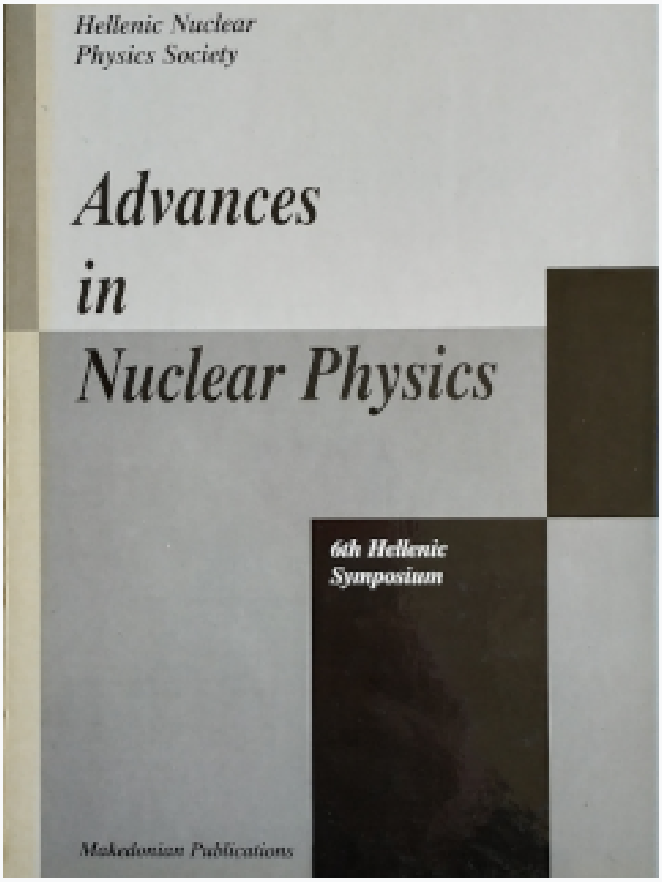
HNPS Advances in Nuclear Physics

Vol 6 (1995)

HNPS1995



Off-shell Nucleons in Intermediate Energy Physics

G. I. Poulis

doi: [10.12681/hnps.2928](https://doi.org/10.12681/hnps.2928)

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Grigorios I. Poulis

National Institute for Nuclear Physics and High-Energy Physics (NIKHEF)
P.O. Box 41882, NL-1009 DB Amsterdam, the Netherlands

Abstract

I discuss some issues pertaining to vertices involving virtual (*i.e.* off-mass-shell) nucleons, namely, in which processes off-shell vertices appear, their transformation properties under field redefinitions, and the attempts that have been made to estimate their effects in electromagnetic processes at intermediate energies.

1 Introduction

In the theoretical description of processes at intermediate energies, the structure of hadrons is described by multiplying the point-like vertex operators by form factors. It is common practice to assume that these vertices - *i.e.* their operator structures and the associated form factors - are in all situations the same as for a free, on-shell hadron. However, many of the processes that are of interest in medium energy nuclear physics involve hadrons as intermediate, and therefore off-mass-shell, particles. This is the case *e.g.* in electron-nucleus scattering or in two-step reactions on a free nucleon, such as Compton scattering or pion photoproduction. In these cases the vertices (electromagnetic or strong) can have a much richer structure: there can be more independent vertex operators and the form factors can depend on more than one scalar variable. The common treatment of such "off-shell" effects is to presume them small and to ignore them by using the free vertices. However, as much of the present effort in intermediate energy physics focusses on delicate effects, such as evidence of quark/gluon degrees of freedom or small components in the hadronic wavefunction, it is mandatory to examine these issues in detail.

