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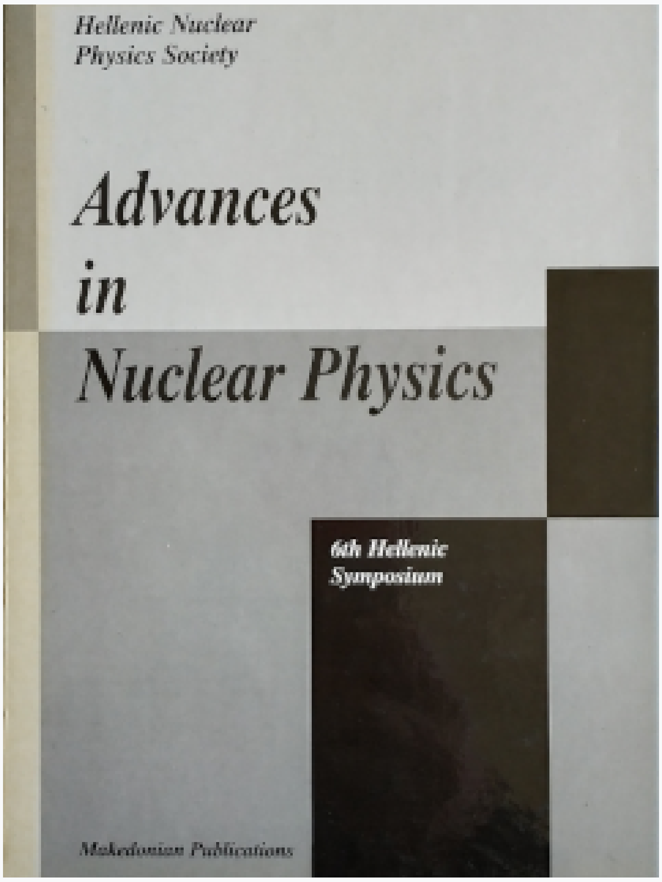

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S. E. Massen

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S. E. Massen

Nuclear Physics Laboratory, Department of Physics
University of Oxford, Oxford OX1-3RH, U.K.
and
Department of Theoretical Physics
Aristotle University of Thessaloniki
GR–54006 Thessaloniki, Greece

Abstract

The effect of short range correlations on the Ca and O isotopes has been studied by using an isospin dependence of the harmonic oscillator spacing, $\hbar\omega$ and of the correlation parameter. The analysis indicates that short range correlations as well as the isospin dependence of the parameters are important to explain the behavior of the differences of the MS charge radii and the differences of the charge densities between the Ca and the O isotopes.

1 Introduction

The Calcium isotopes are of special interest as they are the first long chain of experimentally accessible isotopes in the periodic table and they are bounded by two nuclei, ^{40}Ca and ^{48}Ca, with closed-shell configurations that are suitable for theoretical calculations. The RMS charge radii are very irregular compared with the global $A^{1/3}$ variations. The isotopic shift is characterized by the fact that the two double magic nuclei have almost the same RMS charge radii while electron scattering experiments have revealed that the charge distributions are not identical [1,2]. It has been observed also that the charge radii of the even Ca isotopes increases with the addition of neutrons up to the first half of the neutron $1f_{7/2}$ shell and then decreases while the odd Ca isotopes have always smaller radii than neighboring even ones [3,4]. Although the $n-p$ interaction is behind the modulation, it is not clear how the interaction affects the systematics of the observed isotopic shifts. For the above reasons the study of the Ca isotopes is an important test of nuclear theories to see whether this microscopic structure can be understood.

Hartree - Fock calculations which reproduce the average variation of RMS charge radii against A cannot reproduce the variation beyond shell closures [5]. Brown et al. [6] calculated the charge distribution of the Ca isotopes using a Woods-Saxon state dependent potential with a density dependent symmetry potential which was determined in a self consistent way and using non integer occupation probabilities for the $1d_{3/2}$, $1f_{7/2}$, and $2p_{3/2}$ states. Bhattacharya et al. [7] using an average one-body potential of Woods-Saxon type and experimental occupation probabilities, have reproduced the parabolic variation of the charge radii of the Ca isotopes. Barranco and Broglia [6] were able to explain the parabolic variation of the MS charge radii introducing collective zero-point motion. Finally, Zamick [9] and Talmi [10], in analogy with the binding energies, assumed that the effective radius operator has a two body part as well as one-body part. They were able to explain the odd-even staggering effect observed in Ca isotopes assuming that a mechanism which gives rise to this odd-even variation is the polarization of the core by the valence neutrons.

Similar behavior with that of Ca isotopes has been observed in the O isotopes: ^{16}O, ^{17}O, ^{18}O [11]. Brown et al. have studied these nuclei in the same way as the Ca isotopes using various occupation probabilities for the $1d_{5/2}$ and $2s_{1/2}$ neutron and proton particle states.

