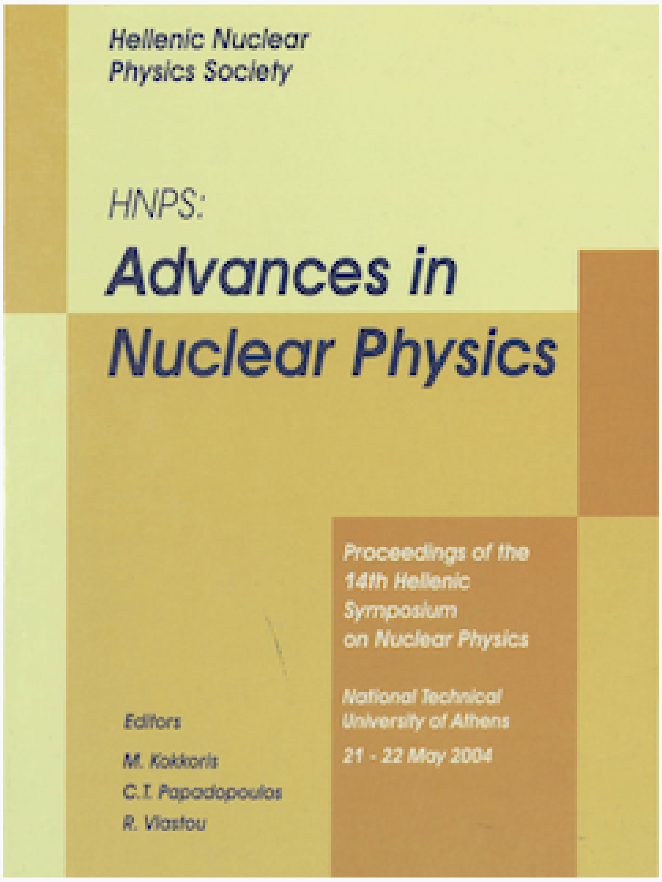
HNPS Advances in Nuclear Physics

Vol 13 (2004)

HNPS2004



On the Possible Stability of Tetraneutron and Hexaneutron

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doi: 10.12681/hnps.2981

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Abstract

A specific cluster approach, in the framework of the Isomorphic Shell Model, is employed to examine the possible stability of ^{2}n, ^{4}n, and ^{6}n. According to this study ^{2}n is definitely unstable, while ^{4}n and ^{6}n could be stable or, at least, exhibit a low-lying resonance. Better knowledge of the strength of the neutron-neutron force is highly desirable.

Key words: Nuclear structure; Neutron nuclei; Neutron clusters; Neutron drops; Tetraneutron; Hexaneutron. PACS numbers: 21.60.Gx, 21.60.-n, 21.30.-x, 21.10.Dr, 27.10.+h, 27.20.+n

1 Introduction

Recently [1], the production and detection of free neutron clusters have been seriously discussed. These clusters (an entirely new form of nuclear matter) were produced during the breakup of beams of very neutron-rich nuclei. The most promising neutron cluster, so far supported by six events, was the tetraneutron (^{4}n) produced in the breakup of ^{14}Be (which represents one of the best possible tools in search of a tetraneutron), most probably in the channel ^{10}Be $+^4$n. The reported lifetime (of the order 100 ns or longer) would indicate that a tetraneutron is particle stable. If this is finally verified, it could challenge our understanding of nuclear few-body systems and nucleon-nucleon (NN) interactions.

Later publications [2,3] deal with the subject of the tetraneutron from the theoretical point of view. It does not seem possible to change modern nuclear Hamiltonians to bind a tetraneutron without destroying many other successful predictions of those Hamiltonians. In general, calculations performed to date suggest that multineutron systems are unbound [4]. Even when an effective

NN potential binds a dineutron, it still cannot bind two dineutrons [4,5]. Even if 3N and 4N forces are considered, the possibility of ^{4}n is still excluded since, by applying the same forces to other well- known very light nuclei (e.g., ^{4}He), unreasonable results are reached [4]. However, it has also been found that subtle changes in NN potentials (which do not affect the phase shift analysis) may generate bound neutron clusters. In addition, the lack of predictive power of the calculations of few-body systems at the 1MeV level [6] does not exclude the possible existence of a very weakly bound ^{4}n.