Consider, in particular, the most general pion-nucleon (strong) vertex, where the incoming nucleon of mass m has momentum p_μ, the outgoing nucleon has momentum p'_μ and the pion has momentum $q_\mu = p'_\mu - p_\mu$. It can be written as [1]

$$\Gamma^5(p', p) = \Big[\gamma_5 G_1 + \gamma_5 \frac{\not p - m}{m} G_2 + \frac{\not p' - m}{m} \gamma_5 G_3 + \frac{\not p' - m}{m} \gamma_5 \frac{\not p - m}{m} G_4\Big] . \tag{1}$$

By sandwiching Γ^5 between on-shell spinors one obtains $G_1(q^2, m^2, m^2)\bar{u}(p)\gamma_5 u(p)$, which shows the much richer structure of the off-shell vertex, in that there are (a) more independent operators, (*i.e.* G_2, G_3, G_4) and (b) each G_i depends not only on the four momentum transfer, q^2, but also on the other kinematical variables p'^2 and p^2. The electromagnetic vertex of the nucleon is more complicated [2]. Its general form is

$$\Gamma^\mu = \sum_{j,k=0,1} (\not p')^j \left[A_1^{jk}\gamma^\mu + A_2^{jk}\sigma^{\mu\nu}q_\nu + A_3^{jk}q^\mu\right](\not p)^k , \tag{2}$$

where the 12 form factors, A_i^{jk}, are again functions of three scalar variables, usually taken to be q^2, p^2, and p'^2. By using the constraints provided by the Ward-Takahashi identity, the number of independent form factors can be shown to reduce to 8. Upon evaluating the vertex between two on-shell spinors, one recovers the familiar form of the electromagnetic current of a free nucleon

$$\bar{u}(p')\Gamma^\mu u(p) = \bar{u}(p')\left[F_1(q^2)\gamma^\mu + i\frac{F_2(q^2)}{2m}\sigma^{\mu\nu}q_\nu\right]u(p) , \tag{3}$$

involving two independent contributions with their associated (Dirac and Pauli) form factors.

2 The Off-Shell Nucleon vertex under field redefinitions

Off-shell vertices are described within the framework of the reduction formalism [3] using interpolating (interacting) fields for the off-shell nucleon. The choice of this interpolating field is not unique. It is well known that on-shell S-matrix elements are oblivious to the choice of interpolating field: any reversible field redefinition will leave the on-shell amplitudes unchanged [4]. However, different interpolating fields in general lead to different off-shell extrapolations [5] and therefore *off-shell form factors cannot be uniquely determined.* This general result can be illustrated by the following simple example for pion-nucleon scattering. Consider the pseudoscalar meson-nucleon Lagrangian

$$L_{ps} = \frac{1}{2}((\partial\phi)^2 - \mu^2\phi^2) + \bar{\psi}(i \not\partial - m)\psi - ig\bar{\psi}\gamma_5\phi\psi , \tag{4}$$

and perform the transformation

$$\psi \rightarrow \exp(i\beta\gamma_5\phi)\psi \ . \tag{5}$$

Then, up to and including order β^2 terms, the new Lagrangian reads

$$\begin{aligned}\tilde{L} = L_{ps} &- 2im\beta\bar{\psi}\gamma_5\phi\psi + \beta\bar{\psi}\gamma_5(\partial\!\!\!/ \ \phi)\psi \\ &+ 2\beta(g + 2m\beta)\bar{\psi}\phi^2\psi + \mathcal{O}(\beta^3) \ .\end{aligned} \tag{6}$$

This transformed Lagrangian has both pseudoscalar (PS) and pseudovector (PV) πNN interaction terms as well as a contact term. For our discussion of the representation dependence we leave β free and show that physical, observable quantities are β-independent.

The meson-nucleon vertex at the tree level for both representations is readily obtained (the dressed vertex at the one-loop level will is discussed in Ref. [6]). From L_{ps} we find for the vertex $\Gamma^5(p', p)_{ps} = g\gamma_5$, corresponding to the trivial half-off-shell vertex function

$$K(q^2, \pm w') = g \ . \tag{7}$$

On the other hand $\tilde{L}$ gives

$$\tilde{K}(q^2, \pm w') = g + \beta(m \mp w') \ , \tag{8}$$

which is β-dependent and clearly has a different off-shell behavior. However, the on-shell matrix element of the vertex operator is the same (*i.e.*, independent of β) for both representations.

