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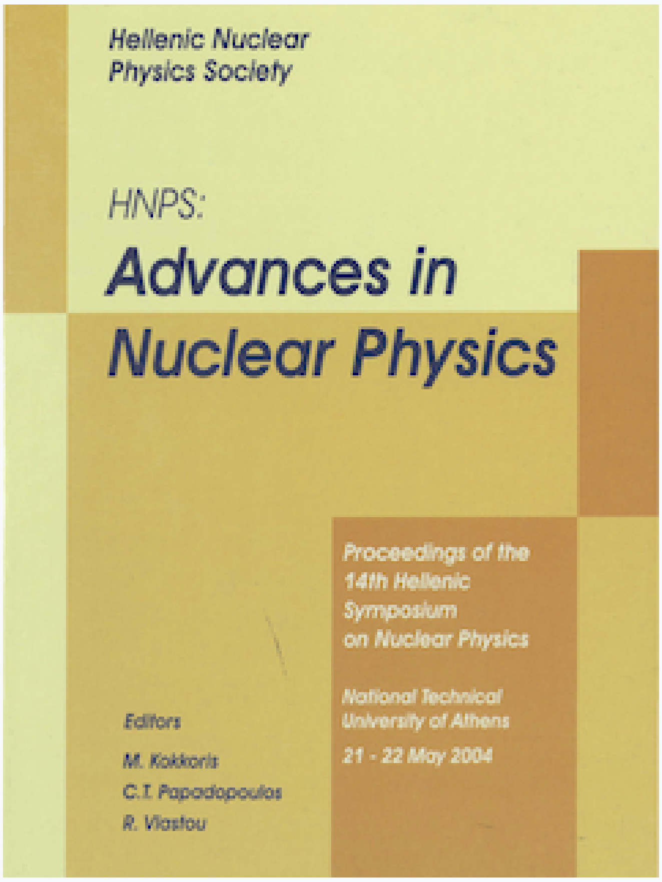

The Role of Λ-Effective Mass in a Phenomenological Analysis of the Λ-Energy in Hypernuclei

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The Role of Λ-Effective Mass in a Phenomenological Analysis of the Λ - Energy in Hypernuclei

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Abstract

A preliminary account is given of an effort undertaken to allow for the effective mass of the Λ - particle in a phenomenological analysis of existing experimental data for the (ground state) energy of the Λ : E_Λ in hypernuclei. The non - relativistic treatment is adopted to describe the motion of the Λ - particle, by considering a central potential well V (r), formed by the nuclear "host" medium of the Λ in the hypernucleus. The Λ - effective mass in the medium is taken to be an r-dependent effective mass $m^*_\Lambda(r)$ representing approximately some non-local effects in a way suggested in the past. Certain preliminary numerical results are obtained and discussed.

Key words: Λ - Energy in hypernuclei; r-dependent effective mass of the Λ;

1. Introduction

$\Lambda-$hypernuclei [1-4] are known and studied for more than half a century. In the beginning, data from nuclear emulsions provided values for the ground state s_Λ binding energies mainly for light hypernuclei. Next, the strangeness exchanging (K^-, π^-) in-flight and at rest reactions provided spectra with mainly p-shell substitutional states at CERN and BNL laboratories. In the last twenty years the associated production reaction (π^+, K^+) has provided spectra from light to heavy $\Lambda-$ hypernuclei with deeply bound $\Lambda-$ states at BNL and KEK laboratories. The peaks of the spectra correspond to various orbital angular momentum states s_Λ, p_Λ, d_Λ, f_Λ,...This method has been improved in statistics and resolution .

Many theoretical efforts have been made for the calculation of the $\Lambda-$ binding energies. Although very sophisticated microscopic methods were used to describe these data, the (semi)phenomenological study with appropriate single particle central potentials and mainly the Woods-Saxon or the Symmetrized Woods-Saxon turns out to be generally successful [5-8]. We mention, in particular, the "Global Least-Squares Fit" (GLSF), phenomenological study of the Λ- energies in a number of hypernuclei (see ref [8] and refs therein).

In a recent paper [9] an approach was proposed on the basis of which the experimentally known two lower energy eigenvalues of the $\Lambda-$ bound in a hypernucleus by a simple central potential, from a fairly wide class of them, are able to provide in certain cases rather direct information on other useful physical quantities, namely on the mean-square radii of the $\Lambda-$ orbitals and the kinetic and potential energies of the Λ. Such an approach although of simplified nature has the attractive feature that approximate analytic expressions for the physical quantities of interest can be derived under certain conditions.

