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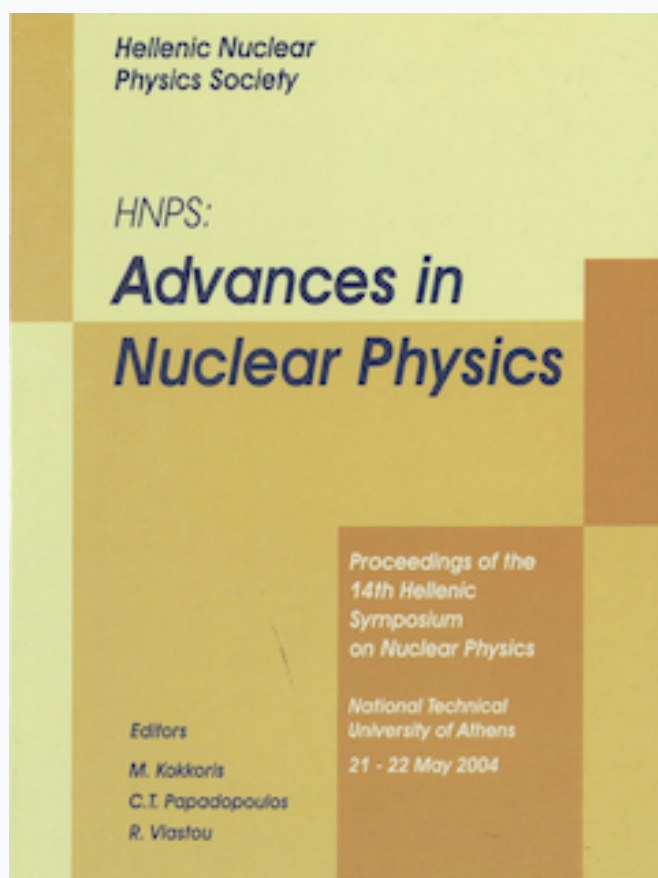
Searches for the Σ-nucleus Potential

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Abstract

The determination of the Σ-nucleus potential is the main orientation of the late Σ-hypernuclear physics. For this purpose, the last (K^-, π^+) light Σ-hypernuclear spectra were studied together with the recent $(\pi^-, K^+)^{28}_{\Sigma}Si$ spectrum in the context of the Green function method with a square well optical potential and a delta function spin-orbit interaction using plane waves. It was found that these spectra with a broad enhancement mainly at large energies are well described by a repulsive potential with nonzero imaginary and spin-orbit parts.

Key words: Σ hypernuclei, repulsive potential, Green function

1   Introduction

The study of Σ-hypernuclei has been done for more than 25 years [1, 2]. The first two decades the light Σ-hypernuclei were studied together with $^{12}_{\Sigma}C$ and $^{16}_{\Sigma}O$ [1-5]. The last 5 years the study has been extended to more heavy nuclei [6, 7]. The first Σ-hypernucleus was discovered at CERN with strangeness exchanging (K, π) reactions [1, 2]. This experiment was closely followed by experiments at BNL and CERN with the in-flight kinematics [1, 2]. A series of at rest experiments at KEK [8] was also done and again in BNL in-flight experiments [1, 2] followed. Subsequently, a series of experiments both in-flight and at rest for $^4_{\Sigma}He$ in KEK [3, 8] and BNL [4] has been done with high statistics, which also included other light nuclei [5]. Finally, the study of medium to heavy Σ-hypernuclei started with (π^-, K^+) reactions [6, 7].

There are three periods in the history of Σ-hypernuclei. Firstly, the discovery of *"narrow width states in the continuum"* [9] which characterized the first decade where many theories were put forward to explain them, but the subsequent

experiments did not show these peaks. Secondly, we had *"the bound state problem"* [9] which characterized the second decade after the cancellation of the narrow width states [1, 2, 8] and the discovery of the ground state of $(K^-, \pi^-)^4_\Sigma He$ [3] which is considered a type of hypernuclear bound state. What is of great interest is the mechanism that creates bound states like the Lane potential for $^4_\Sigma He$ [2-5] which reveals the role of isospin in Σ-hypernuclei. Finally, we have the last period where *"the determination of the Σ-nucleus potential"* is primary. This is the purpose of the new experiments of KEK for medium to heavy nuclei [6, 7]. This is also the quest of the last experiment in BNL in the light hypernuclei where the difference in the (K^-, π^-) and (K^-, π^+) reactions, contrary to the absence of the peaks in the (K^-, π^+) reactions claims for a strong Σ-nucleus interaction [2, 5]. So, the nature of Σ-nucleus potential must be investigated, that is, its value and if it is repulsive or attractive. This is also important for astrophysics [6, 7, 10].

