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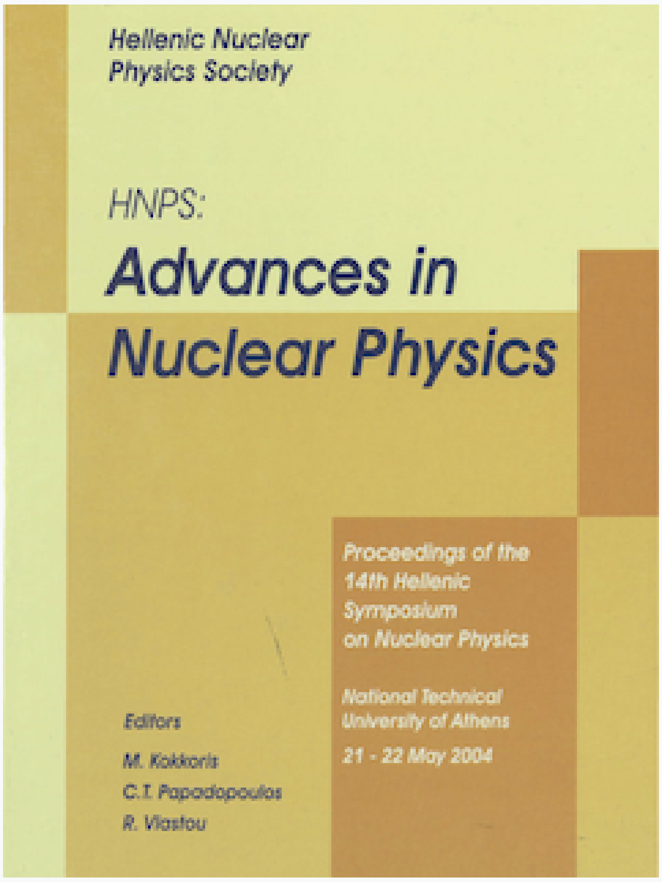

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Dennis Bonatsos, D. Lenis, D. Petrellis, P. A. Terziev

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W(5): Wobbling Mode in the Framework of the X(5) Model

Dennis Bonatsos [a] D. Lenis [a] D. Petrellis [a] P. A. Terziev [b]

[a] *Institute of Nuclear Physics, N.C.S.R. "Demokritos", GR-15310 Aghia Paraskevi, Attiki, Greece*

[b] *Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72 Tzarigrad Road, BG-1784 Sofia, Bulgaria*

Abstract

Using in the Bohr Hamiltonian the approximations leading to the Bohr and Mottelson description of wobbling motion in even nuclei, a W(5) model for wobbling bands, coexisting with a X(5) ground state band, is obtained. Separation of variables is achieved by assuming that the relevant potential has a sharp minimum at γ_0, which is the only parameter entering in the spectra and B(E2) transition rates (up to overall scale factors). B(E2) transition rates exhibit the features expected in the wobbling case.

Key words:
W(5) model, X(5) model, Collective model, Wobbling mode, Triaxial rotator
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1 Introduction

Nuclear wobbling motion [1] is expected to occur for triaxial nuclei at high angular momenta, when the angular momentum is aligned with the axis corresponding to the largest moment of inertia, a situation which classically corresponds to simple rotation without precession of the axes. Although wobbling motion was initially introduced for even nuclei [1], it has been seen experimentally up to now (and only recently) only in odd nuclei (^{163}Lu [2–4], ^{165}Lu [5], ^{167}Lu [6]). Detailed theoretical works have been performed in the cranked shell model plus random phase approximation [7–9], as well as in the particle–rotor model [10,11], which naturally contain free parameters.

In the present work we attempt a nearly parameter-free (up to overall scale factors) description of wobbling in even nuclei, following the methods developed in the E(5) [12], X(5) [13], Y(5) [14], and Z(5) [15] models, which correspond to the U(5)–O(6), U(5)–SU(3), axial–triaxial, and prolate–oblate shape phase transitions respectively. Furthermore, the wobbling nucleus is assumed to possess a relatively rigid triaxial shape, as in Refs. [16–18], with the potential having a sharp minimum at $\gamma = \gamma_0$. γ_0 is the only free parameter entering in the problem. It turns out [19], however, that the results are changing very little with γ_0 within the region of interest. The path we follow is described here:

1) We assume that the ground state band (gsb), which should be Yrast at low angular momentum L, is axial, characterized by $\gamma_0 = 0$. We then use for this purpose the X(5) gsb, which is indeed derived from the original Bohr Hamiltonian [20] after approximately separating variables for $\gamma = 0$ [13].