In another recent reference [7] a review of the whole subject is made with emphasis on the impact the Marques et al experiments [1] had in the scientific society and on the far-reaching implications of the possible existence of a tetraneutron.

From the experimental point of view, however, Marques et al [1] found that, in some reactions where ^{14}Be breaks up to form ^{10}Be, it was difficult to trace the expected four flashes for the four neutrons and, instead, the GANIL team found just one flash of light in one detector as if the four neutrons were arriving together, i.e., in the form of ^{4}n. In addition, the most recent preliminary results of experiments involving the breakup of ^{12}Be into the ^{8}Be+^{4}n channel have revealed more events[8]. Also, the breakingup of ^{8}He into the ^{4}He+^{4}n channel has shown 12 supporting events[9] and the low-energy spectrum in the α particle reaction (d,^{6}Li), in inverse kinematics using incident ^{8}He at 15,8 Mev/A delivered by SPIRAL at GANIL and CD_2 targets, exhibits a deviation consistent with a resonant-like structure at 2.5 MeV supporting again the existense of ^{4}n [10]. From the theoretical point of view in [9], simulations in progress have been used to clarify the origin of ^{4}n events, including a determination of the energy surfaces by a microscopic approach using the Generator Coordinates Method in a four-center model where the neutrons are located at the vertices of a tetrahedral configuration.

Besides the above mentioned experiments and theoretical work, it is interesting for one to recall Ref.[11] which deals with a specific cluster approach (in the framework of the Isomorphic Shell Model) to exotic nuclei and especially to their extreme case, the neutron nuclei. There, all even neutron nuclei up to A=20, i.e., ^{2}n-^{20}n, have been studied. This study was performed exactly a decade ago, that is, much prior to the recent experiments of Ref.[1]. It is worth noting that in Ref.[11] the possibility of particle stable ^{4}n is reported and even the possible stability of other even neutron nuclei, e.g., ^{6}n. In this text, a more detailed study is undertaken with similar results.

2 The model

The model employed here, the Isomorphic Shell Model (ISM), has been previously published in numerous publications, e.g., in Ref.[12-18] and references therein. Here, only a brief outline of the model is given to present the basic concepts and assumptions of the model and to provide the necessary formulas for the present calculations.

The ISM is a microscopic nuclear-structure model that incorporates into a hybrid model the prominent features of single-particle and collective approaches in conjunction with the nucleon finite size. The model employs the following concepts and assumptions

•In nuclear structure, the nucleons have finite (no point) dimensions presented by hard (non-overlapping) spheres of definite sizes.

•Each nucleus is considered isolated due to the short range of nuclear forces and to the relatively large distances among nuclei in a body. Thus, its angular momentum is conserved.

•Antisymmetrization of total wave function for identical nucleons (assuming repulsive or attractive, or even zero forces among them) leads to configurations of most probable nucleon positions identical to those we have when repulsive forces (here of unknown nature) act among nucleons [19]. Each of these configurations, according to Leech [20] for repulsive particles on a sphere as the identical nucleons of a shell, forms a high symmetry polyhedron (i.e., a regular polyhedron or its derivatives.).