From many theoretical works [6,7,12,13,14] it is clear that the occupancies of the single particle states play a crucial role in determining the charge distributions of the Ca and O isotopes. On the other hand it is known that short range correlations (SRC) due to hard collisions between nucleons at relative distances smaller than about $0.5 fm$ may result in a scattering of the nucleons into states of higher energy up to 1 GeV. Calculations for nuclear matter including SRC have shown [15,16] that the depletion of the otherwise filled orbital is $10-20\%$. The effect of SRC on the occupation numbers of the shell model orbits in light nuclei have been studied in ref. [17,18].

In a series of papers [19,20,21], correlated charge form factors, $F_{ch}(q)$, and densities of s-p and s-d shell nuclei were calculated by using correlated wave functions of the relative motion and the factor cluster expansion of Ristig, Ter Low, and Clark [22,23]. The parameters of the method were calculated by fitting the theoretical values of $F_{ch}(q)$ to the experimental ones for the corresponding nuclei.

The aim of the present work is to examine whether SRC can reproduce the behavior of the charge distributions of the Ca and O isotopes. That is to see the importance of the SRC on the isotopic shifts, even if it is known that other effects, in particular ground state correlations [8], contribute to the charge dencities. In the present approach these effects are hide (in a way) in the values of the parameters of the model as they are determined from the systematic of the MS charge radii of the isotopes. In other words, by choosing the SRC parameter in order to reproduce the systematic of MS charge radii of these isotopes [3,11], we examine if the difference of the charge distributions between the isotopes agree with the experimental data [1,2,11].

In Sec.2 the relevant formalism of SRC is presented and an approximate expression of the MS radius as well as the dependence of it on the extra neutron number of the isotopes are given. In Sec.3 an isospin dependence of the correlation parameter is found using an isospin dependence of the harmonic oscillator (HO) energy spacing [24], $\hbar\omega$, as well as the experimental difference of the M-S charge radii between the isotopes. In Sec.4 the results of the method are presented and discussed. Finally Sec.5 summarizes the conclusions.

2 Correlated charge form factor, densities and MS radii for the Ca isotopes

An expression for the charge form factor $F_{ch}(q)$ of ^{16}O and ^{40}Ca nuclei was derived [19,20] using the factor cluster expansion of Ristig, Ter Low and Clark [22,23] and considering normalized correlated wave functions of the relative motion which were parameterized through a Jastrow type wave function of the form:

$$\psi_{nlS}(r) = N_{nlS}[1 - \exp(-\lambda r^2/b^2)]\phi_{nl}(r) \tag{1}$$

where N_{nlS} are the normalization factors, $b = \sqrt{2}b_1$ ($b_1 = \sqrt{\hbar/m\omega}$) and $\phi_{nl}(r)$ are the HO parameter and wave functions of the relative motion. The expression for $F(q)$ is of the form:

$$F(q) = F_1(q) + F_2(q) \tag{2}$$

$F_1(q)$ is the contribution of the one-body term to $F(q)$, which for nuclei up to $1d2s$ shell is written:

$$F_1(q) = \frac{1}{Z}\exp\left[-\frac{b_1^2q^2}{4}\right]\sum_{k=0}^{2} N_{2k}\left(\frac{b_1 q}{2}\right)^{2k} \tag{3}$$

where

$$N_0 = 2(\eta_{1s} + \eta_{2s} + 3\eta_{1p} + 5\eta_{1d})\ ,\ N_2 = -\tfrac{4}{3}(2\eta_{2s} + 3\eta_{1p} + 10\eta_{1d})$$
$$\tag{4}$$
$$N_4 = \tfrac{1}{3}(4\eta_{2s} + 8\eta_{1d})$$

η_{nl} is the occupation probability (0 or 1 in the present case) of the nl state. $F_2(q)$ is the contribution of the two-body term to $F(q)$ and is a function of q^2

through the matrix elements:

$$A_{nlS}^{n'l'S'}(j_k) = < \psi_{nlS}|j_k(qr/2)|\psi_{n'l'S'} >$$

It consists of simple polynomials and exponential functions of q^2 [19,20,21,25].

The correlation parameter λ and the HO parameter b_1 were determined by fitting to the experimental data of $F_{ch}(q)$. From (2) the charge density, $\rho_{ch}(r)$, of the closed shell nuclei ^4He,^{16}O and ^{40}Ca can be found by Fourier transforming of $F_{ch}(q) = f_p(q) f_{CM}(q) F(q)$. $f_p(q)$, $f_{CM}(q)$ are the corrections due to the finite proton size [19] and the centre of mass motion [27] respectively.

