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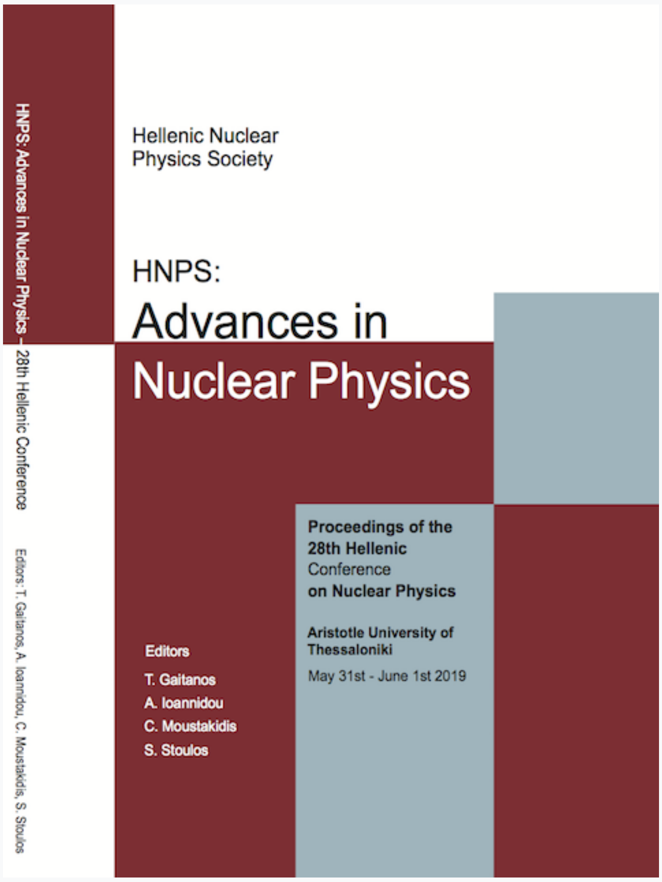


# Towards the Keplerian sequence: Realistic equations of state in rapidly rotating neutron stars

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# Towards the Keplerian sequence: Realistic equations of state in rapidly rotating neutron stars

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**Abstract** *Neutron stars are among the densest known objects in the universe and an ideal laboratory for the strange physics of super-condensed matter. In the present work, we investigate the Keplerian (mass–shedding) sequence of rotating neutron stars by employing realistic equations of state based on various theoretical nuclear models. In particular, we compute the moment of inertia and angular momentum of neutron stars against mass–shedding and secular axisymmetric instability. We mainly focus on the dependence of these properties from the bulk properties of neutron stars. Another property that studied in detail, is the dimensionless spin parameter (kerr parameter) of rotating neutron stars at the mass–shedding limit. In addition, supramassive time evolutionary rest mass sequences, which have their origin in general relativity, are explored. Supramassive sequences have masses exceeding the maximum mass of a non–rotating neutron star and evolve toward catastrophic collapse to a black hole. Important information can be gained from the astrophysical meaning of the kerr parameter and the supramassive sequences in neutron stars. Finally, the effects of the Keplerian sequence, in connection with the latter, may provide us constraints on the high density part of the equation of state of cold neutron star matter.*

**Keywords** *Neutron stars, Equation of state, Keplerian sequence*

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## INTRODUCTION

Among the most important objectives for modern astrophysics are the neutron stars, as they are the ideal laboratory for extremes of densities and gravity. As the nuclear equation of state (EoS) at several times the nuclear saturation density remains unknown, the study of such compact objects, non–rotating or rotating ones, is the key ingredient to understand the constitute of nuclear matter at high densities.

In this work, we study the effects of the mass–shedding frequency on the bulk properties of both non–rotating and rapidly rotating neutron stars [1,2]. For this reason, we employ a total of 23 hadronic EoSs [3] based on various theoretical nuclear models. To be more specific, we calculate their gravitational mass and radius, moment of inertia and kerr parameter. The behavior of these properties at the mass–shedding limit may help to constrain the EoS at high densities [1,2].

In addition, we study the normal and supramassive evolutionary sequences of neutron stars in order to examine the case where they considered to be progenitors of black holes [4]. While normal evolutionary sequences are stable, supramassive ones are not. In fact, the ending of these sequences is the collapse to a black hole through the spin-up effect. Its importance lie with the fact that we could identify when a neutron star would collapse to a black hole by looking its spin up effect.