•The nucleons creating a central potential are the nucleons of each particular nuclear shell alone, instead of all nucleons in a nucleus as assumed in the conventional shell model. In other words, for a harmonic oscillator potential, we consider a multiharmonic nuclear Hamiltonian, as follows.

$$H = H_{1s} + H_{1p} + H_{1d2s} + \ldots, where \tag{1}$$

$$H_i = V_i + T_i = -\overline{V}_i \quad + \frac{1}{2} m(\omega_i)^2 r^2 + T_i. \tag{2}$$

Because of the different hω, wave functions with equal l value are not orthogonal and need to be orthogonalized. In the case of orthogonal wave functions the relevant binding energy equation is

$$\mathrm{E_B} = 1/2 \sum_{i=1}^{A} (\overline{V}_i \cdot N_i) - 3/4 [\sum_{i=1}^{A} h\omega_i (n + 3/2)], \tag{3}$$

where [21]

$$h\omega_i = \frac{\mathrm{h}^2}{m < r_i^2 >}(n + \frac{3}{2}) \tag{4}$$

The coefficients 1/2 and 3/3 take care of avoiding the double counting of nucleon pairs in determining the potential energy and ¡r_i^2¿$^{1/2}$is the average radius of each neutron or proton shell determined by the packing of shells themselves with respect to only two numerical (universal) parameters, i.e., the average size of a neutron (0.974 fm) and that of a proton (0.860 fm). These radii R are written at the bottom of each block of Fig.1.

Given that the $h\omega_i$values are known from above, the V_i are determined with respect to only one parameter, e.g., that of $_n\mathrm{V}_{1s}$ = 86.3 Mev, based on the relationship

$$E_i = -\overline{V}_i + h\omega_i(n_i + 3/2) = -\overline{V}_j + h\omega_j(n_j + 3/2) = E_j \tag{5}$$

For the nuclei up to N, Z≤20, the forms and sizes of polyhedral shells, coming from the above assumptions, are presented by Fig.1 identically published many times, e.g., in [14-16] and references therein. The coordinates of the vertices of the polyhedra involved are also published in Ref.[22] following their numbering in the figure.

In other words, the model implies that at some instant in time (reached periodically) all nucleons could be thought of as residing at their individual average positions, which coincide with vertices of a Leech polyhedron for each shell. This system of particles evolves in time according to each independent particle motion. This is possible, since axes standing for the angular-momenta quantization of directions are identically described by the rotational symmetries of the polyhedra employed [23-26]. Such vectors are shown in Fig.1 for the orbital angular-momentum quantization of directions in all nuclei up to N, Z ≤ 20. Each of these vectors corresponds to an angular momentum vector with given l and m values and labeled by a symbol $n\theta_l^m$ which specifies its angle formed with the common quantization axis z defined by the formula

$$n\theta_l^m = cos^{-1}\frac{m}{\sqrt{l(l+1)}} \tag{6}$$

Specifically, the quantization axis z is defined by the two average positions of the neutron zerohedron numbered 1 and 2 and passes through the middles of two opposite edges of the neutron octahedron or the proton hexahedron. The vectors $_n\theta_l^m$ for l=1 and m=0, ±1 pass through vertices of the neutron octahedron or through the middle of faces of the proton hexahedron, while those for l=2 and m= 0, ±1, ±2 pass through middle of faces of the neutron icosahedron or through vertices of the proton dodecahedron. All these vectors are indeed axes of symmetry of the relevant polyhedra. Based on these vectors, a quantum state may be assigned to each vertex (nucleon average position).

