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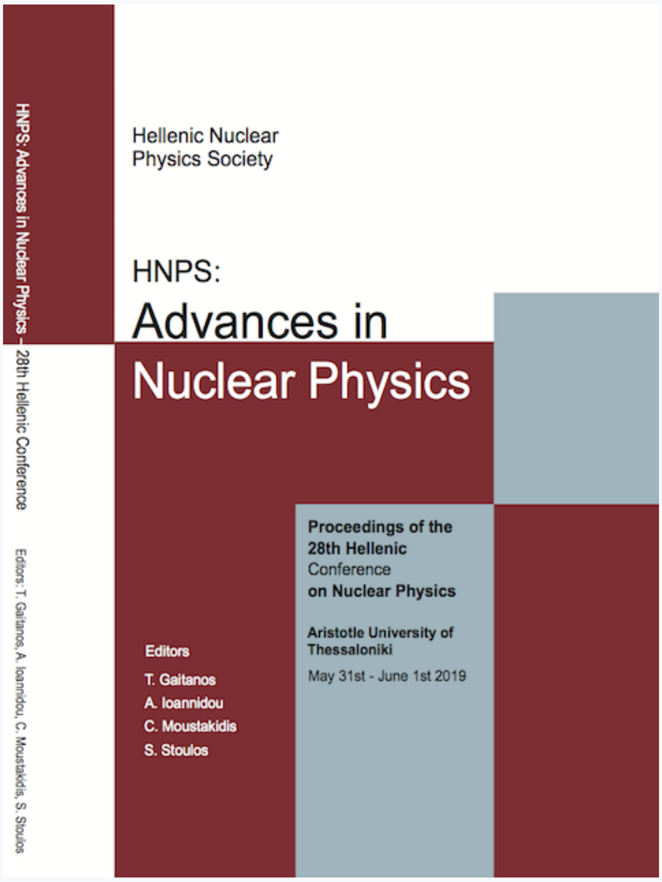

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NLD Equation of State for compressed, hot and relativistic nuclear matter

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Abstract *We investigate the properties of compressed and hot hadronic matter within the Non-Linear Derivative (NLD) formalism. The novel feature of the NLD model is an explicit momentum dependence of the mean-fields, which is regulated by cut-off's of natural hadronic scale. It is covariantly and thermodynamical–consistently formulated at the basis of a field–theoretical level. We show that the NLD model describes adequately all the empirical information of cold nuclear matter as function of density (equation of state) and, in particular, as function of particle momenta (optical potential). Finally, we present predictions of the NLD approach for hot and compressed hadronic matter in terms of the Equation of state as function of density and temperature. These studies are relevant for the forthcoming experiments at FAIR@GSI. They are also important for astrophysical purposes, e.g., static neutron stars and dynamic neutron star binary systems.*

Keywords *optical potential, equation of state*

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INTRODUCTION

Describing nuclear matter in its extreme is gaining more and more attention over the last decades. Experimental data from Heavy Ion Collisions and data from Neutron Stars give a boost to this study, but the same time the theoretical work itself may give directions to the experiments.

A simple tool for the theoretical description of nuclear matter is Relativistic Hydrodynamics (RHD) and a typical approximation in the RHD framework is the relativistic mean-field approximation (RMF), where the meson field operators are replaced by their ground state expectation values, which are classical fields. Taking the standard RHD Lagrangian in RMF approximation, the nucleon self-energies become simple functions of density only and do not depend on momentum of the nucleon explicitly. Therefore the Schrödinger-equivalent optical potential, which is energy dependent (as a consequence of relativistic description), depends linearly on energy and at high energies does not agree with Dirac phenomenology (Figure 1). In order to solve this issue, we introduce the Non-linear derivative (NLD) model, where the nucleon self-energies depend not only on density, but explicitly on momentum of the nucleon [1]. It modifies the behavior of optical potential, eq. (1).

$$U_{opt} = \frac{E}{m}\Sigma_v - \Sigma_s + \frac{1}{2m}\left({\Sigma_s}^2 - {\Sigma_v}^2\right) \tag{1}$$

THE NON-LINEAR DERIVATIVE (NLD) MODEL

The Lagrangian is as in the conventional QHD

$$\mathcal{L} = \frac{1}{2}\left[\bar{\Psi}\gamma_\mu i\vec{\partial}^\mu\Psi - \bar{\Psi}i\overleftarrow{\partial}^\mu\gamma_\mu\Psi\right] - m\bar{\Psi}\Psi - \frac{1}{2}m_\sigma{}^2\sigma^2 + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) + \frac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\rho{}^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu} + \mathcal{L}_{int}$$

but in the interaction term we introduce the momentum dependence with the non-linear derivative operators $\vec{D}$,$\overleftarrow{D}$

$$\mathcal{L}_{int} = \frac{g_\sigma}{2}\left[\bar{\Psi}\overleftarrow{D}\Psi_\sigma + \sigma\bar{\Psi}\vec{D}\Psi\right] - \frac{g_\omega}{2}\left[\bar{\Psi}\overleftarrow{D}\gamma^\mu\Psi\omega_\mu + \omega_\mu\bar{\Psi}\gamma^\mu\vec{D}\Psi\right] - \frac{g_\rho}{2}\left[\bar{\Psi}\overleftarrow{D}\gamma^\mu\vec{\tau}\Psi\vec{\rho}_\mu + \vec{\rho}_\mu\bar{\Psi}\vec{\tau}\gamma^\mu\vec{D}\Psi\right]$$