What happens if we consider a two-step process on a free nucleon, such as pion-nucleon scattering, that involves the propagation of an intermediate off-shell nucleon? Since this is an overall on-shell process, the β-dependent contributions from the off-shell vertices in the pole terms, *i.e.* in the contributions involving two πNN vertices connected by an intermediate nucleon propagator, must be compensated by some other β-dependent contribution. To show that, we consider the *on-shell* pion-nucleon scattering T-matrix at the tree level. Using L_{ps}, it involves s- and u- channel pole terms only and reads

$$\mathcal{T}_{ps} = -ig^2\bar{u}(p')\Big\{\frac{1}{p\!\!\!/ + q\!\!\!/ - m} + \frac{1}{p\!\!\!/ - q\!\!\!/' - m}\Big\}u(p) \ , \tag{9}$$

where p and p' are the intial and final nucleon four-momenta, respectively, and q and q' are the intial and final pion four-momenta, respectively. Using the Dirac equation for the on-shell spinors, the pole term contribution to the T-matrix for the mixed PS and PV Lagrangian can be cast in the form:

$$\tilde{\mathcal{T}}_{pole} = -ig^2\bar{u}(p')\Big\{\frac{1}{p\!\!\!/ + q\!\!\!/ - m} + \frac{1}{p\!\!\!/ - q\!\!\!/' - m}$$

$$+4\beta(g+m\beta)\Big\}u(p)\ , \tag{10}$$

where the β-dependent term can be thought of as an "off-shell" effect. However, there is now also a contribution from the contact term in $\tilde{L}$, the term proportional to $\bar{\psi}\phi^2\psi$, which yields

$$\tilde{T}_{contact} = i\bar{u}(p')\Big\{4\beta(g+m\beta)\Big\}u(p)\ . \tag{11}$$

Clearly the β-dependent terms cancel and the total amplitude remains unchanged,

$$T_{ps} = \tilde{T}_{pole} + \tilde{T}_{contact}\ . \tag{12}$$

This simple example illustrates not only that, as expected [5, 7], total *on-shell* amplitudes for a given process are invariant under field redefinitions, but it also shows the interplay between "off-shell" effects from vertices and contact terms: representation-dependent "off-shell effects" in pole contributions in one representation appear as contact terms [8] in another representation. This makes it impossible to define "off-shell" effects in a unique, representation independent fashion.

Our considerations above concerned only rather simple vertices and at the tree level. The close connection between off-shell effects in a vertex and contact terms also exists when one considers dressed vertices [6]. It can be made plausible with the following example that concerns the dependence of the vertex on the invariant mass p^2. Consider, for simplicity, a scalar vertex for an initially on-shell particle together with the subsequent propagation. Expanding the vertex around the on-shell point

$$\frac{\Gamma(q^2,m^2,p^2)}{p^2-m^2} = \frac{\Gamma(q^2,m^2,m^2)}{p^2-m^2} + \frac{\partial\Gamma}{\partial p^2}(m^2) + \cdots \tag{13}$$

the propagator gets cancelled in the second term. Off-shell effects in the pole terms through the dependence of the vertex on the scalar variable p^2, can therefore also be seen as contact terms. Equation (10) is a specific example of this. The above seems to suggest that it is possible to find a representation for an amplitude where $K(q^2, w)$ has no off-shell dependence, *i.e.* no dependence on w, by keeping enough terms terms in Eq. (13) and introducing the corresponding contact terms. However, the Taylor expansion implicit in Eq. (13) is valid only up to the first branch cut, *i.e.* the pion threshold. Thus, this procedure is, for example, not valid in calculations of pion electroproduction on a nucleon. In this case, the form factors are complex and, as shown in Ref. [6] there is not only a representation dependent change in the real, but also be a concomittant change in the imaginary part of the form factors to be consistent with unitarity.