The correlated proton density and the k moments of the density are separated out into two parts:

$$\rho(r) = \rho_1(r) + \rho_2(r) \tag{5}$$

$$< r^k > = < r^k >_1 + < r^k >_2 \tag{6}$$

where $\rho_1(r)$ and $\rho_2(r)$ are the Fourier transforms of $F_1(q)$ and $F_2(q)$ respectively and $< r^k >_1$ and $< r^k >_2$ are the contributions of $\rho_1(r)$ and $\rho_2(r)$ to the various moments of the density.

An advantage of using HO wave functions and Jastrow wave functions of type (1) is that many calculations can be made analytically and also approximate expressions of the two body term of various quantities can be found if we expand the expression of the form factor in powers of λ and keep powers of λ up to $-3/2$. The approximate expression for the MS charge radius for the above mentioned nuclei is [17]:

$$< r^2 >_{ch} \approx C_{HO}(1 - \frac{1}{A}) b_1^2 + C_{SRC} b_1^2 \lambda^{-3/2} + r_p^2 + \frac{N}{Z} r_n^2 \tag{7}$$

where $C_{HO} = 3$, $C_{SRC} = 12.4673$ for ^{40}Ca and $C_{HO} = 2.25$, $C_{SRC} = 7.1775$ for ^{16}O. r_p^2, r_n^2 are the proton and neutron MS charge radii respectively. In what follows, the simplified notation r^2 is used instead of $< r^2 >_{ch}$.

If the correlation parameter:

$$\mu = \sqrt{\frac{b_1^2}{\lambda}} \tag{8}$$

is used, then expression (7) is written:

$$r^2 \approx C_{HO}(1 - \frac{1}{A}) b_1^2 + C_{SRC} \frac{\mu^3}{b_1} + r_p^2 + \frac{N}{Z} r_n^2 \tag{9}$$

As the isotopes of an element have the same number of protons we assume that the correlated form factors, densities and moments of these isotopes are described by the same formulae (2), (5), and (6). The only difference will be the different values of the parameters b_1 and μ for each isotope. These parameters could be found if the experimental form factors for all the isotopes were known (for large values of the momentum transfer). As these are not known we will try to find an isospin dependence of the parameters b_1 and μ in order to reproduce the known experimental data of the differences of the charge MS radii [3,11] and the available differences of the charge densities between the various Ca and O isotopes [1,2,11].

Expression (9) can be written in the following way:

$$r^2(A_c+n) \approx C_{HO}(1-\frac{1}{A_c+n})[b_1(A_c)+\delta b_1(A_c+n)]^2 + C_{SRC}\frac{[\mu(A_c)+\delta\mu(A_c+n)]^3}{b_1(A_c)+\delta b_1(A_c+n)} + r_p^2 + \frac{A_c-Z+n}{Z}r_n^2 \tag{10}$$

where n is the number of extra neutrons in the A_c+n isotope and A_c is the mass number of the core nucleus.

Expanding this expression in powers of δb_1 and $\delta\mu$ and keeping up to first power of δb_1 and $\delta\mu$, we have:

$$\delta r^2(A_c+n) = r^2(A_c+n) - r^2(A_c) \approx C_b\delta b_1(A_c+n) + C_\mu\delta\mu(A_c+n) + C_{HO}\frac{b_1(A_c)}{A_c+n}\left[\frac{n}{A_c}b_1(A_c) - 2\delta b_1(A_c+n)\right] + \frac{n}{Z}r_n^2 \tag{11}$$

where

$$C_b = 2C_{HO}b_1(A_c) - C_{SRC}\frac{\mu^3(A_c)}{b_1^2(A_c)}, \quad C_\mu = 3C_{SRC}\frac{\mu^2(A_c)}{b_1(A_c)} \tag{12}$$

The term in brackets in the right hand side of expression (11) comes from the correction of the center of mass motion while for the MS charge radius of the neutron the value: $r_n^2 = -0.116 fm^2$ [26] has been used. The values of the coefficients C_b and C_μ for the Ca and O isotopes will be found from the values: $b_1(40) = 1.860 fm$, $\mu(40) = 0.499 fm$ and $b_1(16) = 1.679 fm$, $\mu(16) = 0.470 fm$ which are known from the fit of $F_{ch}(q)$ of ^{40}Ca [20] and ^{16}O [19] to the experimental data.

Because the isotopic shifts come from the different number of the neutrons above the core-nucleus, $\delta b_1(A_c+n)$ and $\delta\mu(A_c+n)$ should depend on this number. In the next section we will try to find an isospin dependence of these quantities using an isospin dependence of $\hbar\omega$ from a recent work of Lalazissis and Panos [24] and from the experimental values of $\delta r^2(A_c+n)$ for Ca [3] and O [11] isotopes. If these expressions of $\delta b_1(A_c+n)$ and $\delta\mu(A_c+n)$ can be found, then the parameters $b_1(A_c+n)$ and $\mu(A_c+n)$ will be determined from the known values of $b_1(A_c)$ and $\mu(A_c)$. In the end, the form factor and the densities of the Ca and O isotopes will be calculated from (2) and (5) respectively.