The Σ story is like a modern fable [2] according to Chrien, who created for the whole situation the following verses: "Sheek and ye shall find; but beware, lest what ye find be only in the eye of the beholder". Also, Yamazaki described his experience with Σ-hypernuclei as: "struggling and straggling with Σ" [11].

The purpose of this paper is to contribute to the search for the Σ-nucleus potential. The new (K^-, π^+) spectra for the light Σ hypernuclei and also the (π^-, K^+) spectrum for $^{28}_\Sigma Si$ are calculated. The common point of these spectra is that they do not contain peaks and have broad enhancement mainly at large energies. The Green function method (GFM) is used for the description of the spectra with a solvable model. The attention is focused mainly on the sign of the potential (repulsive or attractive) because the exact determination of its value demands more complicated models. The analysis favours the repulsive potential.

The structure of this paper is as follows: In section 2, the model is described. In section 3, the calculation of the new hypernuclear spectra is given. Finally, in section 4 the conclusions of this work are given.

2 The model

The GFM has been proved successful for the description of $^{12}_\Sigma C$ and $^4_\Sigma He$ [12, 8]. In the *GFM, the response function or production strength S(E)* [13, 14] is given by the formula:

$$S(E) = -\frac{1}{\pi} Im F(E) \qquad (1)$$

The averaged strength function F(E) over the nuclear spin orientations is given by the formula:

$$F(E) = \sum_{\ell,j} F_{\ell,j}(E) \qquad (2)$$

$$F_{\ell,j}(E) = (2j_\Sigma + 1)(2j_N + 1) \sum_{\{L\}} (2L+1) \begin{pmatrix} j_N & j_\Sigma & L \\ -1/2 & 1/2 & 0 \end{pmatrix}^2 f_{\ell,j}^L(E) \qquad (3)$$

where $\{L\}$ denotes the summation over the permitted values of L such that $\ell_N + \ell_\Sigma + L =$ even (natural parity). The strength function $f_{\ell,j}^L(E)$ is given by:

$$f_{\ell,j}^L(E) = \sum_M \int_0^\infty dr \int_0^\infty dr' \left[f_L^M(r) \right]^* G_{\ell,j}(E; r, r') f_L^M(r') \qquad (4)$$

The weight function $f_L^M(r)$ is:

$$f_L^M(r) = u_{\ell_N, j_N}(r) \int d\Omega \, Y_L^M(\hat{r}) \left[\chi_\pi(\overline{r}) \right]^* \chi_K(\overline{r}) \qquad (5)$$

The $G_{\ell,j}(E; r, r')$ is the radial part of the Green function of the hyperon in the hyperon-nucleus potential corresponding to the $[\ell, j]_\Sigma$ configuration. The $u_{\ell_N, j_N}(r)$ is the radial part of the nucleon wave function (u_N) corresponding to the $[\ell_N, j_N]^{-1}$ hole (in this case proton hole) configuration. The χ_π (or χ_K) is the pion (or the kaon) wave function in the pion-nucleus (or the kaon-nucleus) potential.

The Green function in eq.(4) corresponds to the Schrödinger equation:

$$\frac{d^2 u}{dr^2} + \left\{ \left(\frac{2m}{\hbar^2} \right) [E - V(r)] - \frac{\ell(\ell+1)}{r^2} \right\} u = 0 \qquad (6)$$

where $V(r)$ is an optical potential, which describes the Σ-nucleus interaction and is of the form:

$$V(r) = V_c w(r) + V_{so} (\overline{l} \cdot \overline{s}) r_o^2 (1/r) [dw(r)/dr] \qquad (7)$$

The potential depths V_c and V_{so} are assumed to be complex. The imaginary part of the potential simulates the Σ to Λ conversion. In this paper, the nucleon distribution is represented by a density of rectangular shape:

$$w(r) = \begin{cases} 1 & \text{if } r < R \\ 0 & \text{if } r \geq R \end{cases} \qquad R = r_o A^{1/3} \qquad (8)$$

It can also be verified that:

$$G_{l,j}(E, r, r') = -\frac{2m}{\hbar^2} \frac{\phi(r_<)\psi^{(+)}(r_>)}{W[\phi(t), \psi^{(+)}(t)]_{t=R}} \qquad (9)$$

The functions $\phi(r)$ and $\psi^{(+)}(r)$ are the regular and Jost solutions of the Schrödinger equation (6). The factor $G_{\ell,j}(E; r, r)$ depends only on the hyperon–nucleus potential.