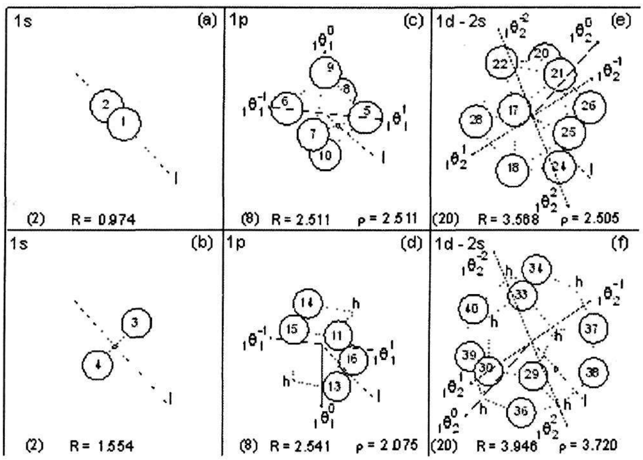

Fig. 1. Co-centric, oriented polyhedral shells of the Isomorphic Shell Model for nuclei with N, Z $\leq$ 20. The angular momentum quantization of directions ${}_n\theta_l^m$with respect to the shown common quantization axis z, the radii of the spheres through the polyhedral vertices R, the distances ρ of the polyhedral vertices from the relevant orbital angular momentum vectors, and the magic numbers 2, 8, 20 formed when a neutron or a proton polyhedral shell is completed are given. The letter h stands for a necessary hole in the proton polyhedra in order to obtain their minimum size which leads to the maximum binding energy.

For a small number of nucleons, as in the present study, the concept of a central potential is not favored. Thus, one can proceed in the semiclassical way (in the spirit of the Ehrenfest theorem [27] that for the average values the laws of Classical Mechanics are valid [28]) by using Fig.1. Each occupied vertex configuration of this figure corresponds to a state configuration with definite angular momentum and energy. The two-body potential employed in such a treatment and the relevant binding energy equation are the following

$$V_{ij} = 1.7(*10^{17})\frac{e^{-(31.8538)r_{ij}}}{r_{ij}} - 187\frac{e^{-(1.3538)r_{ij}}}{r_{ij}}, \text{ and } (7)$$

$$E_B = -\sum_{\substack{\textbf{all}\\ \textbf{nucleon}\\ \textbf{pairs}}} V_{ij} - \sum_{\substack{all\\ nucleons}} A\langle T\rangle_{nlm}, \text{ where } (8)$$

$$\langle \mathbf{T}\rangle_{nlm} = \frac{h^2}{2M}\left[\frac{1}{R_{\max}^2} + \frac{l(l+1)}{\rho_{nlm}^2}\right], (9)$$

$R_{max}$is the outermost polyhedral radius (R) plus the relevant nucleon radius, (i.e., the radius of the nuclear volume in which the nucleons are confined), M is the nucleon mass and $\rho_{nlm}$is the distance of the vertex (n,l,m) from the axis ${}^{n}\theta_{l}^{m}$ (see Fig.1 and Ref.[29]). These distances ρ are also written at the bottom of each block of Fig.1.

In Eq.(8) one should consider extra terms, the same as in Eq.(3), which are given and explained in several publications, e.g., in Refs.[30, 14-15] and references therein. These terms are Coulomb, isospin, even-odd, center of mass motion, and spin-orbit terms. From these terms only the last one is applicable here and will be discussed in the next section.

The most interesting feature of the ISM is that it uses the same assumptions and a few universal (not adjustable) parameters in its successful applications in the whole range of the periodic table, for ground and exited states.

3 Results and Discussion

In Table 1 here the part of Table 2 in Ref.[11] that refers to the even neutron nuclei ^{2}n-^{6}n is precisely repeated here. This part is sufficient for the purpose of the present paper, which is in the line of the experiments of Ref.[1]. In columns 2 and 3 of the present table the ground-state (corresponding to the maximum binding energy) vertex and state configurations, respectively, are given for the nuclei of column1. The average positions numbered 1 and 2 stand for the two neutrons in the 1s1/2 ($m_j = \pm 1/2$) state, while those numbered 5-8 stand for the one to four neutrons in the 1p3/2 ($m_j = \pm 3/2$) state. It is apparent that neutrons assume average positions only on neutron polyhedra and that states of a certain l and $\pm$j values occupy diametrical average positions. In columns 4-6 the potential, the kinetic, and the binding energy, according to the two-body potential of Ref.[22], are given for each nucleus examined. These values are exactly those presented in Ref.[11]. In columns 7-10 the new potential, the kinetic, the spin-orbit (see below the relevant equation), and the new binding energy for the same nuclei, according to the two-body potential of Ref.[30], are listed, as will be explained shortly. In column 11 of the table, remarks on possible stability of each neutron nucleus examined are made based on the sign of the binding energies given in columns 6 and 10. Finally, in column 12 the corresponding rms radii are listed (see below for the relevant equation).