In the above procedure there is only one parameter for each isotope, the correlation parameter $\mu(A_c+n)$ (or $\delta\mu(A_c+n)$) which is determined from the known values of $\delta r^2(A_c+n)$. The other parameters $b_1(A_c)$ and $\mu(A_c)$ (for $A_c = 40$ and 16) are known from the fit of $F_{ch}(q)$ to the experimental data, while the parameter $b_1(A_c+n)$ is determined in sections 3 and 4 from known expressions of $\hbar\omega$.

3 Isospin dependence of the parameters b_1 and μ.

An interesting problem of nuclear physics has been the dependence of the HO energy spacing, $\hbar\omega$, with the mass number. This parameter gives an estimate of the lowest energy level spacing and its variation with the number of neutrons and protons. It represents also the average trend in the variation of the shape of the self consistent nucleon-nucleus potential as function of N and Z. There are various expressions for $\hbar\omega$ as function of A. See for example references [28,29,30]. Lalazissis and Panos [24] have recently determined $\hbar\omega$ as function of N and Z based on a formula for the nucleon charge radius, which was proposed in ref. [31,32], reproducing well the experimentally available RMS charge radii and the isotopic shifts of even-even nuclei. Their expression of $\hbar\omega$ is:

$$\hbar\omega = 38.6A^{-1/3}\left[1+1.646A^{-1}-0.191(N-Z)A^{-1}\right]^{-2} \tag{13}$$

From (13) an expression of the HO parameter $b_1 = \sqrt{\hbar/m\omega}$, depending on N and Z, can be found straight forwardly:

$$b_1 = \sqrt{\frac{\hbar^2}{m}\frac{1}{\sqrt{38.6}}}A^{1/6}\left[1+1.646A^{-1}-0.191(N-Z)A^{-1}\right] \tag{14}$$

This expression, in the region of the isotopes of an element with mass number $A = A_c + n$ is written as follows:

$$b_1(A_c+n) = \sqrt{\frac{\hbar^2}{38.6m}}A_c^{1/6} \times$$
$$(1+\frac{n}{A_c})^{1/6}\left[1+\frac{1.646}{A_c}(1+\frac{n}{A_c})^{-1}-0.191\frac{n}{A_c}(1+\frac{n}{A_c})^{-1}\right] \tag{15}$$

Expanding this in powers of n/A_c and keeping the first three powers of it, we have:

$$\delta b_1(A_c+n) = b_1(A_c+n) - b_1(A_c) \simeq \beta_1 n + \beta_2 n^2 + \beta_3 n^3 \tag{16}$$

The values of the coefficients β_1, β_2 and β_3 for $A_c = 40$ and $A_c = 16$ are given in table 1.

If we substitute equation (16) into (11) an n dependence of $\delta\mu(A_c+n)$ can be found. This is:

$$\begin{aligned}\delta\mu(A_c+n) = & \mu(A_c+n) - \mu(A_c) \\ \simeq & \frac{1}{C_\mu}\Bigg[\delta r^2(A_c+n) - C_b\delta b_1(A_c+n) - \\ & C_{HO}\frac{b_1(A_c)}{A_c+n}\left(\frac{n}{A_c}b_1(A_c) - 2\delta b_1(A_c+n)\right) + \frac{n}{Z}r_n^2\Bigg]\end{aligned} \tag{17}$$

From the known experimental values of $\delta r^2(A_c+n)$ the correlation parameter $\mu(A_c+n)$ can be obtained from the above expression of $\delta\mu(A_c+n)$. $\delta\mu(A_c+n)$ depends on the parameter $\mu(A_c)$ through the coefficients C_b and C_μ given by expressions (12).

The same analysis can be made using various expressions of $\hbar\omega$ which are given in literature. This would be a check for the validity of the given expression of $\hbar\omega$ in the region of nuclei we examined. Also, the Zamick-Talmi expression [9,10] for $\delta r^2(40+n)$, which reproduced the experimental differences of MS charge radii, can be used for the Ca isotopes. This expression is:

$$\delta r^2(40+n) = nC_z + \frac{n(n-1)}{2}A_z + \left[\frac{n}{2}\right]B_z \tag{18}$$

where the coefficients C_z, A_z and B_z and the quantity $\left[\frac{n}{2}\right]$ are given in ref. [10].