The calculations are given using the free wave pion (χ_π) and kaon (χ_K) wave functions. The comparison between plane wave and distorted wave approximation calculations shows that the use of plane waves can give a satisfactory qualitative representation of the in-flight data [15]. The model for the nucleon interaction is the simplest one. We take into consideration only the nucleon in the outer (valence) shell in the harmonic oscillator model, the harmonic oscillator parameter being: $b = \sqrt{\hbar/m\omega}$. For these simplified calculations the Coulomb interaction on the Σ^- is omitted. The parameter r_o in eqn. (8) is taken to be 1.31 fm (see ref. [12]). The calculated spectra of the (K^-, π^+) Σ-hypernuclear production do not depend strongly on the value of r_o, while the $(\pi^-, K^+)^{28}_\Sigma Si$ is sensitive on the value of r_o [6].

In this paper, apart from the theoretical investigation, the appropriate potential (which can be demonstrated by its parameters (ReV_c, ImV_c, V_{so})), representing best the experimental spectrum for each Σ-hypernucleus, was searched. For the comparison between the experimental and the theoretical values, the observed experimental data are normalized to give the integrated cross section equal to unity,

$$\int_{E_{min}}^{E_{max}} \frac{d^2\sigma}{d\Omega dE} \, dE = 1, \qquad (10)$$

where $[E_{min}, E_{max}]$ is the energy range of the available experimental data; the same normalization is applied to the calculated strength functions. With this normalization, the theoretical predictions are scaled in order to give the same integrated cross sections as those obtained experimentally.

In a previous work [13, 14] it was found that the attractive potential with parameters: $V_c = (-5, -i15)\,MeV$, $V_{so} = 15\,MeV$ could represent some data for $^{12}_\Sigma C$, $^{16}_\Sigma O$ and $^{6}_\Sigma Li$. This potential is not good for the new data and in section 3 it is used as a starting point for the variation of the real part of the potential to repulsive values.
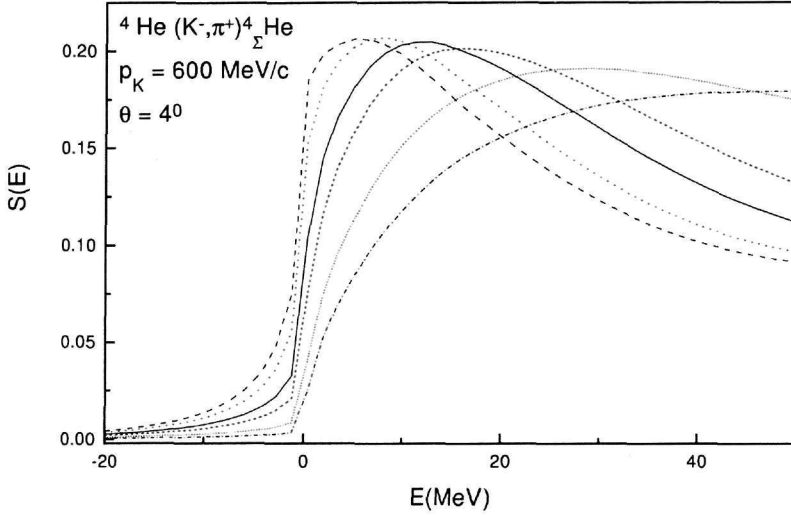
Fig. 1. The calculated spectra for the in-flight BNL experiment [2] for $^4_\Sigma He(K^-, \pi^+)$ and $\theta = 4°$ when the real part of the potential takes the values: ReV_c= -5, 0, 10, 20, 50, 100 MeV from the left to the right respectively and $ImV_c = -15\,MeV$, $V_{so} = 15\,MeV$.

