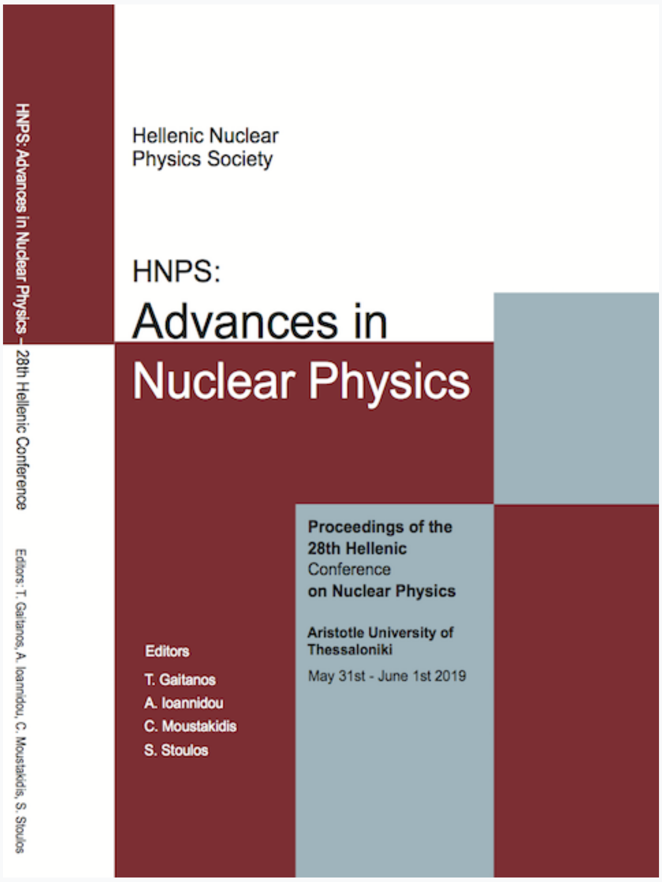
HNPS Advances in Nuclear Physics

Vol 27 (2019)

HNPS2019



Nuclear Symmetry Energy Effects on the Bulk Properties of Neutron-rich Finite Nuclei

Manolis Divaris, Charalampos Moustakidis

doi: 10.12681/hnps.2992

HNPS Advances in Nuclear Physics vol. 27, pp. 91–94 (2020)
M. Divaris and Ch. C. Moustakidis HNPS2019 doi:10.12681/hnps.2992
Page 91

Nuclear Symmetry Energy Effects on the Bulk Properties of Neutron–rich Finite Nuclei

M. Divaris and Ch.C. Moustakidis

Physics Department, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

Abstract *We systematically study the effect of the nuclear symmetry energy in the basic properties of finite, neutron–rich, heavy nuclei where symmetry energy plays a dominant role. We employ a variational method, in the framework of the Thomas–Fermi approximation, to study the effect of the symmetry energy on the neutron skin thickness and symmetry energy coefficients of various nuclei. The isospin asymmetry function a(r) is directly related to the symmetry energy as a consequence of the variational principle. In addition to this, the Coulomb interaction is included in a self–consistent way. The energy density of the asymmetric nuclear matter that is used, has its origins in a momentum-dependent interaction.*

Keywords *symmetry energy, neutron–rich nuclei, neutron–skin*

Corresponding author: M. Divaris (edivar@physics.auth.gr) | *Published online:* May 1st, 2020

Nuclear symmetry energy is the basic regulator of the isospin properties of neutron-rich nuclei. It is expected to affect the neutron skin thickness, the coefficient of the asymmetry energy in the Bethe–von Weizsäcker formula, etc. In addition, the density dependence of the symmetry energy is the main ingredient of the equation of state of neutron-rich nuclear matter. Actually, there are a variety of neutron star properties that are sensitive to this energy.