If we substitute equations (18) and (16) into (17) the expression of $\delta\mu(40+n)$ becomes:

$$\delta\mu(40+n) = \mu(40+n) - \mu(40) \simeq \mu_1 n + \mu_2 n^2 + \mu_3 n^3 + \mu_4\left[\frac{n}{2}\right] \tag{19}$$

The values of the coefficients μ_i ($i = 1, 2, 3$) for the Ca isotopes, which are depended on the coefficients of the Zamick-Talmi expression and the coefficients C_b and C_μ, are given in table 1, while the coefficient $\mu_4 = 0.0468 fm$.

4 Numerical results and discussion

The method described in section 3 has been used to find the isospin dependence of the correlation parameter μ in the Ca and O isotopes using various expressions of $\hbar\omega$. Some of the expressions of $\hbar\omega$, we have used, are the one given by expression (13) as well as the following:

$$\hbar\omega = 35.6A^{1/3}\left[5.31 + 0.6\left(1.217A^{1/3} - \frac{2.783}{A^{1/3}} - 1.047\frac{N-Z}{A}\right)^2\right]^{-1} \quad (20)$$

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3} \quad (21)$$

$$\hbar\omega = 38.87A^{-1/3} - 23.24A^{-1} \quad (22)$$

from ref. [24,30,29] respectively. The last two expressions are isospin independent.

The values of the coefficients β_i ($i = 1,2,3$) of equation (16) (for the Ca and O isotopes) and μ_i ($i = 1,2,3$) of equations (19) (for the Ca isotopes) using the above expressions of $\hbar\omega$ are given in table 1. In this table, the various expressions of $\hbar\omega$ are marked Cases 1 to 4 in the order they have mentioned before.

The parameters $b_1(A_c + n)$ and $\mu(A_c + n)$ for the Ca and O isotopes can be found from equations (16), and (17) using the coefficients β_i from table 1, the experimental $\delta r^2(A_c + n)$ [3,11] and the known values of the parameters $b_1(A_c)$ and $\mu(A_c)$ of the core nuclei ^{40}Ca and ^{16}O. The values of $b_1(A_c + n)$ and $\mu(A_c + n)$ for the Ca and O isotopes and for three cases are given in table 2. From these values of the parameters b_1 and μ, the charge densities and the MS charge radii as well as the difference between these quantities can be found through equation (5) taking into account the corrections due to the finite proton size and the centre of mass motion.

The calculated differences of the MS charge radii for the Ca isotopes and for Cases 1 and 3 of $\hbar\omega$, using expression (11) and the experimental values of $\delta r^2(40 + n)$ in expression (17), are shown in Fig.1. All the used expressions of $\hbar\omega$ give the correct behavior of $\delta r^2(40 + n)$. The better results, that is the better χ^2, correspond to Case 1 where the isospin dependence of $\hbar\omega$ is used. The results of cases 2 and 4 which are not included in the figure are almost the same with the results of Cases 1 and 3 respectively. The same is true when the Zamick-Talmi expression is used for $\delta r^2(40 + n)$ in expression (17).

Even if the correct behavior of $\delta r^2_{theory}(40 + n)$ is expected, as the correlation parameter $\mu(40+n)$ is found from the values of $\hbar\omega(40+n)$ and the experimental

Table 1
The values of the coefficients $\beta_i (i = 1, 2, 3)$ of equation (15) for the Ca and O isotopes and $\mu_i (i = 1, 2, 3)$ of equation (18) for the Ca isotopes calculated for various expressions of $\hbar\omega$. The coefficients in Cases 1 to 6 correspond to the expressions of $\hbar\omega$ (12), and (19) to (23) respectively. In Case 7 the values of the coefficients β_i are the mean values of the Cases 6 and 7. β_i and μ_i in fm.

Isotopes	Case	β_1	β_2	β_3	μ_1	μ_2	μ_3
Ca	1	-0.0028	0.00015	-0.000004	0.0086	-0.00357	0.000007
	2	-0.0033	0.00019	-0.000006	0.0096	-0.00366	0.000010
	3	0.0073	-0.00007	0.000001	-0.0125	-0.00313	0.000000
	4	0.0065	-0.00006	0.000001	-0.0108	-0.00315	-0.000002
	5	-0.0043	0.00030	-0.000012	0.0119	-0.00388	0.000023
	6	-0.0108	0.00071	-0.000033	0.0252	-0.00473	0.000066
	7	-0.0075	0.00050	-0.000023	0.0185	-0.00431	0.000044
O	1	-0.0113	0.00108	-0.000071			
	2	-0.0133	0.00119	-0.000185			
	3	0.0142	-0.00030	0.000009			
	4	0.0129	-0.00029	0.000030			

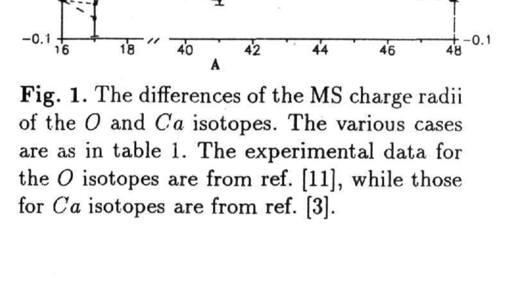

Fig. 1. The differences of the MS charge radii of the O and Ca isotopes. The various cases are as in table 1. The experimental data for the O isotopes are from ref. [11], while those for Ca isotopes are from ref. [3].

