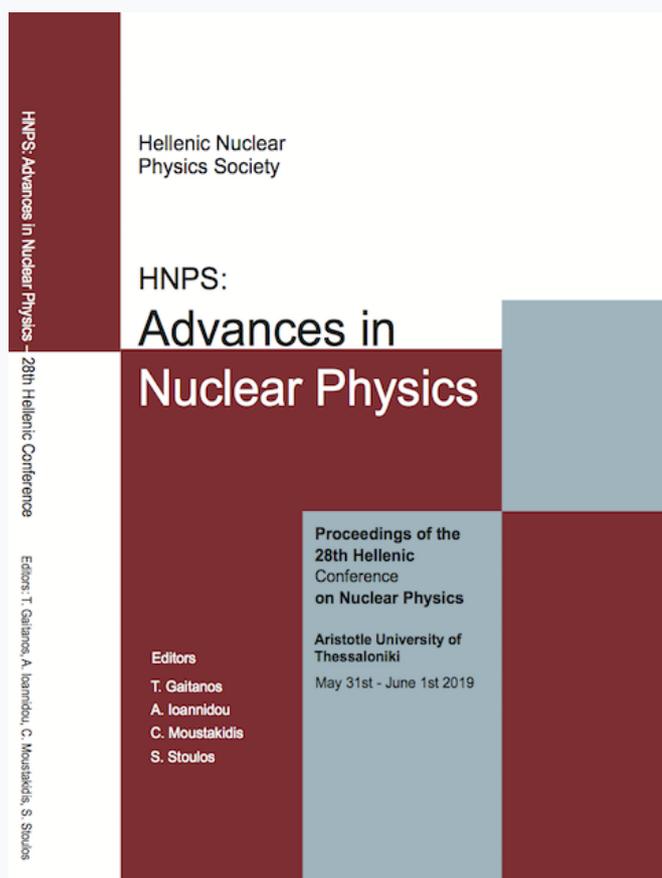


## HNPS Advances in Nuclear Physics

Vol 27 (2019)

HNPS2019



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doi: [10.12681/hnps.2990](https://doi.org/10.12681/hnps.2990)

#### To cite this article:

Margaritis, C., Koliogiannis Koutmiridis, P., & Moustakidis, C. (2020). Speed of sound constraints on maximally-rotating neutron stars. *HNPS Advances in Nuclear Physics*, 27, 155–159. <https://doi.org/10.12681/hnps.2990>

# Speed of sound constraints on maximally-rotating neutron stars

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**Abstract** *In the present work we provide a theoretical treatment concerning the effects of the upper bound of the sound speed in dense matter on the bulk properties of maximally-rotating (at mass-shedding limit) neutron stars. We investigate to what extent the possible predicted (from various theories and conjectures) upper bounds on the speed of sound constrain various key quantities, such as the maximum mass and the corresponding radius, Keplerian frequency, Kerr parameter and moment of inertia. We mainly focus on the lower proposed limit,  $v_s = c/\sqrt{3}$ , and we explore in which mass region a rotating neutron star collapses to a black hole. In any case, useful relations of the mentioned bulk properties with the transition density are derived and compared with the corresponding non-rotating cases.*

**Keywords** *Speed of sound, Dense matter, Rotating neutron stars*

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## INTRODUCTION

It is a common belief that the study (observational and theoretical) of the maximally-rotating neutron stars may offer rich information on the properties of dense nuclear matter. The strong interaction between many bodies has been the main mechanism for the theoretical construction of the equation of state (EoS). The main assumption is that the speed of sound in an EoS cannot exceed the speed of light because of the causality. But a question rises: Is the speed of light the upper bound of the speed of sound in dense matter? In fact, Hartle in [1] pointed out that causality is not enough to constrain the high density part of the EoS. Recently, the effects of the upper bound of the sound speed in neutron star properties have been studied extensively [2–4]. In the present work, we employ in addition two upper bounds, the  $v_s = c/\sqrt{3}$  and the one originated from the relativistic kinetic theory [5]. The main motivation of the present work is to investigate the possibility to provide some universal constraints on the bulk properties of maximally-rotating neutron stars. We also study the constant rest mass sequences of a neutron star in order to provide constraints relative to its collapse to a black hole [6]. Finally, we provide a detailed study about the connection between the minimum period of a rotating neutron star and the maximum neutron star mass of a non-rotating one.