Recently, during the research of Ref.[30], a new two body potential, slightly stronger than that used in Refs.[11,22], was determined. That is, instead of the constants 1.7(*10^{17}), 31.8538, 187, and 1.3538 of the potential in Eq.(7), the new constants 9.93(*10^{16}), 31.23338, 241.193, and 1.45338 was employed. While this new potential changes the potential energies of Ref.[11] (compare columns 4 and 8), the kinetic energies (columns 5 and 8) remain the same. Also, in Ref.[30] and this text, the spin-orbit component of the binding energy is estimated and listed in column 9 of the present Table 1. These new values do not change the conclusions of Ref.[11]. For the spin-orbit coupling one uses (10)

$$\sum_{\substack{all\\nucleons}} V_{LiSi} = \sum_{\substack{all\\nucleons}} \frac{0.03}{h^2/m} (h\omega)^2 l_i . s_i \text{ Mev}, \quad (10)$$

where $h\omega$ comes from Ref.[30] or from (4) here using r_{1p}=2.511 fm from Fig.1.

			Poten. of Ref.[22]			Potential of Ref.[30]				
Nu-clei	Nucleon Average Positions Nos.	State configu-rations	ΣV	<T>	E	ΣV	<T>	E	E	Com. Radii
n	1-2	(1s1/2)	6.9	-10.9	-4.0	7.3	-10.9	0.0	-3.6	unst, 1.33
'n	1-2, 7-8	(1s1/2) (1p3/2)	22.4	-20.0	2.4	23.2	-20.0	0.2	2.2	st. 2.11
n	1-2, 5-8	(1s1/2) (1p3/2)	38.9	-36.6	2.3	40.6	-36.6	0.4	3.7	st. 2.31

Fig. 2. (Table1.) Nucleon average positions and state configurations, potential-, kinetic-, spin-orbit-, binding-energy (in Mev), rms radius (in fm), and comments of stability for each of the even neutron nuclei 2n-6n for two different two-body potentials.

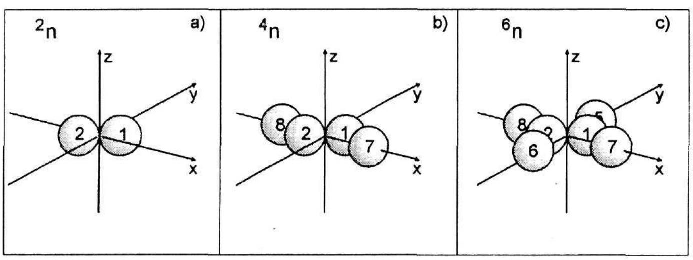

Fig. 3. Possible neutron nuclei, (a) dineutron, 2n (b) tetraneutron, 4n and (c) hexa-neutron 6n. Numbering of neutron average positions as in Fig.1

From columns 6 and 10 of Table 1 it is apparent that ^{2}n is unstable since its binding energy is negative. From the same columns the nuclei ^{4}n and ^{6}n, which appear with positive binding energy, seem particle stable in this study.

An important factor, which could lead to a reduction of the binding energies of column 10, is the following. The two-body potential in both Refs. [22,30] has been derived by assuming that np, pp, and nn forces are all equal. This, of course, is an approximation. There are many reasons to believe that nn and pp forces are weaker than np force. Such an effect would lead to a reduction of the potential energy for all neutron nuclei (see column 7 of Table 1). The question is-would this reduction overcome the positive values of binding energies in column 10 for ^{4}n and ^{6}n? Even in this case, according to the present analysis at least a low-lying resonant state is reasonably expected for both ^{4}n and ^{6}n. Furthermore, from Table 1 it is apparent that ^{6}n is a better candidate for stability than ^{4}n. So far, of course, no relevant experiments have been performed.