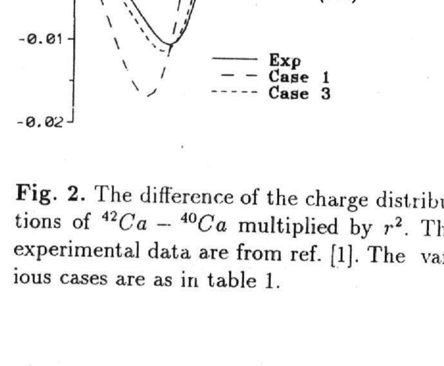

Fig. 2. The difference of the charge distributions of $^{42}Ca - {}^{40}Ca$ multiplied by r^2. The experimental data are from ref. [1]. The various cases are as in table 1.

Table 2
The values of the HO parameter b_1 (in fm) and the SRC parameter μ (in fm) for the Ca and O isotopes when the Cases 1, 3 and 7 and the experimental values of differences of the MS charge radii are used.

	Case 1		Case 3		Case 7	
Nucleus	b_1	μ	b_1	μ	b_1	μ
^{40}Ca	1.8600	0.4990	1.8600	0.4990	1.8600	0.4990
^{41}Ca	1.8573	0.5038	1.8672	0.4832	1.8529	0.5135
^{42}Ca	1.8549	0.5533	1.8743	0.5128	1.8468	0.5703
^{43}Ca	1.8528	0.5370	1.8812	0.4775	1.8413	0.5609
^{44}Ca	1.8508	0.5763	1.8880	0.4985	1.8365	0.6063
^{45}Ca	1.8491	0.5471	1.8947	0.4516	1.8322	0.5825
^{46}Ca	1.8475	0.5506	1.9012	0.4381	1.8282	0.5911
^{47}Ca	1.8461	0.5296	1.9076	0.4007	1.8244	0.5750
^{48}Ca	1.8448	0.5312	1.9139	0.3864	1.8207	0.5816
^{16}O	1.6790	0.4700	1.6790	0.4700		
^{17}O	1.6687	0.4763	1.6929	0.4177		
^{18}O	1.6601	0.6545	1.7063	0.5424		

differences of MS charge radii, the values of χ^2 indicate which expression of $\hbar\omega$ is better in this region of nuclei. The discrepancies that are noted for large values of n and for the isospin independent expressions of $\hbar\omega$ should come from the fact that the truncation made in obtaining expression (11) is not very good and probably higher powers of δb_1 and $\delta\mu$ should be included for these cases.

In Figures 2, 3 and 4 the calculated $\Delta\rho(40+n)r^2 = (\rho(40+n) - \rho(40))r^2$ (for charge or point distribution) for $n = 2$, 4 and 8 and for various cases are shown and are compared with the experimental data. From these figures it is seen that for Case 1 (and also for Case 2 which is not included in the figures) the calculated $\Delta\rho_{ch}(40+n)r^2$ have the correct behavior. That is the charge must flow from the center (and the outer skin in ^{48}Ca) into a region around the half-density radius.

For Case 3 (and also for Case 4 which is not included in the figures) the correct behavior of $\Delta\rho_{ch}(42)r^2$ and $\Delta\rho_{ch}(44)r^2$ is reproduced while for $\Delta\rho(48)r^2$ a wrong behavior is obtained. Also, for these two cases, as can be seen from

Fig.1, $\delta r^2_{theory}(40+n)$ is compared very well with $\delta r^2_{exp}(40+n)$ for $n \leq 4$ but the comperison is not good for $n \geq 5$ (especially for $n = 8$). In Cases 3 and 4, $\hbar\omega$ is isospin independent. The results could be better, for $\Delta\rho(48)r^2$, if another expression of $\hbar\omega$ is used. This expression should lead to a value of the correlation parameter $\mu(48) \approx 0.58fm$. Quite good results for $\Delta\rho(48)r^2$, can be obtained if we use the following two asymptotic expression for $\hbar\omega$ given in ref. [24]:

$$\hbar\omega = 38.6A^{-1/3} - 127A^{-4/3} + 14.75(N - Z)A^{-4/3} \tag{23}$$

$$\hbar\omega = 40.0A^{-1/3} - 56.0A^{-1} - 208.8A^{-5/3} + 68.8(N - Z)A^{-5/3} \tag{24}$$