## EQUATIONS OF STATE OF NEUTRON STARS AND THE RAPID ROTATION

The equilibrium equations for a rotating neutron star, in the framework of General Relativity, can be described a) by the stationary axisymmetric space-time metric [5]

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}(d\varphi - \omega dt)^2 + e^{2\mu}(dr^2 + r^2 d\theta^2) \qquad (1)$$

where the metric functions ν, ψ, ω and μ depend only on the coordinates r and θ, and b) the matter inside the neutron star. If we neglect sources of non–isotropic stresses, as well as viscous ones and heat transport, then the matter inside the neutron star can be fully described by the stress-energy tensor and modeled as a perfect fluid [5]

$$T_{\alpha,\beta} = (\varepsilon + P)u_\alpha u_\beta + P g_{\alpha\beta} \tag{2}$$

where $u_\alpha$ is the fluid's 4-velocity. The energy density and pressure is denoted as ε and P .

## *Keplerian frequency*

We studied the relation between the Keplerian frequency and the bulk properties of both non–rotating and rotating with Keplerian frequency neutron stars according to the relation [6]

$$f_k = C_a \left(\frac{M^a_{max}}{M_\odot}\right)^{1/2} \left(\frac{10\ km}{R^a_{max}}\right)^{3/2} = C_a x^a_{max}\ (\mathrm{Hz}) \tag{3}$$

where α takes the form of the corresponding configuration (static, rotating). The C values are: 1266.68 and 1781.9, respectively. While a linear relation holds on in both cases as Fig. 1 shows, the linear relation at the rotating configuration is almost perfect. However, the importance of the non–rotating case is that it connects two different configurations, the non–rotating and the rotating one.

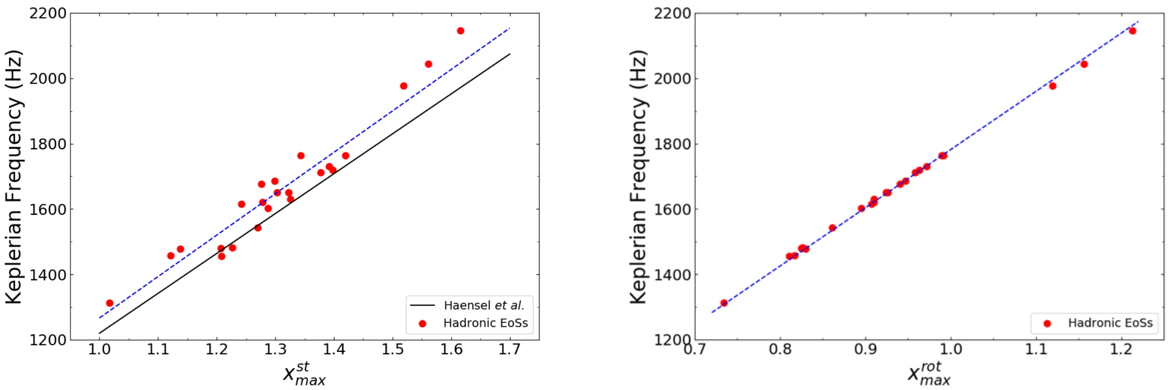


**Figure 1**.*The dependence of the Keplerian frequency on the bulk properties for the (left) non–rotating and (right) rotating configuration for the 23 hadronic EoSs. The black curve marks the work of Haensel et al. [6].*

## *Moment of inertia*

As rapid rotation probes more aspects, we studied the dependence of the moment of inertia on the gravitational mass for the Keplerian sequence. Moment of inertia, which plays an important role in pulsar analysis, is defined as [1,2]

$$I = J/\Omega \tag{4}$$

where J is the angular momentum and Ω is the angular velocity of the star.

From Fig. 2 it is clear that moment of inertia is an EoS-dependent property as its behavior is determined through the specific EoS. The latter can be seen from the relation at the maximum mass configuration at the inside figure of Fig.2.