The above example also showed how a unitary transformation adds *one* power of the nucleon four-momentum to the asymptotic behavior of original vertex Eq. (7). Higher powers in the nucleon momenta can be obtained by using transformations involving higher derivatives [6]. Of course, one can perform transformations acting on the nucleon field that induce not just p'^2 but also a combined p'^2 and q^2 dependence of the half-off-shell vertex, Γ^5. For example, the transformation $\psi \to \exp(i\beta\gamma_5\partial^2\phi)\psi$ induces a new term $\beta(\not{p}' - m)q^2\gamma_5 u(p)$. Observations similar to those made for the strong form factor can be made for the electromagnetic vertex in QED by starting with the QED Lagrangian and transforming the electron field. The electromagnetic vertex obtained at tree level from the QED Lagrangian is simply $-ie\gamma^\mu$ for on and off shell electrons. Applying the transformation $\psi \to \exp(\beta\partial^2\not{A})\psi$ changes for example the half-off-shell vertex to $-i[e + \beta q^2(\not{p}' - m)\gamma^\mu]u(p)$. The β-dependent part of this vertex vanishes on-shell, as expected.

3 Prescriptions and Model Calculations

The remarks made in Sec. II made do not mean that one does not have to worry about the off-shell contributions, only that these contributions are not uniquely defined. In fact, several approaches have been considered for taking off-shell effects into account in two-step electromagnetic processes. They may be divided in attempts to calculate the off-shell vertex functions on the basis of some microscopic model and in purely phenomenological prescriptions.

An example of the latter are the prescriptions introduced by de Forest [9] for the half-off-shell ($p'^2 = m^2$, $p^2 \neq m^2$) electromagnetic vertex. They allow one to use the free current by changing its kinematical variables according to the off-shell situation. Specifically, one defines a four-vector $\bar{p}^\mu \equiv (\sqrt{\mathbf{p}^2 + m^2}, \mathbf{p})$ corresponding to an on-shell particle with the same three-momentum as the off-shell nucleon. However, $p^0 \neq \bar{p}^0$. The nucleon is described by a free spinor $u(\bar{p})$ and the vertex by one of the two forms

$$\Gamma^\mu_{(2)} = F_1\gamma^\mu + i\frac{F_2}{2m}\sigma^{\mu\nu}q_\nu \tag{14}$$

$$\Gamma^\mu_{(1)} = (F_1 + F_2)\gamma^\mu - \frac{F_2}{2m}(\bar{p} + p')^\mu \ . \tag{15}$$

In the on-shell case the second vertex is obtained from the first using the Gordon decomposition and they give identical matrix elements. The difference between results obtained using these two vertex functions can be viewed as a measure of the importance of off-shell effects [9, 10]. Moreover, since the one-body current matrix elements corresponding to Eq. (14) and Eq. (15) are not conserved,

$$q_\mu \, \bar{u}(p')\Gamma_i^\mu u(\bar{p}) \sim (p^0 - \bar{p}^0)\bar{u}(p')\gamma^0 u(\bar{p}) \neq 0 \,, \tag{16}$$

a wider range of results may be obtained by choosing to enforce current conservation by hand, *e.g.* by eliminating $\bar{u}(p')\Gamma^0 u(\bar{p})$ in favor of $\bar{u}(p')\Gamma^3 u(\bar{p})$ or *vice versa*. Using the vertices 15 and 14 with this last option results in the so called $cc1$ and $cc2$ prescriptions, which are widely used in the analysis of electron scattering experiments. However, caution must be exercised when interpreting coincidence $A(e,e'N)B$ measurements with a nucleon N detected with large momentum compared to the typical Fermi momentum scale since, at such (rather extreme) kinematics, the electromagnetic response functions using the different prescriptions may differ by a factor of 2 or 3 [10].