The values of the coefficients β_i for these two cases, marked 5 and 6, are given in table 1 while $\Delta\rho(48)r^2$ for case 5 are shown in Fig.4. These two cases could lead us to the conclusion, that an expression for $\hbar\omega$ with values β_i between the corresponding values of Cases 5 and 6 would give very good $\Delta\rho(48)r^2$. For this reason in Figures 1 and 4 as well as in table 1 the Case marked 7, where the parameters β_i are the mean values of the corresponding parameters of Cases 5 and 6, has been included. For this Case, as can be seen from Fig.4, $\Delta\rho(48)r^2$ is very well compared with the experimental data.

The results for O isotopes are shown in Figures 1, 5 and 6. For the various cases of $\hbar\omega$, it is noted that, the calculated $\delta r^2(16+1)$ is very close to the experimental ones when cases 1 and 2 are used, while this is true for $\delta r^2(16+2)$ when cases 3 and 4 are used. This is shown in Fig.1 where the calculated values of $\delta r^2(16+n)$ for cases 1 and 3 are plotted and compared with the experimental ones. The results for cases 2 and 4 are similar to the corresponding results of cases 1 and 3 respectively. The above behavior of $\delta r^2(16+n)$ is reflected in the behavior of $\Delta\rho_{ch}(16+n)$ which have been calculated for various cases. That is the calculated $\Delta\rho_{ch}(16+1)$ compares better with the corresponding experimental values when the isospin dependent expression of $\hbar\omega$ are used, while for $\Delta\rho_{ch}(16+2)$ better results are obtained when the isospin independent expressions of $\hbar\omega$ are used. These are shown in Figures 5 and 6 where the calculated $4\pi\Delta\rho_{ch}(16+n)$, $n = 1,2$ for Cases 1 and 3 are displayed and compared with the experimental data [11]. The results for Cases 2 and 4 are similar to those for Cases 1 and 3 respectively.

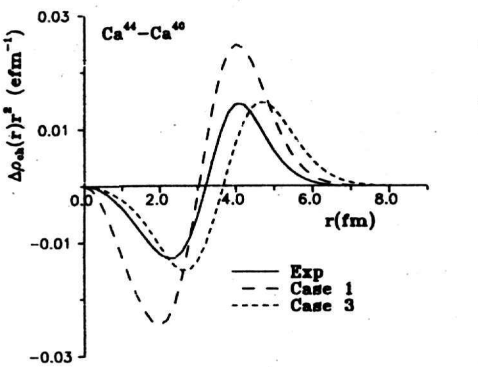

Fig. 3. The difference of the charge distributions of $^{44}Ca - {}^{40}Ca$ multiplied by r^2. The experimental data are from ref. [1]. The various cases are as in table 1.

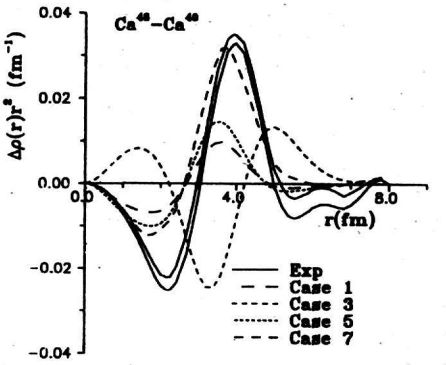

Fig. 4. The difference of the point proton distributions of $^{48}Ca - {}^{40}Ca$ multiplied by r^2. The experimental data are from ref. [2]. The various cases are as in table 1.

The above results of $\Delta\rho_{ch}(16+n)$ can become better if the correlation parameter μ given by (17) will be calculated taking into account the corresponding experimental errors. For $\Delta\rho_{ch}(16+1)$ better results are obtained if instead of $\delta r^2_{exp}(16+1)$ we use $\delta r^2_{exp}(16+1) + error(16+1)$ (see Fig.5 Case 1^+). For $\Delta\rho_{ch}(16+2)$ better results are obtained if instead of $\delta r^2_{exp}(16+2)$ we use $\delta r^2_{exp}(16+2) - error(16+2)$ (see Fig.6 Case 3^-).

5 Conclusion

In the present work the dependence of the SRC on the neutron number of the $1f_{7/2}$ orbital of the Ca isotopes has been studied. From the above analysis it becomes clear that the isospin dependence of the HO parameter b_1 and the correlation parameter μ is important in explaining not only the behavior of the differences of the MS charge radii but also to give information for the differences of the charge densities between the Ca isotopes.

