalar-vector and isovector-vector exchange mesons, respectively. Each baryon-field obeys a Dirac-equation of the following type

$$[\gamma_\mu (i\partial^\mu - \Sigma_B^\mu) - (m_B - \Sigma_{SB})]\psi_B = 0$$

with the selfenergies  $\Sigma_B^\mu$  &  $\Sigma_{SB}$  defined as  $\Sigma_B^\mu = g_{\omega B} \omega^\mu \overleftarrow{\mathcal{D}}_B + g_{\rho B} \vec{\tau}_B \cdot \vec{\rho}^\mu \overleftarrow{\mathcal{D}}_B$  &  $\Sigma_{SB} = g_{\sigma B} \sigma \overleftarrow{\mathcal{D}}_B$ .

Both Lorentz-components of the selfenergy,  $\Sigma^\mu$  and  $\Sigma_s$ , show an explicit linear behavior with respect to the meson fields  $\sigma$ ,  $\omega^\mu$  and  $\vec{\rho}^\mu$  as in the standard RHD. However, they contain an additional dependence on the regulators. If we apply the RMF approximation to static hadronic matter (the spatial

components of the meson fields in Minkowski and isospin-spaces vanish,  $\omega^\mu \rightarrow (\omega^0, \vec{0})$  and  $\vec{\rho}^\mu \rightarrow (\rho_3^0, \vec{0}_3)$ , we end up to the following forms for the NLD meson-field equations:

$$\frac{\partial U}{\partial \sigma} = \frac{1}{2} \sum_B g_{\sigma B} \langle \bar{\Psi}_B \mathcal{D}_B \Psi_B \rangle = \sum_B g_{\sigma B} \rho_{sB}$$

$$m_\omega^2 \omega = \frac{1}{2} \sum_B g_{\omega B} \langle \bar{\Psi}_B \gamma^0 \mathcal{D}_B \Psi_B \rangle = \sum_B g_{\omega B} \rho_{0B}$$

with the scalar and vector density sources

$$\rho_{sB} = \int_{|\vec{p}| \leq p_{FB}} d^3p \frac{m_B^*}{\Pi_B^0} \mathcal{D}_B(p) \quad \text{and} \quad \rho_{0B} = \int_{|\vec{p}| \leq p_{FB}} d^3p \frac{E_B^*}{\Pi_B^0} \mathcal{D}_B(p).$$

The isovector densities are calculated through the standard isospin relations. For a hyperon with a given momentum relative to nuclear matter at rest (at a given nucleon density and isospin asymmetry) the mesonic sources contain only nucleons,  $B = p, n$ .

The meson-field equations of motion show a similar structure as those of the standard RMF approximation. However, the substantial difference between NLD and other conventional RMF models appears in the source terms which now contain in addition the momentum-dependent regulators  $\mathcal{D}_B$ . This is an important feature of the NLD model. The cut-off leads naturally to a particular suppression of the vector field at high densities or high Fermi-momenta in agreement with phenomenology, as discussed in detail in the previous work [2]. This feature is absent in conventional RHD approaches, except if one introduces by hand additional and complicated scalar/vector self-interactions.

The key observable for general discussions related to momentum or energy dependencies of in-medium hadronic potentials is the Schrödinger-equivalent optical potential  $U_{opt}$ , which is a complex quantity. The imaginary part describes the scattering processes of a given particle, e.g. a hyperon, with a nucleon of nuclear matter. The real part of the optical potential is related to the meanfield that a particle, e.g. a hyperon with a given momentum, experiences in the nuclear medium at a given density and isospin-asymmetry. The imaginary part of  $U_{opt}$  cannot be calculated within a conventional RMF prescription. However, in the NLD model (given that it contains an explicit momentum dependence of the mean-fields and thus of the optical potential) we can give, at least, estimations for the imaginary part. We do this in the case of anti-hyperons. The real part of the Schrödinger-equivalent optical potential for hyperons is obtained from a non-relativistic reduction of the Dirac-equation and reads