The empirical Bethe–von Weizsäcker formula provides the binding energy of a finite nucleus with A nucleons and atomic number Z and is given by

$$B(A,Z) = -a_V A + a_S A^{2/3} + a_C \frac{Z(Z-1)}{A^{1/3}} + a_A \frac{(N-Z)^2}{A} + \Delta E_{mic}$$

The energy density functional is an extension to the above formula, where the total energy of finite nuclei is a functional of the total density ρ(r) and the isospin asymmetry function α(r) [1],

$$E[\rho(r),\alpha(r)] = \int_V \varepsilon\big(\rho(r),\alpha(r)\big) d^3r \;\; E[\rho,\alpha] = \int_V \left[\varepsilon_{ANM}\big(\rho(r),\alpha(r)\big) + F_0|\nabla\rho(r)|^2 + \frac{1}{4}\rho(1-\alpha)V_C(r)\right] d^3r$$

In the above expression, the first term is the asymmetric nuclear matter contribution, the second term corresponds to the contribution from the finite size character of the density distribution, while the third term is the Coulomb energy density term. The symmetry energy function is defined as

$$\alpha(r) = \frac{\rho_n(r) - \rho_p(r)}{\rho(r)}$$

with ρ(r) the total density function and $\rho_n(r)$, $\rho_p(r)$ the corresponding neutron and proton density distributions.

We apply the Euler- Lagrange formalism in this situation, by using the langrangian density which is given by [1]

$$L = 4\pi r^2 \left(\varepsilon_{ANM}(\rho,\alpha) + F_0\left(\frac{d\rho}{dr}\right)^2 + \frac{1}{4}\rho(1-\alpha)V_c(r)\right) - \lambda_1 4\pi r^2 \rho - \lambda_2 4\pi r^2 \alpha\rho$$

The corresponding Euler- Lagrange equations are

$$\frac{\partial L}{\partial \rho} - \frac{d}{dr}\left(\frac{\partial L}{\partial \rho\prime}\right) = 0 \qquad \frac{\partial L}{\partial \alpha} - \frac{d}{dr}\left(\frac{\partial L}{\partial \alpha\prime}\right) = 0$$

where the first one gives a differential equation of the density function ρ(r)

$$\rho'' + \frac{2\rho'}{r} - \frac{1}{2F_0}\left[\frac{\partial \varepsilon_{SNM}(\rho)}{\partial \rho} + \alpha^2\left(S(\rho) + \rho\frac{\partial S(\rho)}{\partial \rho}\right) + \frac{1}{4}(1-\alpha)V_c(r) - \lambda_1 - \lambda_2\alpha\right] = 0$$

Instead of solving this differential equation, we are using a trial function given by the Fermi-type formula [1]

$$\rho(r) = \frac{n_0}{1 + exp[(r-d)/w]}$$

The second Euler–Lagrange equation provides the asymmetry function

$$\alpha(r) = \frac{1}{8S(\rho)}(V_c(r) + 4\lambda_2)$$

Since the asymmetry function obeys the constraint

$$0 \leq a(r) \leq 1$$

and as the above expression of the asymmetry function does not comply with this constraint, we use the assumption

$$\alpha(r) = \begin{cases} \frac{1}{8S(\rho)}(V_c(r) + 4\lambda_2), & r \leqslant r_c \\ 1, & r \geqslant r_c \end{cases}$$

where r_c is the cutoff radius. If we integrate this expression according to the normalization condition

$$\int_V \alpha(r)\rho(r)d^3r = N - Z$$

where we integrate over the entire volume V that the nucleus occupies, we obtain the Lagrange-multiplier

$$\lambda_2 = 2\left(\int_{V_c}\rho(r)d^3r - \frac{e^2}{8}\int_{V_c}\frac{V_c(r)\rho(r)}{S(\rho)}d^3r - 2Z\right)\left(\int_{V_c}\frac{\rho(r)}{S(\rho)}d^3r\right)^{(-1)}$$