## SPEED OF SOUND BOUNDS, MAXIMUM MASS CONFIGURATION AND ROTATING NEUTRON STARS

We have constructed the maximum mass configuration by considering the following two structures for the neutron star EoS [7]:

- (a) Maximum angular velocity for known low-density EoS
- (b) Maximum angular velocity from the relativistic kinetic theory

Both cases are presented in [8]. In case (a), the speed of sound takes the values  $c$  and  $c/\sqrt{3}$ , while in case (b) it is self-constrained in the range  $\frac{1}{3} \leq \left(\frac{v_s}{c}\right)^2 \leq 1$ . The EoS, that is used, is predicted by the Momentum-Dependent-Interaction (MDI) model in correlation with data from Akmal [9] and predicts the currently observed maximum neutron star masses (for more details, see [6]). The cases

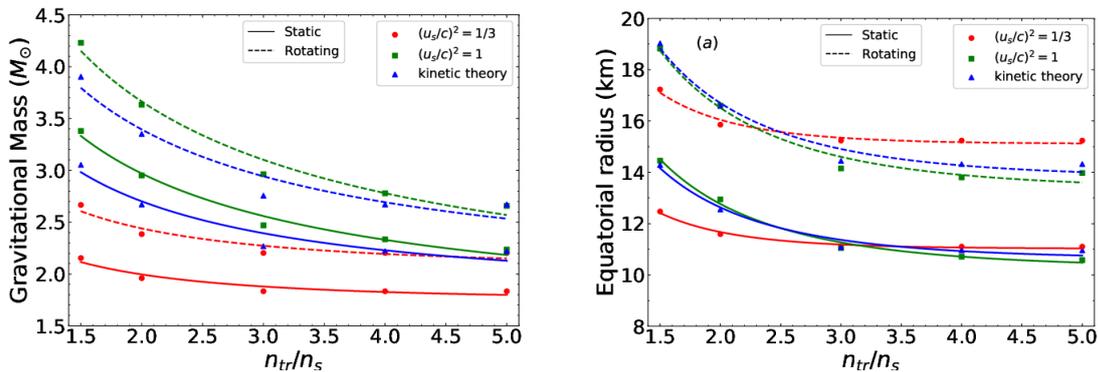
which took effect in this study were the ones where the fiducial transition density is  $n_{tr} = p_n n_s$ , where  $n_s$  is the saturation density of symmetric nuclear matter ( $n_s = 0.16 \text{ fm}^{-3}$ ) and  $p_n$  takes the values 1.5, 2, 3, 4, 5. In the specific case where  $n_{tr} = 1.5n_s$ , when the speed of sound is equal to  $c$ , then we have the EoS-maxstiff scenario and when the speed of sound is equal to  $c/\sqrt{3}$ , then we have the EoS-minstiff scenario.

In approach (a) the continuity on the EoS is well ensured. However, approach (a), due to its artificial character, does not ensure continuity in the speed of sound at the transition density. In particular, we avoided the discontinuities in the speed of sound appeared at the transition densities by employing a method presented in [10]. Moreover, in the approach (a) we studied the method where discontinuities are presented in the EoS and the one where continuity exhibits based on the previously mentioned method. In approach (b), these effects on the bulk properties of neutron stars at the maximum mass configuration are insignificant.

For the numerical integration of the equilibrium equations, we used the public RNS code by Stergioulas and Friedman [11].

### CONSTRAINTS ON THE BULK PROPERTIES

In case of the gravitational mass, as Fig.1. shows, both in nonrotating and maximally rotating configuration, a reduction on the gravitational mass along the transition density occurs until it reaches a constant value.



**Figure 1.** Dependence of the maximum gravitational mass (left) and the equatorial radius (right) on the transition density for various speed of sound bounds.