Another often used prescription for the nucleon vertex operator was introduced by Gross and Riska [11] and is given by

$$\Gamma_\mu(q^2) = \gamma_\mu F_1(q^2) + \frac{q_\mu \not{q}}{q^2}\left[1 - F_1(q^2)\right] + \sigma_{\mu\nu} q^\nu \frac{F_2(q^2)}{2m} \,. \tag{17}$$

It also only involves on-shell information, the free Dirac and Pauli form factors, F_1 and F_2, but has a more general Dirac structure. The second term on the right-hand side of Eq. (17) vanishes when the vertex is evaluated between on-shell spinors, but contributes when one or both are off-shell. It is easily seen that this vertex satisfies the Ward-Takahashi identity when free Feynman propagators are used for the nucleon. In pion electroproduction, this prescription amounts to adding a contact term to the Born amplitude which is needed to restore gauge invariance [12].

The validity of these and other recipes can only be assessed on the basis of a realistic microscopic calculation and will depend on the kinematics of the process. Studies in the context of meson loop models have been performed *e.g.* in Refs. [13-15]. In these studies the nucleon is "dressed" with a meson cloud using pseudoscalar or pseudovector pion-nucleon coupling and computing the half-off-shell vertex $\Gamma^\mu_{\gamma NN} u(p)$ to one pion-loop level, that is, to $\mathcal{O}(eg^2)$. Of course, such a simplified model is not able to predict even the on-shell form factors. However, it is reasonable to expect that the *relative* importance of off-shell effects in this simple model is a good measure of what would happen in a more realistic model. Typically effects of the order of $5-15\%$ are found for the dependence of the form factors on the variable p'^2.

Recently, off-shell form factors have been calculated using chiral perturbation theory which is the effective theory of QCD at low energies. The nucleon electromagnetic form factor was calculated by Bos and Koch [15] and the pion electromagnetic vertex was computed by Rudy *et al.* [16]. Scherer and Fearing [8] have performed the pion form factor computation using two different chiral Lagrangians, related through a unitary transformation of the fields, which leaves the observables unchanged. It was shown explicitly how in the description of Compton scattering off a pion the off-shell form factors are not the same while the observable on-shell form factor and the amplitude are the same in the two representations. The investigations in Sec. II have in fact been triggered by the findings of Ref. [8].

4 Sidewise Dispersion relations

A different approach for learning something about the off-shell vertex functions was suggested by Bincer [2]. He showed, using the reduction formalism that one may analytically continue both the electromagnetic and the strong nucleon form factor $K(q^2, w)$ not only as a function of q^2 but also of the invariant mass of the off-shell nucleon $w = \sqrt{p^2}$. He showed that it is a real analytic function of w with cuts along the real axis, starting at $w = \pm(m + \mu_\pi)$ and extending to $\pm\infty$. Furthermore, $K(q^2, w)$ is purely real along the real axis in the interval $-(m+\mu_\pi) < w < (m+\mu_\pi)$. Thus, $K(q^2, w)$ satisfies dispersion relations, termed "sidewise" to emphasize that the dispersion variable is now the nucleon four momentum, $w = \sqrt{p^2}$. Consider again the strong (*i.e.* pion-nucleon) vertex with the pion on its mass shell, *i.e.* in $K(m_\pi^2, w)$. Assuming that $|K(q^2, w)|$ approaches a constant as $|w| \to \infty$, the once-subtracted sidewise dispersion relation for $K(q^2, w)$ can be cast in the form [2]

$$\frac{|K(m_\pi^2, w)|}{|K(m_\pi^2, m)|} = \exp\left\{ \frac{(w-m)}{\pi} \mathcal{P} \int\limits_{m+\mu_\pi}^{\infty} dw' \left[\frac{\phi(w')}{(w'-m)(w'-w)} - \frac{\phi(-w')}{(w+m)(w'+w)} \right] \right\}, \tag{18}$$