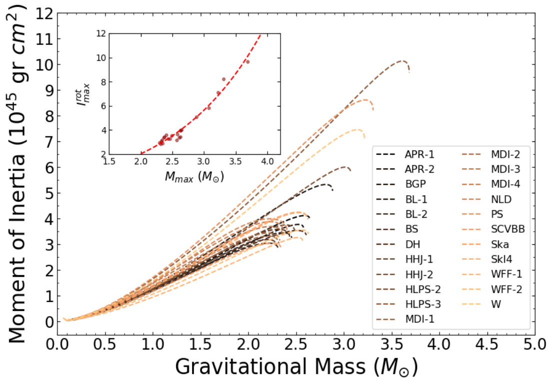


**Figure 2**. *The dependence of the moment of inertia on the gravitational mass for the 23 hadronic EoSs. (inside) The data at the maximum mass configuration are also presented.*

The relation at the maximum mass configuration is

$$I_{max}^{rot} = -1.568 + 0.883exp\left[0.7\left(\frac{M_{max}}{M_{\odot}}\right)\right](10^{45}grcm^2) \quad (5)$$

*Kerr parameter*

In neutron stars analysis, an important parameter to study is the dimensionless angular momentum which is known as spin parameter. Using this parameter we can define the kerr parameter for neutron stars as [1,2,7]

$$K = \frac{cJ}{GM^2} \quad (6)$$

We studied the dependence of this parameter on the gravitational mass for the Keplerian sequence in Fig. 3. There is a maximum value at the kerr parameter at around 0.75, much lower than the 0.998 for the maximally-rotating Kerr black holes [7]. This limit can help us to constrain the compactness parameter at neutron stars and also can be a criteria to determine the final fate of the collapse of a rotating compact object.

A relation for the maximum mass configuration has been also found and given by the equation

$$K_{max} = 0.488 + 0.074\left(\frac{M_{max}}{M_{\odot}}\right) \quad (7)$$

*Rest mass sequences*

The constant rest mass sequences of a neutron star are lines that extend from the Keplerian sequence to the non–rotating end point or at the axisymmetric instability limit [4]. Sequences that are below the rest mass value that corresponds to the maximum mass configuration at the non–rotating model are stable and terminate at the non–rotating model sequence. Above this value, all the sequences are unstable and terminate at the axisymmetric instability limit. The latter ones, are called supramassive sequences as their mass exceeds the maximum mass of the non–rotating configuration.

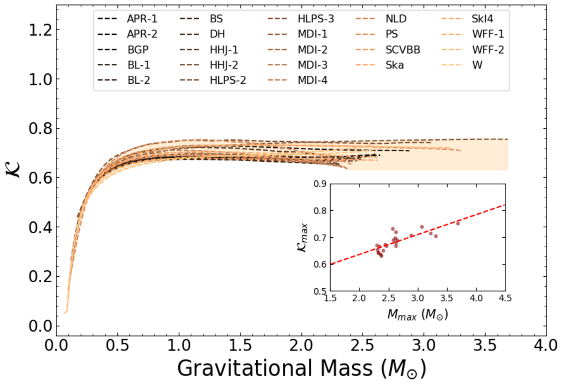


**Figure 3**. *The dependence of the kerr parameter on the gravitational mass for the 23 hadronic EoSs. (inside) The data at the maximum mass configuration are also presented.*

In all cases, neutron stars which evolve along normal evolutionary sequences, never spin-up as they lose angular momentum. In contradiction to them, neutron stars on supramassive ones, because their unstable portion is always at higher angular velocity than the stable one, at the same value of angular momentum, must spin-up with angular momentum loss in the neighborhood of the stability limit. If the neutron star is massive enough, then the evolutionary sequence (supramassive) exhibits an extended region where spin-up is allowed. The latter can be shown in Fig.4 (left). This effect may provide us an observable precursor to gravitational collapse to a black hole [4].

As a follow up to Fig. 4 (left), we have constructed the last stable rest mass sequence (LSRMS) for the 23 hadronic EoSs, as shown in Fig. 4 (right). This sequence is the one that divides the normal from supramassive evolutionary sequences. In Fig. 4 (right), we present a window (colored area) where the LSRMS can lie and because this sequence is the one that corresponds to the maximum mass configuration at the non–rotating model, this is also the region where the EoS can lie.

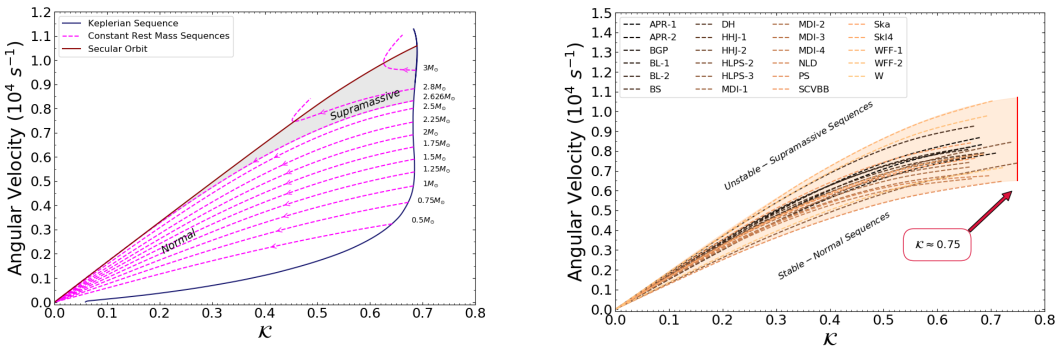


**Figure 4**. *(left) Normal and supramassive rest mass sequences and (right) the LSRMS sequence for the 23 hadronic EoSs as the dependence of the angular velocity on the kerr parameter.*