3   Description of the new Σ hypernuclear spectra

3.1   The repulsion in the Σ^--nucleus interaction

Since the main result of this work is the use of a repulsive potential, it should be useful to see when and why this idea was introduced and used.

Firstly, Myint, Tadokoro and Akaishi [16] introduced the idea of the repulsion in the Σ^--nucleus interaction in the theoretical study of $^{208}Pb(K^-, \pi^+)\,^{208}_\Sigma Pb$. When the Σ^- particle is in the nuclear center, it feels repulsion from the near nucleons and attraction from the distant nucleons, which balance each other. But when it is in the nuclear surface, the repulsion remains the same but the attraction is smaller. So, the result is repulsion.

Secondly, Harada, who studied $^4_\Sigma He$ for about ten years of discussion for its existence [17-21], has claimed that in the case of (K^-, π^+) spectra which corresponds to the isospin $T = 3/2$ case, the potential must be repulsive [19]. He has drawn this relation, and he first used the potential which contained the Lane term U^τ [17-19]:
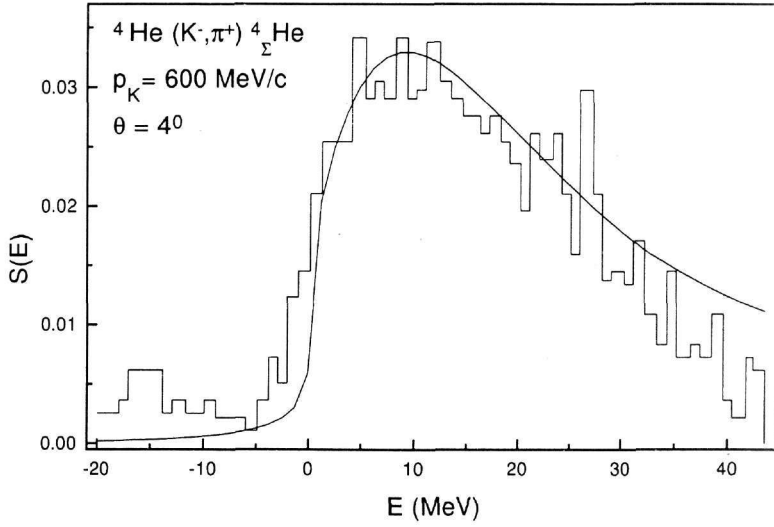
$$U_{C\Sigma}(R) = U^0(R) + U^\tau(R)(T_C\,t_\Sigma)/A \tag{11}$$

Fig. 2. The calculated spectrum for $V_c = (5, -i7) \, MeV$ and $V_{so} = 20 \, MeV$ for the in-flight BNL experiment for $^4_\Sigma He(K^-, \pi^+)$ and $\theta = 4°$ together with the experimental spectrum of ref. [2].

In the case of (K^-, π^+) reaction: $U_{C\Sigma} = U^0 + 1/2U^\tau$ [17]. Harada also considered the (K^-, π^+) spectra ideal for the determination of the Σ-nucleus potential [19-20].

Thirdly, the optical potential study from Σ^--atoms must be mentioned. Gal, Batty and Friedman [22, 10] have made a new fit to the Σ^--atomic data considering a non linear dependence of the potential $V_{opt}^{\Sigma^-}(r)$ from the nuclear density ρ. They found that the attraction outside the nucleus turns to repulsion near the nuclear surface. This is strongly related to the masses of the neutron stars, since the Σ^- appear first in the dense nuclear matter [10].

3.2 Description of the light (K^-, π^+) Σ hypernuclear spectra

The study of light Σ-hypernuclei (see also [23]) offers a chance to reconcile shell model and cluster aspects [2]. The hypernuclei considered are: $^4_\Sigma He$, $^6_\Sigma Li$ and $^9_\Sigma Be$ from the last BNL data [5].