Finally, it is interesting for one to estimate the rms radius of the nuclei in Table 1 by using the formula [30]

$$\langle r^2 \rangle^{1/2} = \{\Sigma r_i^2/N + (0.91)^2\}^{1/2} fm, (11)$$

where the different r_i come from Fig.1 and where 0.91fm is a crude estimation of the contribution to the radius of the neutron finite size [30]. The radii found are listed in column 12 of Table 1. Of course, one should compare the value 2.11 fm given here for ^{4}n with the value 11.5 fm reported by Pieper [3]. This tremendous difference, at least, is consistent with the opposite conclusions of the present and that work concerning the possible stability of ^{4}n. Also, the underlying structure of the tetraneutron here (see Fig.3) is not that of two well-separated dineutrons as mentioned in [3].

It is instructive for one to visualize the average structure of the even neutron nuclei ^{2}n-^{6}n shown in Fig.3 and coming from Fig.1 by considering the occupied vertex configurations given in column 2 of Table 1 as apparent from the common numbering in both of these figures. The unusual shapes and deformations assumed by these nuclei are obvious from Fig.3. Specifically, the average cluster shape of ^{2}n is linear, while that of ^{4}n and ^{6}n is planar.

The fact that, according to Ref. [31], the angular speed for the independent particle motion and that for the collective rotational motion are almost equal for the nuclei examined makes these two motions to be coupled. Thus, due to the relationship $\omega_{i.p.} \approx \omega_{coll}$, the adiabatic approximation is not valid. That is, the above mentioned comparable size of rotational speeds [31] results in mixing of independent particle and collective motions, and such a mixing should be considered, e.g., in determining the ground state energies of ^{2}n-^{6}n nuclei. This mixing of motions is the subject of a future work.

In conclusion, it is interesting to state clearly that the model employed [32,33] and the two-body potential used [22] have been already published since 1985 and 1992, and 1982, respectively. The model employed [32,33] uses no adjustable parameters. Even more interesting is that a great part of the present paper comes from the conference proceedings of Ref.[11] published in 1993, that is, a decade prior to Ref. [1], where the production and detection of neutron clusters are seriously discussed.

4 Conclusions

In the present and previous studies [12-18] the cluster approach, in the framework of the Isomorphic Shell Model, has proved useful in calculating properties of nuclei all the way from the usual nuclei up to the most exotic nuclei, that is, the neutron nuclei.

From the present study it is concluded that while a dineutron is definitely unstable, a tetraneutron and a hexaneutron could be particle stable. The strength of the neutron-neutron potential is crucial for a final conclusion of their possible stability. From the present analysis, however, at least a low-lying resonant state is reasonably expected for both ^{4}n and ^{6}n, where the second nucleus is more favored than the first.

The fact that the angular speed of the independent particle motion and that of the collective rotational motion (for the nuclei here examined) are almost equal [31] results in the violation of the adiabatic approximation. Consequences of this violation will be studied in a future work.

References

[1] F.M. Marques et al, Phys. Rev. C 65 (2002) 044066.

[2] N.K. Timofeyuk, J. Phys. G 29 (2003) L9.

[3] S.C. Pieper, Phys. Rev. Lett. 90 (2003) 252501.

[4] D.R. Tilley, H.R. Weller, and G.M. Hale Nucl. Phys. A 541 (1992) 1 and references therein.

[5] A.B. Volkov, Nucl. Phys. 74 (1965) 33.

[6] J.J. Bevelacqua, Phys. Rev. C 33 (1986) 699.

[7] NewScientist 26 October 2002, p. 30.

[8] F.M. Marques, The search for neutral nuclei, in: Book of abstracts, International Symposium on Exotic Nuclei, Exon-2004, p. 50 (Peterhof, Lake Ladoga, Russia, July 5-12, 2004) organized by JINR, GSI, RIKEN, and GANIL.