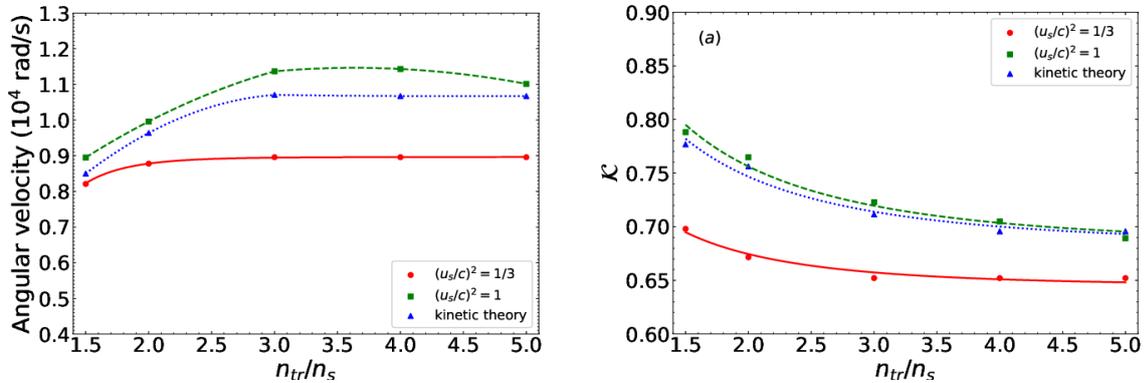
In the case of the equatorial and polar radius as a function of the transition density, while similar dependence with the gravitational mass is presented, the bound  $c/\sqrt{3}$  differs. In particular, after  $3n_s$  the  $c/\sqrt{3}$  case leads to higher values of radius than the other two bounds.

The angular velocity at the maximum mass configuration for the maximally rotating case is an increasing function of the transition density, as displayed in Fig. 2. This effect remains valid only for the bound  $c/\sqrt{3}$ . In the rest cases, it reaches a maximum at  $3n_s$  and then decreases along the transition density.

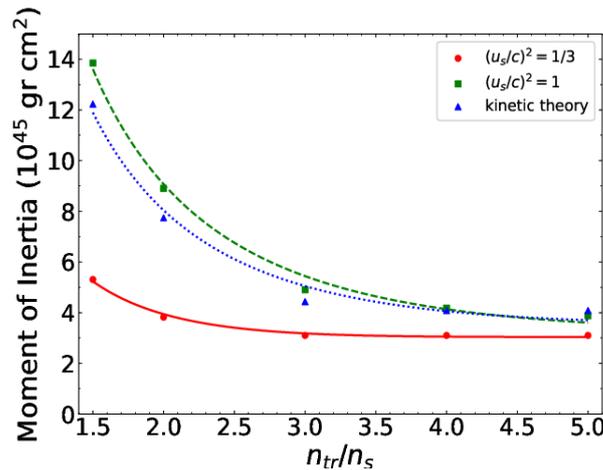
The Kerr parameter can lead to possible limits for the compactness on neutron stars and it can be a criterion for determining the final fate of the collapse of a rotating compact star [6]. It follows the mass-density relation as a decreasing function of the transition density.

We studied the moment of inertia in correlation with the transition density for the various bounds. In Fig. 3, moment of inertia follows a decreasing trajectory along the transition density until it reaches a constant value.

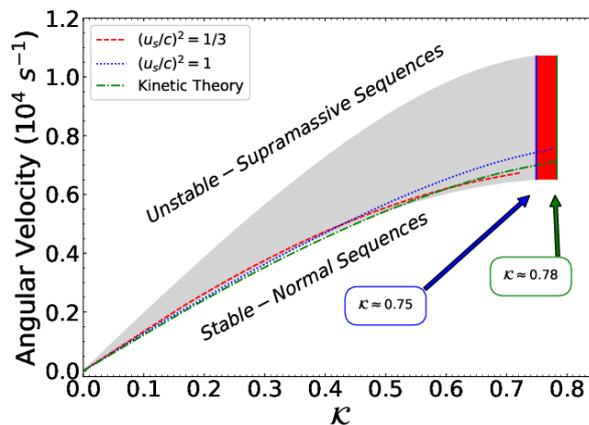
We plot in Fig. 4, the last stable rest mass sequence (LSRMS) for the various bounds at  $n_{tr} = 1.5n_s$ , which corresponds to the maximum mass configuration at the nonrotating model and defines the upper limit to the stable region. It is remarkable, that the three bounds, define the lower limit on the LSRMS.