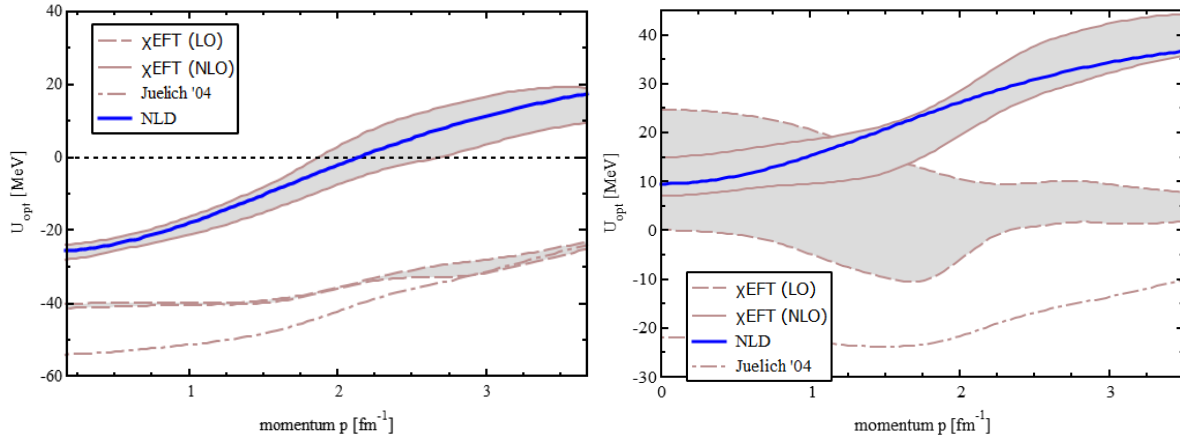
$$U_{opt}^B = -S_B + \frac{E_B}{m_B} V_B + \frac{1}{2m_B} (S_B^2 - V_B^2)$$

It describes the in-medium interaction of a baryon species B with a momentum p, or single-particle energy  $E_B = E_B(p)$ , relative to nuclear matter at rest at a given density and isospin asymmetry. We will compare the NLD results with recent microscopic calculations from  $\chi$ -EFT [3, 4] and Lattice-QCD [5] for the hyperon in-medium potentials.

## RESULTS

We consider nuclear matter at rest, at a given density, isospin asymmetry and at zero temperature, in which hyperons are situated at a given momentum relative to the nuclear matter at rest. The quantity of interest is the optical potential  $U_{opt}$  of the in-medium hyperons. There is no experimental information on the momentum dependence of the inmedium hyperonic potentials, so, as it has been mentioned before, we use for comparison some recent microscopic calculations. For  $\Lambda$  and  $\Sigma$  hyperons we compare with calculations based on the chiral-Effective Field Theory [3,4] and for the  $\Xi$  hyperon we compare with calculations based on Lattice-QCD [5]. In the NLD calculations we take as a parameter only the

strangeness cut-off of the hyperons and it has a monopole form  $\mathcal{D}_Y(p) = \frac{\Lambda_{Y1}^2}{\Lambda_{Y2}^2 + p^2}$ , with  $\gamma = \sigma, \omega, \rho$  indicating the cut-off values for the hyperon-nucleon  $\sigma, \omega$  and  $\rho$  vertices, respectively, and  $Y = \Lambda, \Sigma, \Xi$  denotes the hyperon type. At first, the cut-offs of the hyperons have to be determined. The strangeness  $S=1$  cut-offs are adjusted to the corresponding hyperonic optical potentials at saturation density  $\rho_0$  of symmetric and cold nuclear matter from  $\chi$ -EFT calculations. This is shown in Fig. 1 for the optical potential of both  $\Lambda$ - and  $\Sigma$ -hyperons. The NLD calculations can describe the microscopic  $\chi$ -EFT results in NLO very well. The corresponding values for the strangeness cut-off's are tabulated in Table 1.