[9] V. Bouchat et al, Search for tetraneutron in the breakup of ^{8}He, in: Book of abstracts, International Symposium on Exotic Nuclei, Exon-2004, p. 58 (Peterhof, Lake Ladoga, Russia, July 5-12, 2004) organized by JINR, GSI, RIKEN, and GANIL.

[10] D. Beaumel et al, Search for "tetraneutron" via the d(^{8}He, ^{6}Li)^{4}n transfer reaction, in: Book of abstracts, International Symposium on Exotic Nuclei, Exon-2004, p.59 (Peterhof, Lake Ladoga, Russia, July 5-12, 2004) organized by JINR, GSI, RIKEN, and GANIL.

[11] G.S. Anagnostatos, Cluster approach to exotic nuclei: Neutron nuclei, Proc. International school-seminar on heavy ion physics (Dubna, June 1992), Vol. 2, ed. Yu.Ts. Oganessian, Yu.E. Penionzhkevich and R. Kalpakchieva (JINR, Dubna, 1993) p. 548.

[12] P.E. Hodgson, Contemporary Physics 35 (1994) 329.

[13] P.E. Hodgson, Contemporary Physics 43 (2002) 461.

[14] P.K. Kakanis and G.S. Anagnostatos, Phys. Rev. C 54 (1996) 2946.

[15] G.S. Anagnostatos, P. Ginis, and J. Giapitzakis, Phys. Rev. C 58(1998) 3305

[16] G.S. Anagnostatos, Phys. Rev. C 51 (1995) 152.

[17] G.S. Anagnostatos, A.N. Antonov, P. Ginis, J. Giapitzakis, M.K. Gaidarov, A. Vassiliou, Phys. Rev. C 58 (1998) 2115.

[18] M. Avrigeanou, G.S. Anagnostatos, A.N. Antonov, J. Giapitzakis, Phys. Rev. C 62 (2000) 017001.

[19] C. W. Sherwin,Introduction to Quantum Mechanics (Holt, Rinehart and Winston, Inc., New York, 1959, p.205).

[20] J. Leech, Math. Gaz. 41 (1957) 81.

[21] W.F. Hornyak, Nuclear Structure (Academic, New York, 1975).

[22] G.S. Anagnostatos and C.N. Panos, Phys. Rev. C 26 (1982) 260.

[23] G.S. Anagnostatos, Lett. Nuovo Cimento 22 (1978) 507.

[24] G.S. Anagnostatos, Lett. Nuovo Cimento 28 (1980) 573.

[25] G.S. Anagnostatos, Lett. Nuovo Cimento 29 (1980) 188.

[26] G.S. Anagnostatos, J. Giapitzakis, and A. Kyritsis, Lett. Nuovo Cimento 32 (1981) 332.

[27] E. Merzbacher, Quantum Mechanics (John Wiley and Sons, Inc., New York, 1961, p. 42).

[28] C. Cohen-Jannoudji, B. Diu, F. Laloee, Quantum Mechanics (John Wiley and Sons, Inc., New York, 1977, p. 240).

[29] C.N. Panos and G.S. Anagnostatos, J. Phys. G 8 (1982) 1651.

[30] J. Giapitzakis, Symmetries in nuclear matter and in Σ hypernuclei, Dissertation (2003), University of Patras, Greece, General Section (available on request).

[31] G.S. Anagnostatos, Intrinsic-collective coupling in ^{6}He and ^{8}He, in: Proc. International conference on exotic nuclei (Foros, Crimea, October 1991), ed. Yu.E. Penionzhkevic and R. Kalpakchieva (World Scientific, Singapore, 1992) p. 104.

[32] G.S. Anagnostatos, Int. J. Theor. Phys. 24 (1985) 579.

[33] G.S. Anagnostatos, Can. J. Phys.70 (1992) 361.