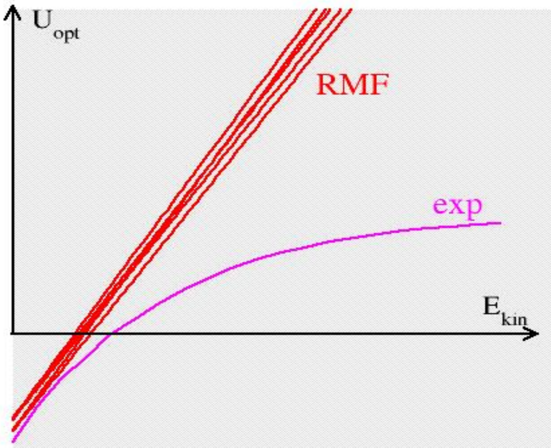

Figure 1. *In-medium proton Schrödinger-equivalent Re(U_{opt})*

which are assumed to be generic functions of partial derivative operator and supposed to act on the nucleon spinors Ψ and $\bar{\Psi}$ respectively. The Taylor expansion of the operator functions (supposing that they are smooth functions) in terms of partial derivatives generates an infinite series of higher-order derivative terms

$$\vec{D} := D(\vec{\xi}) = \sum_{j=0}^{n\to\infty} \frac{\partial^j}{\partial\vec{\xi}^i} D|_{\vec{\xi}\to 0} \frac{\vec{\xi}^j}{j!}$$

$$\overleftarrow{D} := D(\overleftarrow{\xi}) = \sum_{j=0}^{n\to\infty} \frac{\overleftarrow{\xi}^j}{j!} \frac{\partial^j}{\partial\overleftarrow{\xi}^i} D|_{\overleftarrow{\xi}\to 0}$$

The expansion coefficients are given by the partial derivatives of D with respect to the operator arguments $\vec{\xi}$ and $\overleftarrow{\xi}$ around the origin. The operators are defined as $\vec{\xi} = -\frac{v^\alpha i\vec{\partial}_\alpha}{\Lambda}$ and $\overleftarrow{\xi} = \frac{i\overleftarrow{\partial}_\alpha v^\alpha}{\Lambda}$, where Λ is a cut-off parameter (its value is supposed to be of natural hadronic scale of around 1 GeV) and v^α is an auxiliary vector [2].

Applying the generalized Euler-Lagrange equations to the full Lagrangian density with respect to the spinor field Ψ, leads to a Dirac equation with self-energies, which in the RMF approximation to infinite nuclear matter are

$$\Sigma_{vi}^\mu = g_\omega\omega^\mu D + g_\rho\tau_i\rho^\mu D \qquad \text{and} \qquad \Sigma_{si} = g_\sigma\sigma D.$$

The meson-field equations are taken from the standard Euler–Lagrange equations

$$m_\sigma{}^2\sigma + \frac{\partial U}{\partial\sigma} = g_\sigma \sum_{i=p,n} \langle\bar{\Psi}_i D\Psi_i\rangle = g_\sigma\rho_s$$

$$m_\omega{}^2\omega = g_\omega \sum_{i=p,n} \langle \overline{\Psi}_i \gamma^0 D \Psi_i \rangle = g_\omega \rho_0$$

$$m_\rho{}^2\rho = g_\rho \textstyle\sum_{i=p,n} \tau_i \langle \overline{\Psi}_i \gamma^0 D \Psi_i \rangle = g_\rho \rho_I \ .$$

It must be noticed that the cut-off Λ regulates both the density and momentum dependence of self-energies, and the density dependence of meson-field sources (particularly for ω-field).

Applying the Noether theorem for translational invariance to the NLD Lagrangian gives us the energy-momentum tensor, from which the energy density $\varepsilon \equiv T^{00}$ and the pressure P are

$$\varepsilon = \sum_{i=p,n} \frac{\kappa}{(2\pi)^3} \int_{|\vec{p}|\le p_{F_i}} d^3p E(\vec{p}) - \langle \mathcal{L} \rangle$$

$$P = \frac{1}{3} \sum_{i=p,n} \int_{|\vec{p}|\le p_{F_i}} d^3p \frac{\vec{\Pi}_i \cdot \vec{p}}{\Pi_i^0} + \langle \mathcal{L} \rangle$$

NLD RESULTS

Model Parameters

On table 1 there is the form of operator D we choose (it's a monopole form) and the values of parameters, which have been extracted from fit to known properties (saturation density, binding energy per nucleon, compressibility etc.), and on table 2 there are the values of these properties in comparison with values from other theoretical models.