2) We assume (as in Ref. [21]) that triaxiality should appear at higher L. Starting then from the original Bohr Hamiltonian, we approximately separate variables following the steps of Bohr and Mottelson [1] in the definitive description of wobbling and keeping γ close to γ_0. The resulting model, in which only γ_0 appears as a parameter, we call W(5). The spectrum of W(5) is measured from the ground state of X(5) and normalized to the first excited state of the gsb of X(5), in order to be directly comparable to the X(5) spectrum.

3) The $n_w = 0$ band of W(5) (where n_w is the number of wobbling phonons [1]) is found to cross the gsb of X(5) at certain L, depending (very weakly within the region of interest) on γ_0. Thus the $n_w = 0$ band of W(5) becomes Yrast beyond some specific L. Bands with $n_w = 1, 2, \ldots$ exist at higher energies.

4) The $n_w = 0, 1, 2$ bands of W(5) are connected by intraband and interband B(E2) transitions which exhibit the characteristic features expected in the case of wobbling [22].

In Sections 2 and 3 of the present work the β-part and the γ-part of the W(5) spectrum are derived respectively, while the results are discussed in Section 4 and plans for further work are presented in Section 5.

2 The β-part of the spectrum

The original Bohr Hamiltonian [20] is $H = T + V(\beta, \gamma)$ with

$$T = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{k=1,2,3} A_k Q_k^2\right], \quad (1)$$

where β and γ are the usual collective coordinates, B is the mass parameter, Q_k ($k = 1, 2, 3$) are the components of angular momentum, and $A_k = 1/\sin^2(\gamma - 2\pi k/3)$.

Introducing [13] reduced energies $\epsilon = 2BE/\hbar^2$ and reduced potentials $u = 2BV/\hbar^2$, one aims at an approximate separation of variables by assuming that the reduced potential can be separated into two terms, one depending on β and the other depending on γ, i.e. $u(\beta, \gamma) = u(\beta) + u(\gamma)$.

In the X(5) model [13,23], approximate separation of variables is achieved by assuming that the potential $u(\gamma)$ has a minimum around $\gamma_0 = 0$, guaranteeing that K, the projection of angular momentum on the body-fixed $\hat{z}'$-axis, is a good quantum number. Similarly, in the Z(5) model [15], approximate separation of variables is achieved by assuming that the potential $u(\gamma)$ has a minimum around $\gamma_0 = \pi/6$, guaranteeing [24] that α, the projection of angular momentum on the body-fixed $\hat{x}'$-axis, is a good quantum number.

The X(5) and Z(5) solutions, briefly mentioned above, are obtained for specific values of γ_0 (0, $\pi/6$ respectively), and are valid for any value of the angular momentum L. A different approximate solution, which for brevity we are going to call W(5), can be obtained by following the steps of Bohr and Mottelson [1] for the description of wobbling motion. This solution will be obtained for a range of γ_0 values, but it will be valid only for large values of the angular momentum L, which is supposed to be aligned along the axis corresponding to the largest moment of inertia.

Using the A_k ($k = 1, 2, 3$) appearing in Eq. (1), one sees that in the region $0 < \gamma < \pi/6$ one has $A_1 < A_2 < A_3$. Therefore the largest moment of inertia corresponds to $k = 1$. In what follows we are going to restrict ourselves to the $0 < \gamma < \pi/6$ region.

For large angular momenta L aligned along the $k = 1$ axis, following Bohr and Mottelson [1] one can see that the eigenvalues of the $A_1 Q_1^2 + A_2 Q_2^2 + A_3 Q_3^2$ term in Eq. (1) take the form

$$\varepsilon(n_w, L) = A_1 L(L+1) + 2A_1 L \left(n_w + \frac{1}{2}\right) A_w, \qquad (2)$$

with $A_w = \sqrt{(A_2/A_1 - 1)(A_3/A_1 - 1)}$, where n_w is the number of the wobbling excitation quanta, for which the approximate (in the present case) relation $n_w = L - \alpha$ (where α is the projection of angular momentum on the $k = 1$ body-fixed axis, as before) holds. Since $\alpha \approx L$ and L is a good quantum number, α can be approximately treated as a good quantum number, too.