Harada first, studying the at rest $(K^-, \pi^+)^4_\Sigma He$ spectrum [19] found that a phenomenological potential of 10 MeV fits well the data. He also studied the in-flight spectrum with his own very sophisticated model based on microscopic calculations using the Nijmegen potentials and found a perfect fit [20].
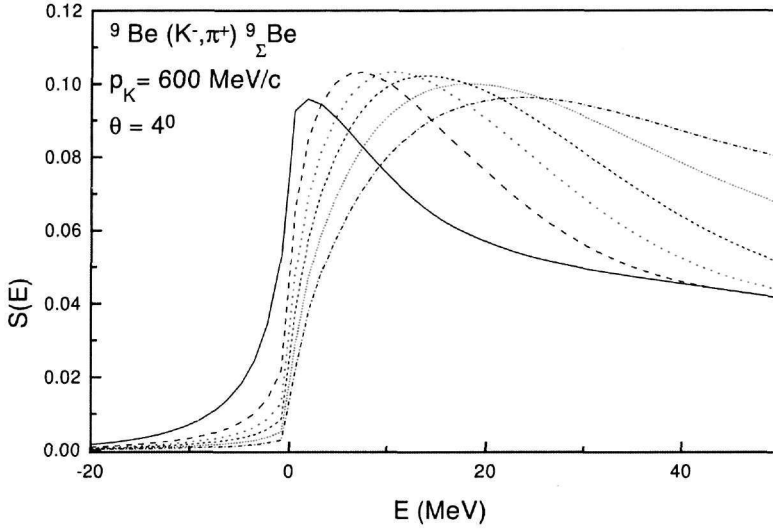
Fig. 3. The calculated spectra for the in-flight BNL experiment [2] for $^9_\Sigma Be(K^-, \pi^+)$ and $\theta = 4°$ when the real part of the potential takes the values: $ReV_c = 5, 10, 20, 30, 45, 70$ MeV from the left to the right respectively and $ImV_c = -15\,MeV$, $V_{so} = 15\,MeV$.

Following a similar procedure with the model of section (2), the ReV_c is varied from a shallow attractive [13, 14] to a strong repulsive potential. In fig. 1 the spectra can be seen when ReV_c takes the values -5, 0, 10, 20, 50, 100 MeV and $ImV_c = -15\,MeV$, $V_{so} = 15\,MeV$. Then the spectrum and the maximum are shifted to the right and there is a broadening of the shape of the spectrum. The change that the variation of ImV_c and V_{so} brings to the spectrum is also studied; the fitting between the theoretical and the experimental spectrum is better with smaller (absolute) ImV_c and larger V_{so} part. In fig. 2 the best spectrum for the potential parameters $(5, -7, 20)$ is shown. The fit with the data [2] is quite satisfactory.

Next, the $(K^-, \pi^+)^9_\Sigma Be$ spectrum is studied. It must be pointed out that the new BNL [5] experiment with the Moby Dick spectrometer did not show any peaks in the case of the $(K^-, \pi^-)^9_\Sigma Be$ spectrum which was considered as a type of Σ-hyperatom [23].

In fig. 3 the ReV_c takes the values 5, 10, 20, 30, 50, 70 MeV. It is seen that there is again broadening of the $(K^-, \pi^+)^9_\Sigma Be$ spectrum and the maximum is shifted to the right. The change of the spectra with the variation of ReV_c and V_{so} is small. In fig. 4 the best spectrum is shown for the potential parameters $(50, -15, 15)$. Dabrowski [24] studied this spectrum with a very naive model
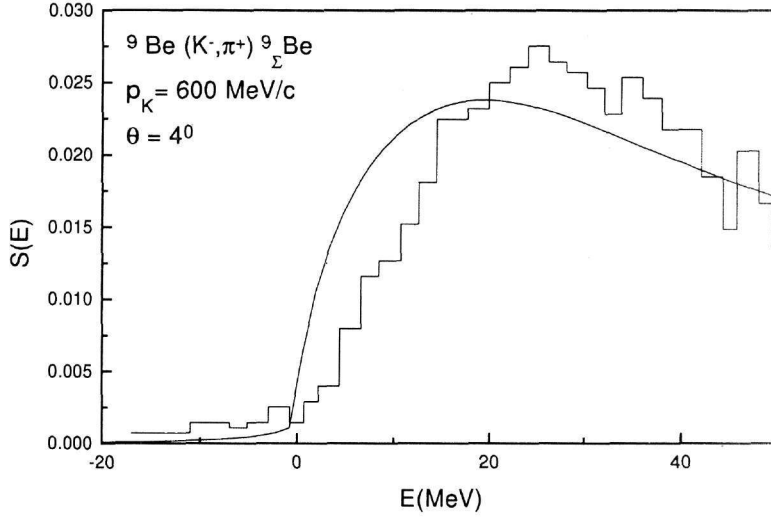
Fig. 4. The calculated spectrum for $V_c = (50, -i15)\,MeV$ and $V_{so} = 15\,MeV$ for the in-flight BNL experiment for $^9_\Sigma Be(K^-, \pi^+)$ and $\theta = 4°$ together with the experimental spectrum of ref. [2].

and found again that a repulsive potential with real part of 20 MeV is closer to the data.