Table 1. *Form of D & Parameters*

$\vec{D}$	D in NM	Λ_s Λ_v [GeV]	g_σ	g_ω	g_ρ	b [fm^{-1}]	c	m_σ	m_ω	m_ρ [GeV]
$\frac{1}{1+\sum_{j=1}^{4}\left(\frac{v_j^\alpha i\overrightarrow{\partial_\alpha}}{\Lambda}\right)^2}$	$\frac{\Lambda^2}{\Lambda^2+\vec{p}^2}$	0.95 1.125	10.08	10.13	3.50	15.341	-14.735	0.592	0.782	0.763

In our fit to bulk properties of nuclear matter we use different cut-off parameters Λ_s and Λ_v for the scalar and meson-nucleon vertices, respectively. The parameters of NLD are: the meson-nucleon couplings g_s , g_ω and g_ρ, the parameters b and c of the self-interactions of the σ-meson, the mass m_σ of the σ-meson and the cut-offs Λ_s and Λ_v (for m_ω and m_ρ we take the bare masses, because in all the calculations concerning the fit the results for these two masses turned out to be always around their free values).

The NLD equation of state

The density dependence affects the equation of state i.e. the binding energy per nucleon as function of nucleon density. In Figure 2 this is demonstrated for isospin-symmetric $(\alpha = 0)$[1] and pure neutron matter $(\alpha = -1)$ in comparison with DBHF calculations. The momentum-dependent

[1] The isospin-asymmetry parameter α is defined as $\alpha = \frac{J_p^0 - J_n^0}{J_p^0 + J_n^0}$,where $J_{p,n}^0$ denote the proton and neutron density, respectively.

monopole form factor D regulates the high-density dependence of the fields such that the NLD EoS agrees with the DBHF calculations for both symmetric nuclear and pure neutron matter. It must be noted that the NLD parameters are not fitted to the calculations of DBHF models, but to the empirical data at ground state density only.

Table 2. *Bulk saturation properties for NLD in comparison with other theoretical models*

Model	ρ_{sat} [fm^{-3}]	E_b [MeV/A]	K [MeV]	α_{sym} [MeV]	L [MeV]	K_{sym} [MeV]	K_{asy} [MeV]
NLD	0.156	-15.30	251	30	81	-28	-514
NL3*	0.150	-16.31	258	38.68	125.7	104.08	-650.12
DD	0.149	-16.02	240	31.60	56	-95.30	-431.30
D^{3}C	0.151	-15.98	232.5	31.90	59.30	-74.7	-430.50
DBHF	0.185	-15.60	290	33.35	71.10	-27.1	-453.70
	0.181	-16.15	230	34.20	71	87.36	-340
empirical	0.167±0.019	-16±1	230±10	31.1±1.9	88±25	–	-550±100

In-medium nucleon optical potential

Figure 3 shows the in-medium proton optical potential as function of the in-medium single particle kinetic energy for typical RMF models and NLD model in comparison with results of Dirac phenomenology [3]. Obviously NLD is consistent with Dirac phenomenology.

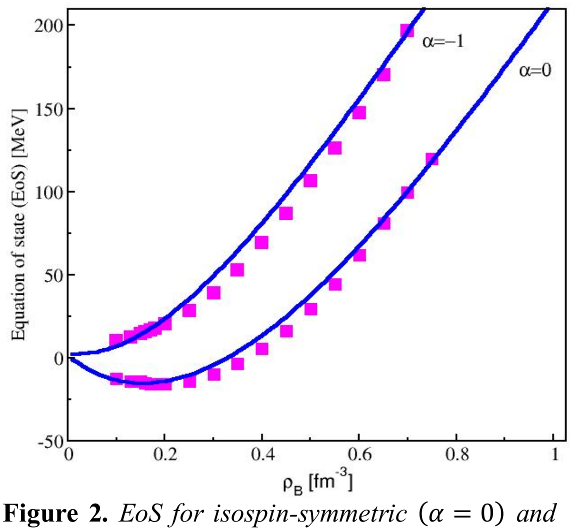

Figure 2. *EoS for isospin-symmetric* $(\alpha = 0)$ *and pure neutron matter* $(\alpha = -1)$. *NLD model (blue solid line) and DBHF calculations (filled squares).*