Using this result in the Schrödinger equation corresponding to the Hamiltonian of Eq. (1), one can separate it into two equations

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{4\beta^2}A_1\left(L(L+1)+2L\left(n_w+\frac{1}{2}\right)A_w\right)+u(\beta)\right]\xi_{L,n_w}(\beta)$$

$$=\epsilon_\beta\xi_{L,n_w}(\beta), \tag{3}$$

$$\left[-\frac{1}{\langle\beta^2\rangle\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}+u(\gamma)\right]\eta(\gamma)=\epsilon_\gamma\eta(\gamma), \tag{4}$$

where $\langle\beta^2\rangle$ is the average of β^2 over $\xi(\beta)$, and $\epsilon=\epsilon_\beta+\epsilon_\gamma$. Here we assume, as in Refs. [16–18], that the potential $u(\gamma)$ has a deep minimum at $\gamma=\gamma_0$, and that the variable γ remains "frozen" at the value γ_0 in A_1 and A_w, appearing in Eq. (3).

The total wave function should have the form

$$\Psi(\beta,\gamma,\theta_i)=\xi_{L,n_w}(\beta)\eta(\gamma)\mathcal{D}^L_{M,\alpha}(\theta_i), \tag{5}$$

where θ_i (i = 1, 2, 3) are the Euler angles, $\mathcal{D}(\theta_i)$ denote Wigner functions of them, L and M are the eigenvalues of angular momentum and the eigenvalues of the projection of angular momentum on the laboratory-fixed $\hat{z}$-axis respectively, and $\alpha=L-n_w$.

In the case in which $u(\beta)$ is an infinite well potential ($u(\beta)=0$ for $\beta\leq\beta_W$, $u(\beta)=\infty$ for $\beta>\beta_W$) one can use the transformation [13] $\tilde{\xi}(\beta)=\beta^{3/2}\xi(\beta)$, as well as the definitions [13] $\epsilon_\beta=k_\beta^2$, $z=\beta k_\beta$, in order to bring Eq. (3) into the form of a Bessel equation

$$\frac{d^2\tilde{\xi}}{dz^2}+\frac{1}{z}\frac{d\tilde{\xi}}{dz}+\left[1-\frac{\nu^2}{z^2}\right]\tilde{\xi}=0, \tag{6}$$

with

$$\nu=\frac{\sqrt{A_1L(L+1)+2A_1L\left(n_w+\frac{1}{2}\right)A_w+9}}{2}. \tag{7}$$

Then the boundary condition $\tilde{\xi}(\beta_W)=0$ determines the spectrum

$$\epsilon_{\beta;s,\nu}=\epsilon_{\beta;s,n_w,L}=(k_{s,\nu})^2,\qquad k_{s,\nu}=\frac{x_{s,\nu}}{\beta_W}, \tag{8}$$

and the eigenfunctions

$$\xi_{s,\nu}(\beta) = \xi_{s,n_w,L}(\beta) = \xi_{s,\alpha,L}(\beta) = c_{s,\nu}\beta^{-3/2}J_\nu(k_{s,\nu}\beta), \tag{9}$$

where $x_{s,\nu}$ is the sth zero of the Bessel function $J_\nu(z)$, while the constants $c_{s,\nu}$ are determined from the normalization condition $\int_0^\infty \beta^4\xi^2_{s,\nu}(\beta)d\beta = 1$. The notation for the roots has been kept the same as in Ref. [13], while for the energies the notation $E_{s,n_w,L}$ will be used. The lowest band corresponds to $s = 1$, $n_w = 0$ with $L = 0, 2, 4, \dots$, while the next bands are $s = 1$, $n_w = 1$ with $L = 1, 3, 5, \dots$, and $s = 1$, $n_w = 2$ with $L = 2, 4, 6, \dots$ [24].

In the special case of $\gamma_0 = \pi/6$ one can easily see that $A_1 = 1$, $A_2 = A_3 = 4$, $A_w = 3$. Then Eq. (7) takes the form $\nu = \frac{1}{2}\sqrt{L(L+4) + 6n_wL + 9}$, which is in agreement with the corresponding Z(5) expression [15], up to terms of order n_w^2.