## *A limit on the central energy density*

Following the work of Lattimer and Prakash [8], we have used the 23 hadronic EoSs in order to reproduce their result that Tolman VII analytical solution describes the upper limit on the central energy density inside any compact object. We have confirmed that the Tolman VII solution defines the upper limit to the energy density inside a compact star but without taking into account the rotation. If we add rotation to our models, then the Tolman VII solution cannot describe the rotating data. For this reason, we propose here the existence of an analytical solution which describes both the non–rotating and the maximally-rotating configuration and is given by the form

$$\frac{M}{M_{\odot}} = 4.25 \sqrt{\frac{10^{15} gr cm^{-3}}{\varepsilon_c / c^2}} \tag{8}$$

The above data are presented in Fig. 5.

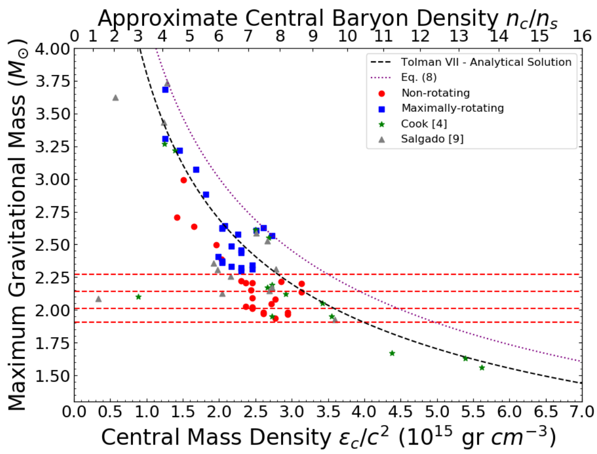


**Figure 5**. *The maximum gravitational mass versus the central energy density and the central baryon density for the 23 hadronic EoSs. The non–rotating data are presented with red circles and the rotating ones with blue squares. The data from Cook [4] and Salgado [9] are also presented with the stars and triangles. The Tolman VII analytical solution is marked with the black dashed line while the Eq. (8) with the purple dotted line. The red horizontal dashed lines mark the maximum observed neutron star mass.*

## CONCLUSIONS

Uniformly rotating neutron stars for a majority of hadronic EoSs [3] have been studied. For the numerical integration of the equilibrium equations, we used the public RNS code by Stergioulas and Friedman [10]. In particular, we have calculated their gravitational mass, moment of inertia and kerr parameter. Relations between these bulk properties have been established and shown in the corresponding figures. We have also studied the normal and supramassive evolutionary sequences of constant rest mass for a specific EoS and determining the stability region of a neutron star.

From an astrophysical point of view, we have established a universal empirical relation between the Keplerian frequency and the bulk properties of both non–rotating and rotating with the Keplerian frequency neutron stars. In addition, we have shown that moment of inertia is an EoS–dependent property as it is highly depending on the employed EoS. Furthermore, we have computed the maximum possible value of the kerr parameter in neutron stars at 0.75, and concluded with this way that the gravitational collapse of a rotating neutron star, constrained to mass–energy and angular momentum conservation [4,7], cannot lead to a maximally–rotating Kerr black hole ($K_{B.H.} = 0.998$). An important finding is also the LSRMS. This sequence, as it is the one that corresponds to the maximum mass configuration of the non–rotating model, can give us useful insight of the region where the EoS can lie and constrain both the angular velocity and kerr parameter.

The gravitational waves, as the powerful tool to study compact objects such as neutron stars, will provide us with the mass–shedding limit (Keplerian frequency) of these objects. To be more specific, the remnant formed in the immediate aftermath of the GW170817 merger contains sufficient angular momentum to be near its mass–shedding limit [11]. The observational measurement of the Keplerian frequency, along with the theoretical predictions, would provide us with strong constraints on the constitution of the dense nuclear matter.

## Acknowledgments

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