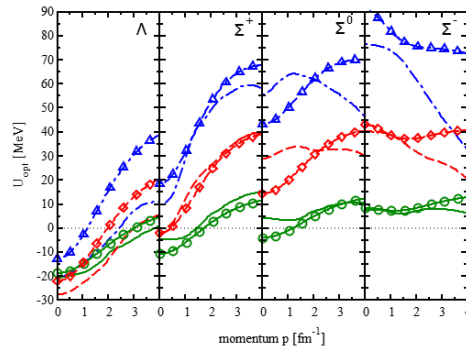


**Fig. 1.** Optical potential of  $\Lambda$  and  $\Sigma$  hyperons as function of their momentum  $p$  in Symmetric Nuclear Matter (SNM) at  $\rho_0$ . The NLD-results (thick-solid curve) are compared with  $\chi$ -EFT microscopic calculations at different orders LO (band with closed dashed borders) and NLO (band with closed solid borders). Further microscopic calculations of the Jülich group (dot-dashed curve) are shown too.

**Table 1.**  $\Lambda$ ,  $\Sigma$  and  $\Xi$  regulators for  $\sigma$ -( $\Lambda_\sigma$ ),  $\omega$ -( $\Lambda_{\omega 1,2}$ ) and  $\rho$ -hyperon ( $\Lambda_{\rho 1,2}$ ) vertices in units of eV. In the cases for  $\Sigma$  and  $\Xi$  the isospin cut-offs ( $\Lambda_{p1,2}$ ) are relevant for the charged particles only. For the  $\Sigma$  hyperon different cut-offs values are used for  $\Sigma^-$  (upper line) and for  $\Sigma^+$  (bottom line).

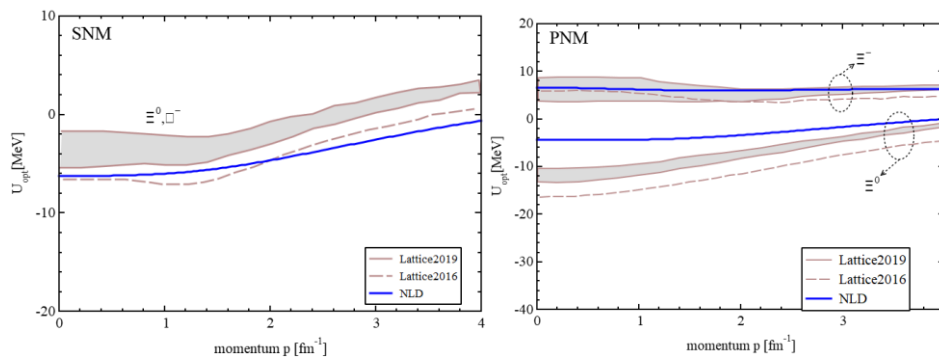
$\Lambda$ cut-off					$\Sigma$ cut-off					$\Xi$ cut-off				
$\Lambda_\sigma$	$\Lambda_{\omega 1}$	$\Lambda_{\omega 2}$	$\Lambda_{\rho 1}$	$\Lambda_{\rho 2}$	$\Lambda_\sigma$	$\Lambda_{\omega 1}$	$\Lambda_{\omega 2}$	$\Lambda_{\rho 1}$	$\Lambda_{\rho 2}$	$\Lambda_\sigma$	$\Lambda_{\omega 1}$	$\Lambda_{\omega 2}$	$\Lambda_{\rho 1}$	$\Lambda_{\rho 2}$
0.7	0.85	0.79	-	-	0.67	0.95	0.79	0.47	0.47	0.6	0.8	0.71	1.3	1.2

In Fig.3 there are results for pure neutron matter (PNM) at different baryon densities. The NLD model does predict the general microscopic trends.