3 The γ-part of the spectrum

The γ-part of the spectrum is obtained from Eq. (4). We consider a harmonic oscillator potential having a sharp minimum at $\gamma = \gamma_0$ ($0 < \gamma_0 \leq \pi/6$), i.e.

$$u(\gamma) = \frac{1}{2}c(\gamma - \gamma_0)^2 = \frac{1}{2}c\tilde{\gamma}^2, \qquad \tilde{\gamma} = \gamma - \gamma_0. \tag{10}$$

The minimum is sharp as long as the constant c is taken to be sufficiently large. Considering only small oscillations around γ_0, and

$$\eta(\tilde{\gamma}) = \tilde{\eta}(\tilde{\gamma})e^{-3(\cot 3\gamma_0)\tilde{\gamma}/2}, \tag{11}$$

Eq. (4) is brought into the form

$$\left(-\frac{\partial^2}{\partial\tilde{\gamma}^2} + \frac{1}{2}c\langle\beta^2\rangle\tilde{\gamma}^2\right)\tilde{\eta}(\tilde{\gamma}) = \left(\epsilon_{\tilde{\gamma}}\langle\beta^2\rangle - \frac{9}{4}(\cot 3\gamma_0)^2\right)\tilde{\eta}(\tilde{\gamma}), \tag{12}$$

which is a simple harmonic oscillator equation.

The total energy in the case of the W(5) model is then

$$E(s, n_w, L, n_{\tilde{\gamma}}) = E_0 + A(x_{s,\nu})^2 + Bn_{\tilde{\gamma}}, \tag{13}$$

where $n_{\tilde{\gamma}}$ denotes the number of quanta of the above oscillator.

4 Results

For low angular momentum the nucleus is expected to have $\gamma_0 = 0$. As angular momentum rises, at some point the nucleus will "jump" (as in Ref. [21]) to the large L limit corresponding to wobbling motion. As a consequence, the ground state band (gsb) of the nucleus should correspond to the gsb of X(5). The X(5) gsb should be the Yrast band up to some value of L, beyond which the $n_w = 0$ wobbling band should become Yrast, while additional wobbling bands with $n_w = 1, 2, \ldots$ should be seen further up in energy.

The calculation of B(E2) transition rates resembles the one of the Z(5) model [15] and has been carried out in Ref. [19]. It is worth comparing the results to the main features expected to be exhibited by B(E2)s in wobbling bands [22].

1) In region 2 of the Lund convention, which corresponds to the present case, the interband $(n_w = 1) \to (n_w = 0)$ transitions are expected to be strong for $L \to L + 1$ and weak for $L \to L - 1$ [7]. This is exactly the situation seen in Ref. [19].

2) The ratio

$$\frac{B(E2)_{out}}{B(E2)_{in}} = \frac{B[E2; L_1 \to (L+1)_0]}{B[E2; L_1 \to (L-2)_1]}, \tag{14}$$

where the notation L_{n_w} is used, is expected [22] to be of the order 0.2–0.3, i.e. much larger than what is expected for typical interband transitions. The $(n_w = 1) \to (n_w = 0)$ transitions in Ref. [19] do exhibit this behaviour.

3) The $B(E2)_{out} = B[E2; L_1 \to (L+1)_0]$ values are expected to go as $1/L$ and not as $1/L^2$ [22]. The results in Ref. [19] do exhibit this feature.

5 Discussion

In summary, a W(5) model describing the wobbling bands coexisting with a X(5) ground state band in even nuclei has been introduced. Separation of variables is achieved by assuming that the potential has a sharp minimum at $\gamma = \gamma_0$. The model predictions for given value of γ_0 are parameter-free (up to overall scale factors). The W(5) predictions for intraband and interband B(E2) transition probabilities exhibit the features expected for wobbling bands. A characteristic feature of the model is that the $n_w = 0$ wobbling band is not coinciding with the gsb, but with the superband crossing the gsb.

Concerning further work, the following comments can be made:

1) It is clear that the W(5) model should be tested against experiment in nuclei of which the gsb at low L appears to be close to X(5). A summary of such nuclei in the rare earth region is given in Ref. [25]. It is indeed seen [19] that existing experimental spectra on ^{156}Dy [26,27] correspond very well to the $n_w = 0$ and $n_w = 1$ bands of the W(5) model for $\gamma_0 = 20^\circ$, a value which has been found of interest [4] in the framework of "Ultimate Cranker" [28] calculations.