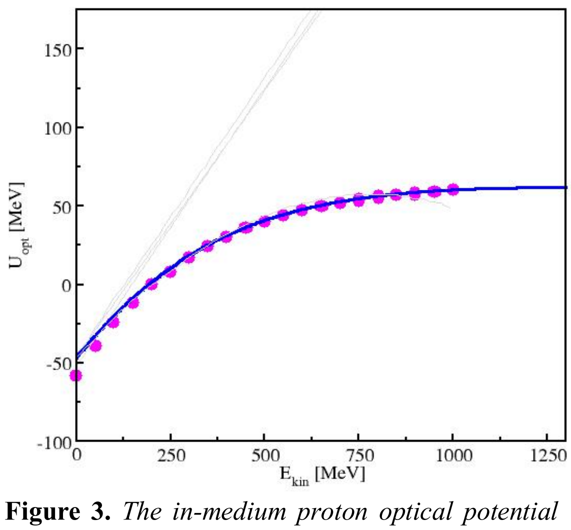

Figure 3. *The in-medium proton optical potential as function of the in-medium single particle kinetic energy. Typical RMF models (grey straight lines), NLD model (blue solid curve) and results of the Dirac phenomenology (filled circles).*

Equation of state for hot matter

All the previous study was for temperature equal to zero. In Figures 4 and 5 are shown predictions of the NLD approach for hot hadronic matter for symmetric and pure neutron matter, respectively. The reason for this extrapolation is our willing to apply this model to dynamic systems, like binary Neutron Stars, where the temperatures are higher.

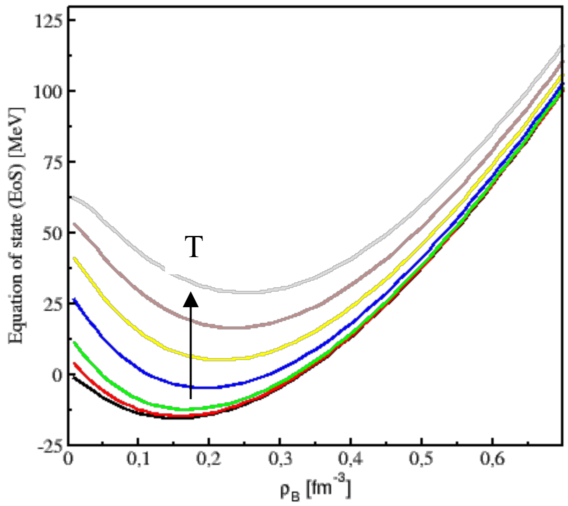

Figure 4. *EoS for different values of temperature. Symmetric matter* ($\alpha = 0$). *Starting from T=0 (black curve) the second curve is for T=10 MeV (red curve), the third for T=20 MeV (green curve) etc.*

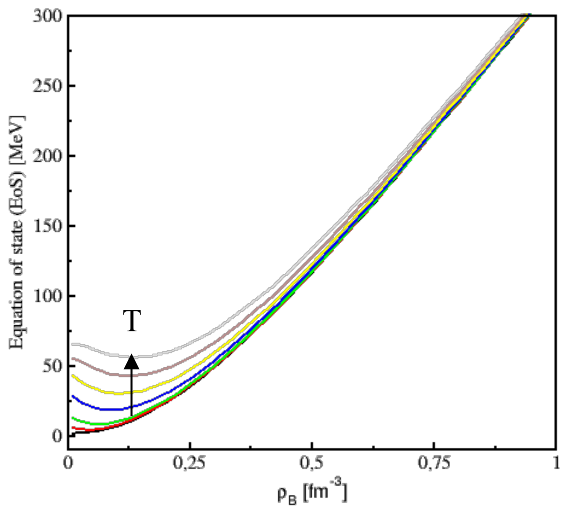

Figure 5. *EoS for different values of temperature. Pure neutron matter* ($\alpha = -1$). *Starting from T=0 (black curve) the second curve is for T=10 MeV (red curve), the third for T=20 MeV (green curve) etc.*

CONCLUSIONS

The advantage of NLD model is that keeping the simplicity of RMF approximation, it can describe complex features (non-liner density and momentum dependences), only by introducing appropriate regulators on a Lagrangian level covariantly, which regulate the high density and momentum components of mean fields.

The equation of state we get is soft at low densities (the compressibility is around 250 MeV, Table 2), but becomes stiffer at high densities a remarkable agreement with microscopic DBHF calculations (Figure 2). The momentum dependence we introduced seems to be correct, if we compare with results from Dirac phenomenology (Figure 3). All these are compatible with all recent observations of high density relevant equations of state (Neutron Stars).

Regarding future developments, we will apply our model to new experiments, which will be done at HADES collaboration for pion induced reactions. This will give us many experimental data not only for nucleons but for other particles as well, so we can expand this formalism to strangeness. Also we will do more systematic comparison with results from transport calculations.

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