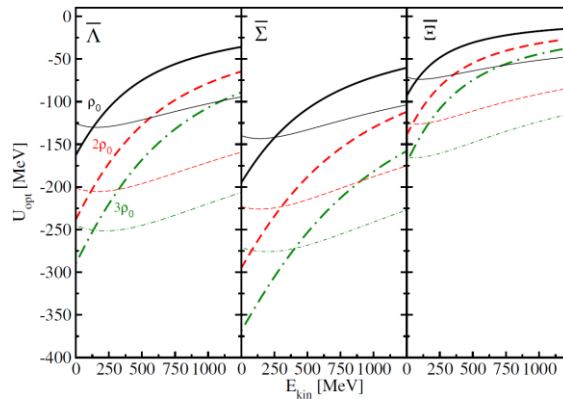


**Fig. 3.** Optical potentials for hyperons versus their momentum for PNM. Solid curves with symbols indicate the NLD calculations while pure curves without symbols are the microscopic  $\chi$ -EFT results from [3]. Green pairs (circles-solid for NLD and solid for  $\chi$ -EFT) refer to low density of  $p_F = 1 \text{ fm}^{-1}$ , red pairs (diamonds-solid for NLD and dashed for  $\chi$ -EFT) refer to  $\rho_0$  of  $p_F = 1.35 \text{ fm}^{-1}$  and blue pairs (triangles-solid for NLD and dashed-dotted for  $\chi$ -EFT) refer to a density of  $p_F = 1.53 \text{ fm}^{-1}$ .

Finally, in Fig. 4 we have the comparison of NLD model with recent microscopic calculations from Lattice-QCD for the  $\Xi$  hyperons, for both SNM and PNM. In Fig. 5 the predictions of NLD for antihyperon in-medium potentials are presented as well. The motivation for this study of antihyperons is the future experiments, such as those at FAIR, where the in-medium properties of anti-hadrons will be investigated too. In general for anti-particles the imaginary part of  $U_{\text{opt}}$  may be also important, so it is showed in the same figure as well. In the NLD approach the  $\text{Im}(U_{\text{opt}})$  has been estimated from dispersion relations [2] and it is noticeable that it is comparable with its corresponding real part, particularly at very low energies. Possible physical explanations for that is that in antinucleon-nucleon scattering annihilation can occur through the production of light pions or the interaction of anti-hyperons with nucleons can happen via the production of the heavier kaons due to strangeness conservation (it may affect the  $\text{Im}(U_{\text{opt}})$  at low energies).



**Fig. 4.** Optical potential of cascade hyperons versus their momentum for SNM & PNM at  $\rho_0$ . The solid curve indicates the NLD predictions while the dashed curve and the gray band refer to recent Lattice calculations from Refs. [5].



**Fig. 5.** Optical potentials (real part) for anti-hyperons versus their kinetic energy for SNM at various densities: at  $\rho_0$  (thick-solid),  $2\rho_0$  (thick-dashed) and  $3\rho_0$  (thick-dashed-dot). Estimates for the imaginary part at  $\rho_0$  (thin-solid), at  $2\rho_0$  (thin-dashed) and at  $3\rho_0$  (thin-dashed-dot).

## CONCLUSION

On the whole, results from NLD are in agreement with calculations based on  $\chi$ -EFT and Lattice-QCD.

Regarding the estimations for the Imaginary part of the Optical potential for anti-hyperons, they may be tested in anti-proton induced reactions or in reactions with secondary  $\Xi$  beams (e.g. at PANDA experiment at FAIR).

Now about future work, we may apply the NLD model to beta-equilibrated compressed matter. It will be an investigation of the hyperon-puzzle to Neutron Stars.

## References

- [1] T. Gaitanos, A. Chorozidou, Nucl. Phys. A 1008, 122153 (2021)
- [2] T. Gaitanos, M. Kaskulov, Nucl. Phys. A 899, 133 (2013), T. Gaitanos, M. Kaskulov, Nucl. Phys. A 940, 181 (2015)
- [3] S. Petschauer et al., Eur. Phys. J. A 52, 15 (2016)
- [4] J. Haidenbauer, U.G. Meißner, A. Nogga, Eur. Phys. J. A 56 (3), 91 (2020)
- [5] T. Inoue, LATTICE-HAL QCD PoS INPC2016, 277 (2016), T. Inoue, HAL QCD AIP Conf. Proc. 2130(1), 020002 (2019)