The aim of this work is to give a preliminary report of an effort undertaken to investigate what would be the result of taking approximately into account some non-local effects in using a "GLSF" phenomenological analysis mentioned previously. In section 2 the simplified formalism used is described, while in section 3 certain of our first preliminary results are given and discussed.

2. Consideration of an $r-$ dependent effective mass for the Λ in the hypernucleus.

In the present approach, as in ref. [9], the non-relativistic motion of the Λin the hypernucleus is governed by the radial Schrödinger equation in which the Λ-nucleus potential well V is assumed to be a local central potential of the form

$$V(r) = -Df(r/R) \tag{1}$$

, where $D > 0$ is the depth of the potential well, R its "radius" and $f$its potential form factor ($f(0) = 1$) which determines its shape [10]. The radius R is related to the mass number of the nuclear core $A_c = A - 1$ (A is the mass number of the hypernucleus) and is given by

the ("rigid-core" model) expression:

$$R = r_0 A_c^{1/3} \tag{2}$$

Generalized expressions for R were also considered in [9].

In this work we shall attempt to take into account approximately non-local effects by using an r-dependent effective mass approximation for the Λ:$m^*_\Lambda(r)$ [11]. In fact, we take into account some of these effects by considering, as in ref [12] the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m_\Lambda^*(r)} + V(r) \tag{3}$$

Then, the radial Schrödinger equation is:

$$-\frac{\hbar^2}{2m_\Lambda^*(r)}\frac{d^2u_{n\ell}(r)}{dr^2} + \frac{\hbar^2}{2m_\Lambda^*(r)}\frac{\ell(\ell+1)}{r^2}u_{n\ell}(r) + V(r)\,u_{n\ell}(r) = E_{n\ell}u_{n\ell}(r) \tag{4}$$

Furthermore, use is made of the assumption for the effective mass $m_\Lambda^*(r)$, as suggested by Lombard et al. [12]:

$$m_\Lambda^*(r) = m_\Lambda(1 + \frac{V_M(r)}{m_\Lambda}) \tag{5}$$

where

$$V_M(r) = V(r) \tag{6}$$

We note that for large r the effective mass $m_\Lambda^*(r)$ approaches m_Λ, the free mass of the Λ.

On the basis of relations (5) and (6), equation (4) becomes:

$$-\frac{\hbar^2}{2m_\Lambda}\frac{d^2u_{n\ell}(r)}{dr^2} + \frac{\hbar^2}{2m_\Lambda}\frac{\ell(\ell+1)}{r^2}u_{n\ell}(r) + (1 + \frac{V_M(r)}{m_\Lambda})V(r)u_{n\ell}(r) =$$

$$(1 + \frac{V_M(r)}{m_\Lambda})E_{n\ell}u_{n\ell}(r) \tag{7}$$

or

$$-\frac{\hbar^2}{2m_\Lambda}\frac{d^2u_{n\ell}(r)}{dr^2} + \frac{\hbar^2}{2m_\Lambda}\frac{\ell(\ell+1)}{r^2}u_{n\ell}(r) + \left[V(r) - \frac{V_M(r)}{m_\Lambda}(-V(r) + E_{nl})\right]u_{n\ell}(r) =$$

$$E_{n\ell}u_{n\ell}(r) \tag{8}$$

Setting

$$\tilde{V}_M(r, E_{n\ell}) = V(r) - \frac{V_M(r)}{m_\Lambda}(-V(r) + E_{n\ell}) \tag{9}$$

eq (8) is written:

$$-\frac{\hbar^2}{2m_\Lambda}\frac{d^2u_{n\ell}(r)}{dr^2}+\frac{\hbar^2}{2m_\Lambda}\frac{\ell(\ell+1)}{r^2}u_{n\ell}(r)+\tilde{V}_M(r,E_{n\ell})\,u_{n\ell}(r)=E_{n\ell}u_{n\ell}(r)(10)$$

It is seen that, in the above treatment of the effective mass, the radial Schrödinger equation is modified. Instead of the potential$V(r)$, a modified (effective) potential, which is energy dependent, $\tilde{V}_M(r,E_{n\ell})$ appears. We note finally that central potentials $V(r)$ more general than those given by (1) may also be considered.