where $\phi(\pm w)$ is the phase of $K(m_\pi^2, w)$ along the positive (+) or negative (-) cut. Thus, $K(m_\pi^2, w)$ can be determined if these phases are known. If $|K(q^2, w)|$ grows like $|w|^n$ (n an integer) as $w \to \infty$, n+1 subtractions must be performed which introduce the same number of *a priori* unknown subtraction constants into the dispersion relation. The role of subtractions in the sidewise dispersion method is important: since we only know $K(q^2, w)$ at the on-shell point, $w = m$, a need for more than one subtraction will spoil any possible predictive power. In cases where the vertex function is not known at the on shell point, as *e.g.* in the case of the $\sigma^{\mu\nu}$ electromagnetic vertex of the nucleon, even one subtraction will destroy predictive power.

So far, dispersion relations just reflect analyticity properties of Greens functions and are void of any predictive power. This changes when one makes use of *unitarity* constraints that provide additional relations between the real and imaginary parts of the Green's function and therefore on ϕ. The consequences of unitarity for $K(m_\pi^2, w)$ may be obtained by looking at its absorptive part which receives contributions from ***physical*** on-shell intermediate states. Unitarity provides for K, which is now a vector in the space of the different reaction channels, the constraint [2, 17, 18].

$$\mathrm{Im}\, K = F^{-1} T F K^* \ , \tag{19}$$

where F^{-1} and F are phase space factors [6]. For the πNN form factor, this constraint can be written as

$$\begin{aligned} \mathrm{Im}\, K_\pi(m_\pi^2, w) = & \ \theta(|w| - m - \mu_\pi) T_{\pi\pi}(w) K_\pi^*(m_\pi^2, w) \\ & + \theta(|w| - w_T) A(w) \ , \end{aligned} \tag{20}$$

where w_T is the threshold energy of the first inleastic channel. The first term on the right-hand-side of Eq. (20) arises from the intermediate pion-nucleon two-body state and the second term represents contributions from intermediate states with higher mass, *e.g.* $\pi\pi N$, ηN, $K\Lambda$, etc.

For $w < w_T$, the last term in Eq. (20) does not contribute and one sees from the above equation that the phase of the form factor for the πNN vertex, ϕ_π = $\mathrm{Arg}(K_\pi)$, is determined by the elastic πN phase shift

$$\phi_\pi = \delta_\pi^l = \frac{1}{2} \tan^{-1} \left(\frac{2\mathrm{Re} T_{\pi\pi}}{1 - 2\mathrm{Im} T_{\pi\pi}} \right) \ . \tag{21}$$

Thus, for $w < w_T$, the experimental πN phase shift allows one in principle to calculate the vertex function, $K(m_\pi^2, w)$ from Eq.(18); since the phase shift is a representation independent observable quantity, both the real and imaginary parts of the representation dependent off-shell form factor, $\mathrm{Re}K_\pi$ and $\mathrm{Im}K_\pi$, must change under a field transformation such that the phase, $\phi_\pi \equiv \arctan(\mathrm{Im}\, K_\pi / \mathrm{Re}\, K_\pi)$, remains unchanged.

Above this threshold some assumptions must be made for the phase [2, 19, 17] which may lead to quite different predictions for the half-off-shell form factor. In Ref. [6] two such assumptions have been tested using a unitary model. It is found that the validity of either approximation is model dependent and therefore neither may be trusted. Even in the hypothetical case of a single channel system with no open channels, though, one faces the following puzzle: it seems to be possible to determine the phase ϕ and thus also the function $K(w)$ for the off-shell vertex in a model independent fashion using the *observable* phase shift of the on-shell T-matrix. This appears to be in contradiction with the observation in Section II that the off-shell form factor changes when we carry out field transformations. How can this be reconciled with the sidewise dispersion relations that express $K(w)$ in terms of *observable* quantities?