2) The β-equation [Eq. (3)] obtained above in the W(5) framework is also exactly soluble [29,30] for the Davidson potentials [31] $u(\beta) = \beta^2 + \beta_0^4/\beta^2$, where β_0 is the position of the minimum of the potential. In analogy to earlier work in the E(5) and X(5) frameworks [32] it is expected that $\beta_0 = 0$ will correspond to a "wobbling vibrator", while $\beta_0 \to \infty$ will lead to the original wobbling rotator of Ref. [1].

3) Using the variational procedure developed recently in the E(5) and X(5) frameworks [32], one should be able to prove that the W(5) model can be obtained from the Davidson potentials by maximizing the rate of change of various measures of collectivity with respect to the parameter β_0, thus proving that W(5) corresponds to the critical point symmetry of the transition from a "wobbling vibrator" to a wobbling rotator.

Work in these directions is in progress.

References

[1] A. Bohr and B. R. Mottelson, Nuclear Structure, Benjamin, New York, 1975, Vol. II, section 4-5e.

[2] S. W. Ødegård et al., Phys. Rev. Lett. **86** (2001) 5866.

[3] D. R. Jensen et al., Phys. Rev. Lett. **89** (2002) 142503.

[4] D. R. Jensen et al., Nucl. Phys. A **703** (2002) 3.

[5] G. Schönwaßer et al., Phys. Lett. B **552** (2003) 9.

[6] H. Amro et al., Phys. Lett. B **553** (2003) 197.

[7] Y. R. Shimizu and M. Matsuzaki, Nucl. Phys. A **588** (1995) 559.

[8] M. Matsuzaki, Y. R. Shimizu, and K. Matsuyanagi, Phys. Rev. C **65** (2002) 041303.

[9] M. Matsuzaki, Y. R. Shimizu, and K. Matsuyanagi, Phys. Rev. C **69** (2004) 034325.

[10] I. Hamamoto, Phys. Rev. C **65** (2002) 044305.

[11] I. Hamamoto, Phys. Rev. C **67** (2003) 014319.

[12] F. Iachello, Phys. Rev. Lett. **85** (2000) 3580.

[13] F. Iachello, Phys. Rev. Lett. **87** (2001) 052502.

[14] F. Iachello, Phys. Rev. Lett. **91** (2003) 132502.

[15] D. Bonatsos, D. Lenis, D. Petrellis, and P. A. Terziev, Phys. Lett. B **588** (2004) 172.

[16] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8** (1958) 237.

[17] A. S. Davydov and V. S. Rostovsky, Nucl. Phys. **12** (1959) 58.

[18] A. S. Davydov, Nucl. Phys. **24** (1961) 682.

[19] D. Bonatsos, D. Lenis, D. Petrellis, and P. A. Terziev, nucl-th/0406005.

[20] A. Bohr, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **26** (1952) no. 14.

[21] R. J. Turner and T. Kishimoto, Nucl. Phys. A **217** (1971) 317.

[22] R. F. Casten, E. A. McCutchan, N. V. Zamfir, C. W. Beausang, and J.-Y. Zhang, Phys. Rev. C **67** (2003) 064306.

[23] R. Bijker, R. F. Casten, N. V. Zamfir, and E. A. McCutchan, Phys. Rev. C **68** (2003) 064304.

[24] J. Meyer-ter-Vehn, Nucl. Phys. A **249** (1975) 111.

[25] E. A. McCutchan et al., Phys. Rev. C **69** (2004) 024308.

[26] C. W. Reich, Nucl. Data Sheets **99** (2003) 753.

[27] F. G. Kondev et al., Phys. Lett. B **437** (1998) 35.

[28] T. Bengtsson, Nucl. Phys. A **496** (1989) 56; **512** (1990) 124.

[29] J. P. Elliott, J. A. Evans, and P. Park, Phys. Lett. B **169** (1986) 309.

[30] D. J. Rowe and C. Bahri, J. Phys. A **31** (1998) 4947.

[31] P. M. Davidson, Proc. R. Soc. **135** (1932) 459.

[32] D. Bonatsos, D. Lenis, N. Minkov, D. Petrellis, P. P. Raychev, and P. A. Terziev, Phys. Lett. B **584** (2004) 40; Phys. Rev. C **70** (2004) 024305.