**Figure 2.** Dependence of the angular velocity (left) and the Kerr parameter (right) on the transition density at the maximum mass configuration for various speed of sound bounds.

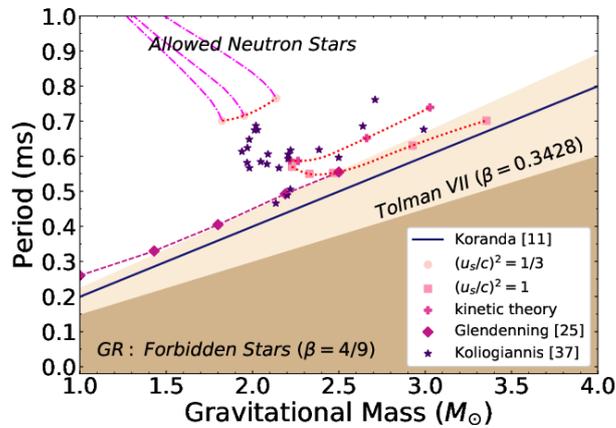


**Figure 3.** Dependence of the moment of inertia on the transition density at the maximum mass configuration for the various speed of sound bounds.



**Figure 4.** Last stable rest mass sequence as the dependence of the angular velocity on the Kerr parameter for the various speed of sound bounds at  $n_{tr}=1.5n_s$ .

For a given EoS the maximum gravitational mass in the sequence of gravitationally bound neutron stars has the minimum Keplerian period. Therefore, in order to provide useful constraints on the EoS, we present Fig. 5.



**Figure 5.** Minimum rotational period of rotating neutron stars as a function of the mass of the maximum mass spherical star allowed by the EoS of the stellar matter.

## DISCUSSION AND CONCLUSIONS

Sequences of both nonrotating and rotating neutron stars with different speed of sound bounds and transition densities have been studied. In this paper we have studied the bulk properties of maximally rotating neutron stars in correlation with the transition density and the gravitational mass.

The gravitational mass and radius are decreasing functions along the transition density for all bounds until they reach a constant value. Another interesting effect is provided through the  $c/\sqrt{3}$  bound, where the radius exceeds the other two bounds after  $3n_s$ . The study of the angular velocity showed that, in all cases, after  $3n_s$  a decrease is observed, except in  $c/\sqrt{3}$  case where it stabilizes after  $3n_s$  in a constant value. In the case of the Kerr parameter, the bound  $c$  leads to higher values than the other two bounds. At the most extreme configuration studied in this paper, meaning at  $1.5n_s$  and bound  $c$ , the Kerr parameter reaches a maximum value at around 0.8, which means that the gravitational collapse of a uniformly rotating neutron star, constrained to mass–energy and angular momentum conservation, cannot lead to a maximally rotating Kerr black hole [6]. The effects of the speed of sound bounds are enhanced for the moment of inertia, where the decrease is up to 2.5 times between the  $c$  and  $c/\sqrt{3}$  bound. This can lead to possible constraints on the spin frequency on neutron stars. From the LSRMS sequence it is obvious that the normal region of neutron stars is extended downwards concerning the angular velocity. The latter one leads us to lower spin frequencies on neutron stars. Furthermore, the minimum rotating period of a neutron star as a function of the spherical gravitational mass has been studied. The bound  $c/\sqrt{3}$  significantly limits the allowed area of neutron stars, excluding also a majority of realistic EoSs. This bound can provide strong constraints on the maximum gravitational mass, as well as on the minimum rotating period of neutron stars.

The observational measurement of the Keplerian frequency as well as of the rest bulk properties (mass, radius, Kerr parameter and moment of inertia and etc.), along with the theoretical predictions, would provide us with strong constraints on the high–density part of the EoS. Moreover, these observations will help also to check the validity of the proposed upper bounds of the speed of sound in dense nuclear matter (for example, see the recent [4]).

## Acknowledgments

The authors thank Prof. K. Kokkotas for his constructive comments on the preparation of the manuscript, Prof. N. Stergioulas for providing the RNS code and for fruitful discussions and Dr. S. Typel for useful corresponds and discussions. This work was partially supported by the COST action PHAROS (CA16214) and the DAAD Germany–Greece grant ID 57340132.

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