(V_c is the volume that corresponds to the cutoff radius r_c). The Coulomb potential that appears in the energy density functional expression

$$V_c(r) = \frac{e^2}{2}\int\frac{\rho(r')[1-\alpha(r')]}{|r-r'|}d^3r'$$

can be decomposed into two parts [1] as follows

$$V_c^A(r) = 2\pi e^2\left[\frac{1}{r}\int_0^r[1-\alpha(r')]\rho(r')r'^2dr' + \int_r^{r_c}[1-\alpha(r')]\rho(r')r'dr'\right], \quad r \leqslant r_c$$

$$V_c^B(r) = \frac{2\pi e^2}{r}\int_0^{r_c}[1-\alpha(r')]\rho(r')r'^2dr', \quad r \geqslant r_c$$

We consider the following expansion of the symmetry energy [1]

$$S(\rho) = S(\rho_0) + L\delta + \frac{K_{sym}}{2!}\delta^2 + O(\delta^3)$$

where L is the slope parameter

$$L = 3\rho_0 \frac{dS(\rho)}{d\rho}\bigg|_{\rho=\rho_0}$$

and the coefficient K_{sym} is given by

$$K_{sym} = 3\rho_0 \frac{d^2S(\rho)}{d\rho^2}\bigg|_{\rho=\rho_0}$$

In order to use an expression for the symmetry energy, we consider the expansion of the energy per particle of asymmetric nuclear matter [3]

$$E_b(\rho,\alpha) = E_b(\rho,\alpha=0) + E_{sym,2}(\rho)\alpha^2 + E_{sym,4}(\rho)\alpha^4 + \ldots + E_{sym,2k}(\rho)\alpha^{2k} + \ldots$$

and the symmetry energy can be obtained from the following relation

$$S(\rho) \equiv E_{sym,2}(\rho) = \frac{1}{2!}\frac{\partial^2 E_b(\rho,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0}$$

For the energy per particle of nuclear matter E_b, we employ the expression from the momentum-dependent interaction model (MDI), as given by [3].

The proton and neutron radii are given by the following expressions

$$R_p = \left(\frac{1}{Z}\int r^2\rho_p(r)d^3r\right)^{1/2} \quad R_n = \left(\frac{1}{N}\int r^2\rho_n(r)d^3r\right)^{1/2}$$

and they are used together to define the neutron-skin thickness

$$R_{skin} = R_n - R_p$$

The asymmetry coefficient a_A that appears in the Bethe-Weizsacker formula, is given by the expression

$$\alpha_A = \frac{A}{(N-Z)^2}\int \rho(r)S(\rho)\alpha^2(r)d^3r$$

In the present study, we are making calculations for the ^{208}Pb nucleus. The asymmetry function a(r) is also known as the asymmetry parameter I, and J is the symmetry energy at the saturation density ρ_0=0.16144 fm^{-3}, J=S(ρ_0). From computational analysis, we have extracted the following figures, which show the relations between the neutron skin R_{skin} and the slope parameter L (Fig. 1), between R_{skin} and the asymmetry parameter I (Fig. 2) and the relation between the asymmetry coefficient a_A and I (Fig. 3).