An isospin dependence of the parameters b_1 and μ can be given (see table 1, Case 7) which reproduce very well both the systematic of the MS charge radii as well as the differences of the charge densities between the Ca isotopes. It is observed also that when the isospin dependence expression of $\hbar\omega$ is used, the variation of the HO parameter b_1 is a decreasing function of n while the correlation parameter μ follows somehow the variation of $\delta r^2(40+n)$. See for example the values of the parameters $b_1(40+n)$ and $\mu(40+n)$ for Cases 1 and 7 in table 2. From this behavior of the correlation parameter μ it could be concluded that the polarization of the core by the valence $1f_{7/2}$ neutrons in the Ca isotopes effect the strength of the SRC while this parameter remains almost constant (or it is not changed very much) for (closed shell) nuclei ^4He to ^{40}Ca [19,20,18]. For the isospin independent expressions of $\hbar\omega$ the parameter b_1 is an increasing function of n while the parameter μ follows the variation of $\delta r^2(40+n)$ only up to $n=5$. After this nucleus, the calculated values of $\delta r^2(40+n)$ are quite far from the experimental data. This should be the reason that the calculated $\Delta\rho(48)r^2$ is not good in Cases 3 and 4.

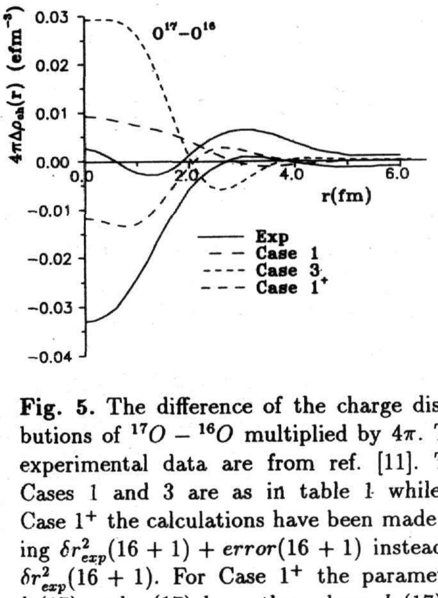

Fig. 5. The difference of the charge distributions of $^{17}O - {}^{16}O$ multiplied by 4π. The experimental data are from ref. [11]. The Cases 1 and 3 are as in table 1 while in Case 1^+ the calculations have been made using $\delta r^2_{exp}(16+1) + error(16+1)$ instead of $\delta r^2_{exp}(16+1)$. For Case 1^+ the parameters $b_1(17)$ and $\mu(17)$ have the values: $b_1(17) = 1.6687 fm$ and $\mu(17) = 0.4940 fm$

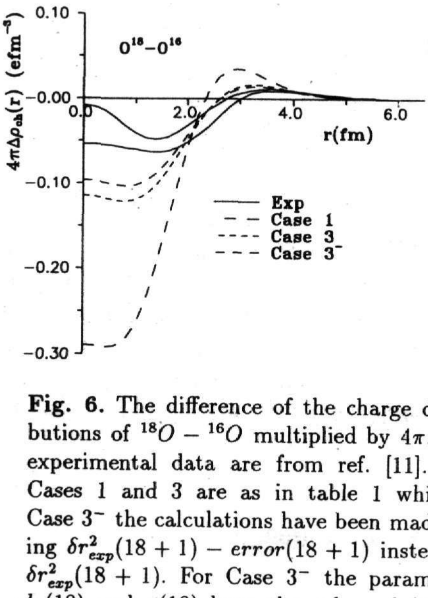

Fig. 6. The difference of the charge distributions of $^{18}O - {}^{16}O$ multiplied by 4π. The experimental data are from ref. [11]. The Cases 1 and 3 are as in table 1 while in Case 3^- the calculations have been made using $\delta r^2_{exp}(18+1) - error(18+1)$ instead of $\delta r^2_{exp}(18+1)$. For Case 3^- the parameters $b_1(18)$ and $\mu(18)$ have the values: $b_1(18) = 1.7063 fm$ and $\mu(18) = 0.5283 fm$

Similar behavior for the parameter μ has been observed for the O isotopes. There is a difference that the isospin dependence of the correlation parameter is not very clear as we have examined only two isotopes. Nevertheless, the obtained results for $\Delta\rho_{ch}(16+1)$ which compared quite well with the experimental data could lead to the conclusion that SRC and the isospin dependent of the parameters b_1 and μ might be important in understanding the odd-even staggering. Certainly, this conclusion would be more clear if experimental data for the charge densities of the odd Ca isotopes were available.

The simplicity of the present analysis makes the method attractive enough to apply to other isotopic sequences and also examine what the effect of the SRC is on the relative depletion of the Fermi sea of the isotopes. Also, it could be used to predict the differences of the charge densities of the isotopes when $\delta r^2(A_c+n)$ are known from experiment while $\Delta\rho_{ch}(A_c+n)$ are not known.

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