Last, the spectrum of $^6_\Sigma Li$ is examined. What is interesting in this case is that the new high statistics BNL data [5] did not show the peaks that existed in the old BNL data and corresponded to the p and s states. These peaks were clear in the angle $\theta = 4°$, while in the angles 9° and 13° were not seen [14]. Chrien [2] claimed that these two peaks were due to inscattered pions at this angle.

In fig. 5, the $(K^-, \pi^+)^6_\Sigma Li$ spectra are shown where ReV_c is varied from shallow attractive to strong repulsive potential taking the values -5, 7, 45, 150, 1000 MeV and are compared to the experimental data. The same features are observed. The best spectrum is for the strongest ReV_c. The change with the variation of the other parameters of the potential is negligible. The correspondence is not so good as for $^4_\Sigma He$ and partly for $^9_\Sigma Be$ and ReV_c is very strong. One possible reason of that is the value of b for the nucleon wave function [25]; the investigation showed that the spectrum is very sensitive to its value. In any case, this spectrum must be further investigated.
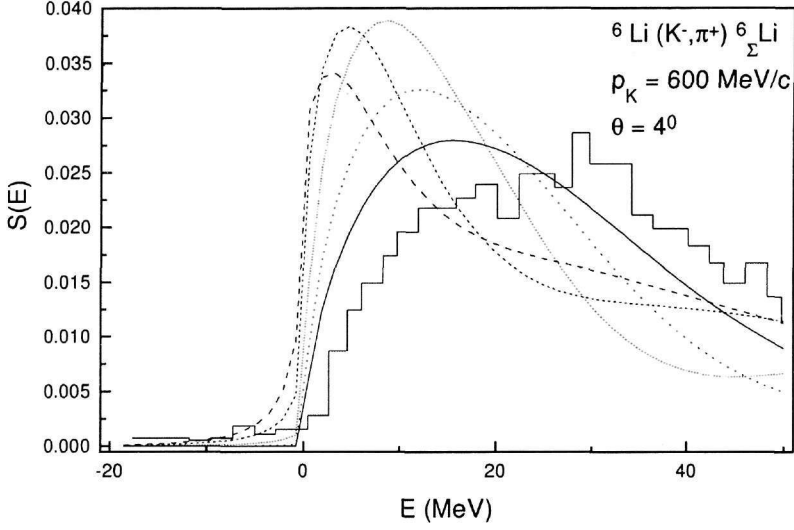
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Fig. 5. The calculated spectra for the in-flight BNL experiment for $^6_\Sigma Li(K^-, \pi^+)$ and $\theta = 4°$ when the real part of the potential takes the values: $ReV_c = -5, 7, 45, 150, 1000\, MeV$ from the left to the right respectively and $ImV_c = -15\, MeV$, $V_{so} = 15\, MeV$, together with the experimental spectrum of ref. [2].

3.3 Description of the $(\pi^-, K^+)^{28}_\Sigma Si$ spectrum

The last years a new series of experiments conducted by Noumi has been started in KEK with the SKS (Superconducting Kaon Spectrometer) [6, 7] via the (π, K) reaction, which has been proved successful in Λ hypernuclei [26]. Their main purpose was to study the Σ-nucleus potential taking data for medium to heavy Σ hypernuclei. First they took for $^{28}_\Sigma Si$ [6] for which many theoretical studies were preceded [27-29] and they continued with Ni, In, Bi [7]. They also made theoretical calculations with the GFM with a Woods-Saxon potential using the eikonal distortions. They found that the potential with parameters (150,-15,0) reproduced best the data.