3. Some numerical results and discussion

In this section we report some preliminary numerical results in the framework of a GLSF phenomenological analysis of experimental Λ-binding energies $B_{n\ell}=-E_{n\ell}$ for a number of hypernuclei in the region $9\leq A_c\leq 88$ on the basis of the formalism of the previous section using the ground state binding energies $B_\Lambda=-E_\Lambda\equiv -E_{00}$ and a Gaussian potential $V(r)$, that is,

$$V(r)=V_G(r)=-D_G\cdot e^{-\frac{r^2}{R_G^2}} \quad (11)$$

and

$$V_M(r)=V_G(r) \quad (12)$$

with the rigid-core model expression for $R_G=r_0^G A_c^{1/3}$. Therefore the modified (effective) potential $\tilde{V}_M$(expression (9)) in the radial Schrödinger equation becomes now: (where we omit the upperscript G in r_0^G)

$$\tilde{V}_M=\tilde{V}_{MG}=-D_G\cdot e^{-\frac{r^2}{(r_0\cdot A_C^{1/3})^2}}+(D_G/m_\Lambda)\cdot e^{-\frac{r^2}{(r_0\cdot A_C^{1/3})^2}}\cdot(D_G\cdot e^{-\frac{r^2}{(r_0\cdot A_C^{1/3})^2}}-B_{n\ell}) \quad (13)$$

Such a least-squares fit procedure with two adjustable parameters D_G, r_0 led to the following best fit values:

$$D_G=38.23MeV, r_0=1.088fmwith\chi^2=68.72 \quad (14)$$

It should be also noted that in obtaining our numerical results the reduced Λ-core masses μ_Λ and $\mu_\Lambda^*(r)$ were used instead of m_Λ and $m_\Lambda^*(r)$.

Using the above best fit values of the parameters the plots obtained for $m^*_\Lambda(r)$,$V_G\,(r)$, $\tilde{V}_{MG}\,(r)$ are displayed in figures 1, 2 and 3 for the hypernucleus with A_c = 88. It is seen that the effective potential is slightly deeper than potential $V_G(r)$.

The estimated value of B_Λ for this hypernucleus is 23.36 MeV . This is quite close to the

experimental one (23.11 MeV).

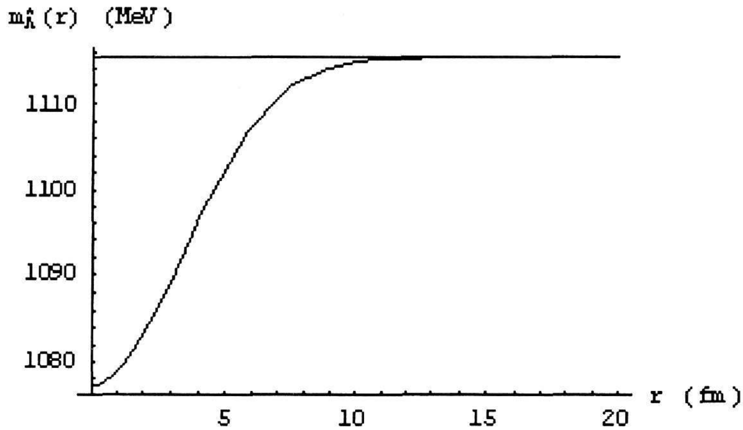

Fig. 1 The r-dependent effective mass of the Λ in the hypernuclei $m^*_\Lambda(r)$. The straight line indicates the

free Λ mass m_Λ = 1115.4 Mev.

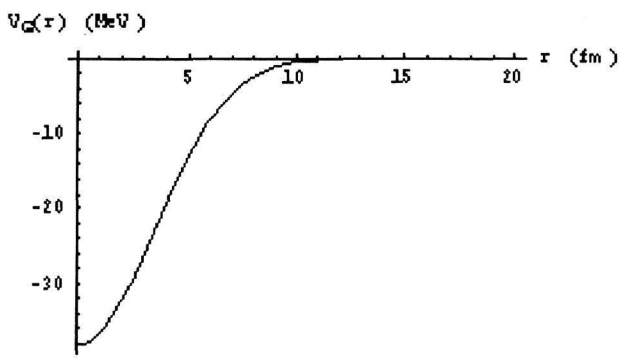

Fig 2 The Gaussian potential $V_G(r)$.

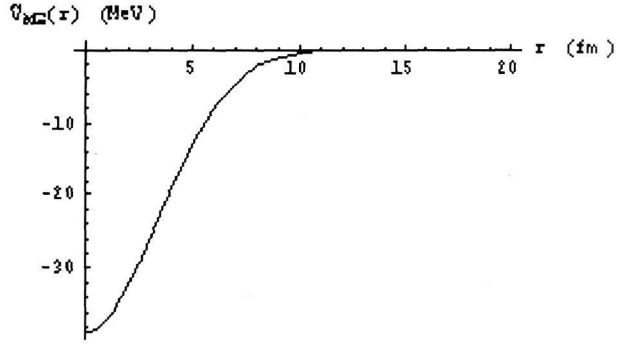

Fig 3 The effective potential $\underset{M}{\tilde{V}}(r, E_{00}) = \underset{MG}{\tilde{V}}(r)$.

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