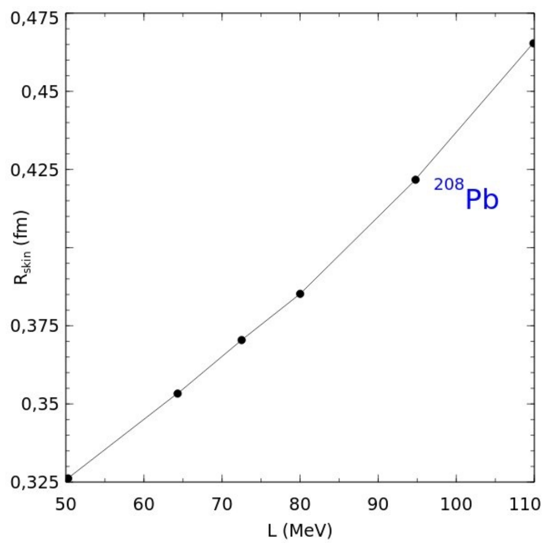


Figure 1. *Neutron skin thickness R_{skin} – slope parameter L diagram*

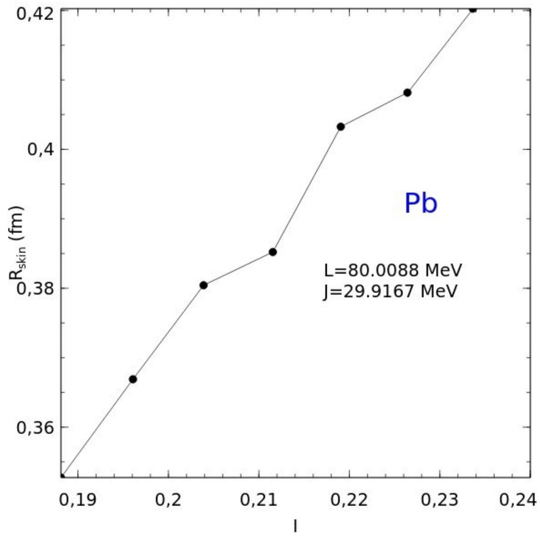


Figure 2. *Neutron-skin thickness R_{skin} – asymmetry parameter I*

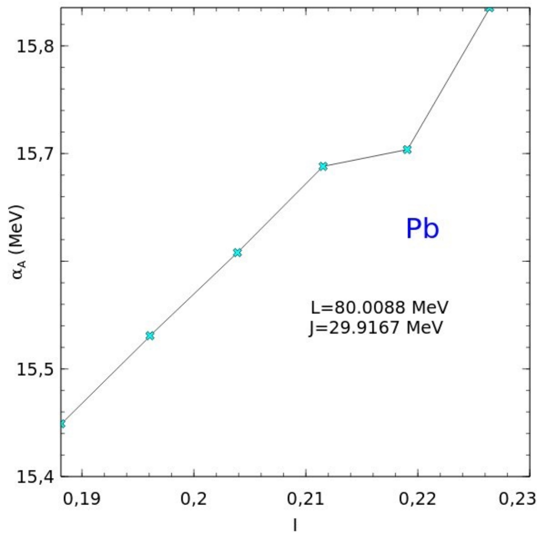


Figure 3. *Asymmetry coefficient a_A – asymmetry parameter I*

There appears to be a linear correlation between the neutron skin R_{skin} and the slope parameter L, as seen from Fig.1 above. Also, from Fig. 2, R_{skin} seems to be linearly dependent on the asymmetry parameter I, which means that, as the difference between the neutron and proton distributions increases, the neutron skin of the nucleus also increases. Furthermore, from Fig. 3, the asymmetry coefficient a_A also appears to increase as the asymmetry increases.

From the present work we can see that symmetry energy is indeed a key regulator, affecting various nuclear properties, such as the nuclear skin thickness R_{skin}, the asymmetry coefficient a_A, the total energy of the nucleus E. One future goal is to introduce temperature in our study.

References

[1] M.C. Papazoglou, Ch. C. Moustakidis, Phys. Rev. C 90, 014305 (2014), doi: 10.1103/PhysRevC.90.014305
[2] G.A. Lalazissis, S. Raman, P. Ring, At. Dat. Nucl. Dat. Tabl. 71, 1 (1999), doi: 10.1006/adnd.1998.0795
[3] L. Tsaloukidis, Ch. Margaritis, Ch.C. Moustakidis, Phys. Rev. C 99, 015803 (2019), doi: 10.1103/PhysRevC.99.015803
[4] A. Akmal, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C 58, 1804 (1998), doi: 10.1103/PhysRevC.58.1804