Using the proposed model a similar investigation was made taking many potential values. In fig. 6, the best $(\pi^-, K^+)^{28}_\Sigma Si$ spectrum is shown with the potential (70,-15,12). What is interesting is the small hump in the left side of the spectrum. This is due to the spin-orbit potential because it vanishes when $V_{so} = 0$. Varying again the ReV_c the same characteristics are valid as in light Σ-hypernuclei. Additionally, in small values of ReV_c the spectrum is double humped due to V_{so}; a small investigation showed that this is possibly a mixture of g, f and h states.
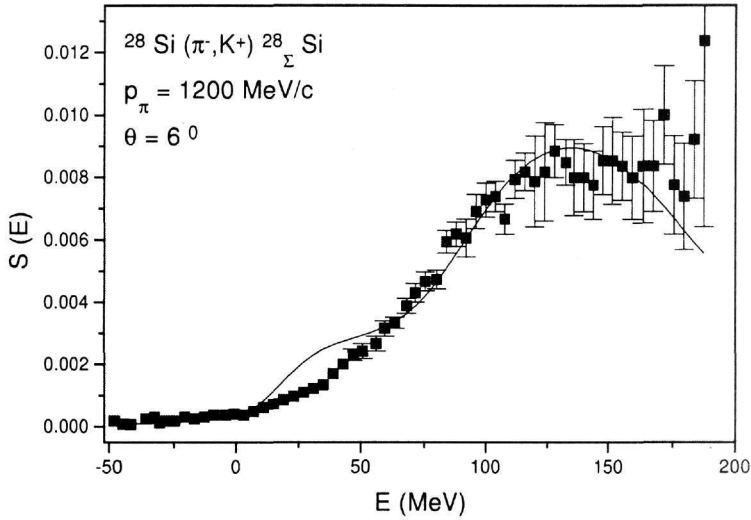
Fig. 6. The calculated spectrum for $V_c = (70, -i15)\, MeV$ and $V_{so} = 12\, MeV$ for the KEK experiment for $(\pi^-, K^+)^{28}_\Sigma Si$ and $\theta = 4°$ together with the experimental spectrum of ref. [6].

These results are in agreement with the other theoretical models [27-29], especially that of Bandō, Motoba and Žofka [27]. They claimed that a comparison with the (K, π) spectra shows that a large V_{so} makes the spectrum more dense. They also stated that the influence of the Coulomb potential is larger in heavier nuclei. This explains why it was not included in this primary study. A future more thorough investigation must take it into consideration.

As a general statement, it is remarked that a preliminary study of the new Σ hypernuclear spectra was made, focused mainly on the sign of the potential (choice between attractive and repulsive one) and not the exact value of it. This study favours the repulsive potential versus the attractive one. This must be followed by a more sophisticated and detailed study for each hypernucleus separately.

4 Conclusions

The GFM with a solvable model constitutes a useful tool for the study of the new Σ^--hypernuclear spectra; this study has led to the following statements:

When the feature of a spectrum is a broad enhancement at great energies, this is described by a repulsive potential (meaning the real part of the potential).

The behaviour of the real part of the optical potential defines the spectrum, when it is augmented from a shallow attractive to a strong repulsive value. Then the spectrum is broadened and the maximum is shifted to larger energies.

In this case the imaginary and spin-orbit parts of the potential cause generally small quantitative changes.

More specifically, for $^{4}_{\Sigma}He$ the best representation of the data exists for the potential parameters: $V_c = (5, -i7)\, MeV$, $V_{so} = 20\, MeV$. For $^{9}_{\Sigma}Be$, the best potential is: $V_c = (50, -i15)\, MeV$, $V_{so} = 15\, MeV$ and for $^{6}_{\Sigma}Li$ the more possible potential is: $V_c = (1000, -i15)\, MeV$, $V_{so} = 15\, MeV$. Last, for $^{28}_{\Sigma}Si$ the best fit is for $V_c = (70, -i15)\, MeV$, $V_{so} = 12\, MeV$; in this case, the spin-orbit potential plays some role. It must be pointed out that these values are preliminary (especially for $^{6}_{\Sigma}Li$).

The *perspectives* of this work in order to have a more definite conclusion about the strength of the potential parameters is the inclusion of the Lane potential for the light Σ hypernuclei and of the Coulomb potential for $^{28}_{\Sigma}Si$.

There are also other options for a better model, like use of the eikonal distortions or the distorted waves, another model for the nucleon interaction, use of the Woods-Saxon potential, inclusion of the elementary interaction....

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