The answer lies in the fact that in the sidewise dispersion relation approach the number of necessary subtractions is *a priori* unknown. Indeed, different choices of the nucleon interpolating field will in general lead to different asymptotic behaviors of the off-shell form factor. The examples given in Sect. II illustrate this point. From Eq. (7) we see that $K(w)$ is of order $\mathcal{O}(1)$ as $w \to \infty$. On the other hand, the vertex function, Eq. (8), obtained from the transformed Lagrangian, is of order $\mathcal{O}(w)$ at infinity. Thus, the "representation dependence" in sidewise dispersion relations shows up in the *a priori* unknown needed number of subtractions. As previously remarked, any predictive power of the sidewise dispersion relations method will be lost if two (or more) subtractions are necessary since we only know the form factor at the physical point $w = m$.

As dispersion relations do not depend on a particular Lagrangian, it is useful to look at the above discussion for the vertex function K in a different way and to contrast it with the dispersion relations in q^2 for the pion–nucleon scattering amplitude. Consider the unitarity constraint, Eq. (19): evidently, it remains valid under the replacement $K \to f(w)K$, where $f(w)$ is a real function of w, reflecting a different off-shell behavior. If $f(w)$ is a polynomial in w and one has $f(m)$ =1, then the analytic properties of K are not changed and K still satisfies a dispersion relation. However, in general (additional) subtractions will be needed, and these subtractions have to be done at an unphysical points, $w \neq m$, and therefore cannot be done model independently. When using the dispersion relation approach for the T-matrix, we also may need subtractions to make the integrals converge. However, for this purpose we can do these subtractions at different energies where we have experimental information about the T-matrix. In some cases, this makes it possible to determine the pion-nucleon T matrix at an unphysical point through dispersion relations, while the off-shell *form factors* can never be uniquely determined. Notice also that our discussion does not imply that dispersion relations for the electromagnetic form factor, with the *momentum transfer* q^2 as the dispersion variable, show any representation dependence. In this case the form factor $F(q^2)$ can be measured for a number of values of the four momentum transfer, q^2.

5 Summary

In many interesting nuclear physics processes such as electron-nucleus scattering, Compton scattering, pion photoproduction *etc.*, nucleons may also appear as intermediate, virtual particles, off the mass shell. The operator structure, as well as the functional dependence of the vertex function on the available kinematical variables are much richer for vertices involving such off-shell hadrons. Generic field theoretical arguments suggest that these off-shell vertices cannot be uniquely determined. We have elaborated on this point using simple pion nucleon Lagrangians and transforming the nucleon field. We have mentioned some of the attempts that have been made to include off-shell effects in theoretical calculations. Unfortunately, simple meson-loop models are inadequate for obtaining a realistic description of vertex functions. Thusly, one has to resort to phenomenological prescriptions that are supposed to give an estimate of the systematic uncertainties that off-shell effects introduce in the interpetation of experiments in medium energy physics. However, the non-uniqueness of off-shell effects renders this approach dubious. A different aproach pursued in the literature is to use sidewise dispersion relations that relate the off-shell vertex functions to observable quantities such as pion-nucleon phase shifts. Apart from shortcomings related to the influence of inelastic channels, there is a more fundamental problem with this method, which has been overlooked in the past: if off-shell vertex functions are not unique, how is it possible that using sidewise dispersion relations they may be obtained from observable (and therefore uniquely defined) quantities? We have shown that the answer lies in the fact that under field redefinitions the number of required subtractions changes and the predictive power of sidewise dispersion relations is lost. Based on our discussion, we conclude that in practice sidewise dispersion relations cannot provide reliable and unique information about the structure of off-shell nucleons. The off-shell vertex, which has a much more complicated structure than the free vertex, thus cannot be extracted from experimental data, but should instead be *consistently* calculated within the framework of a microscopic theory. Such a calculation will yield the dressed off shell vertices and the concommitant contact terms. The proper interpretation of future high precision measurements of intermediate energy processes depends crucially on our ability to carry out such consistent calculations in realistic microscopic models.

Acknowledgements

I am indebted to the organizers for providing partial support towards my participation in this workshop. The research presented here has been done in collaboration with Richard Davidson and Justus Koch.

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