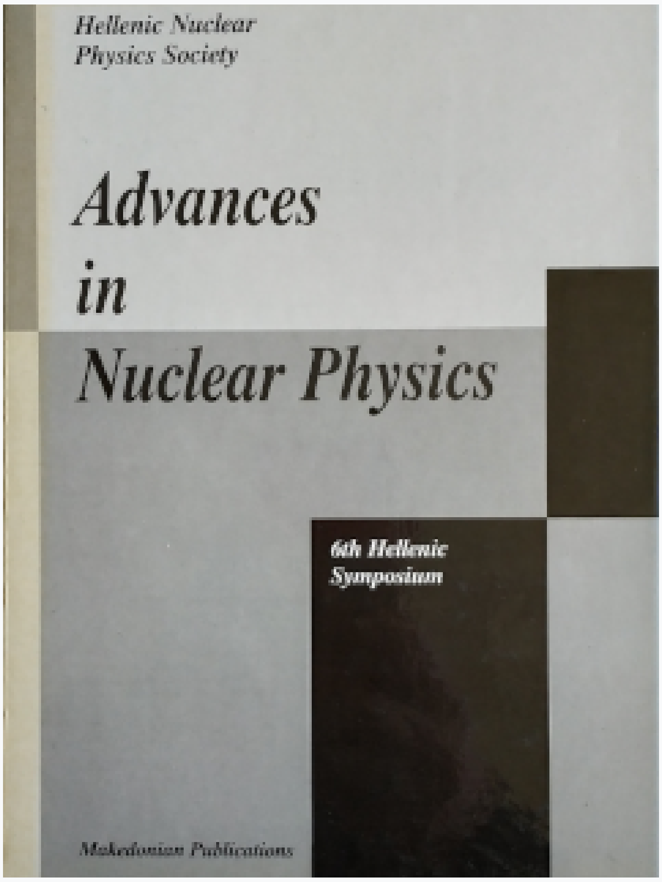
HNPS Advances in Nuclear Physics

Vol 6 (1995)

HNPS1995



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doi: [10.12681/hnps.2913](https://doi.org/10.12681/hnps.2913)

Cold Dark Matter in SUSY Theories. The Role of Nuclear Form Factors and the Folding with the LSP Velocity.

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Abstract

The momentum transfer dependence of the total cross section for elastic scattering of cold dark matter candidates, i.e. lightest supersymmetric particle (LSP), with nuclei is examined. We find that even though the energy transfer is small ($\leq 100 KeV$) the momentum transfer can be quite big for large mass of the LSP and heavy nuclei. The total cross section can in such instances be reduced by a factor of about five. The spin induced cross section for odd-A nuclei is less reduced since the high multipoles tend to enhance the cross section as the momentum transfer increases (for LSP mass $< 200\ GeV$). We present calculations of the event rates for a number of nuclear targets in a relatively wide phenomenologically allowed SUSY parameter space. These event rates are folded with a Maxwellian velocity distribution of the LSP consistent with its density in the halos. We thus find that the event rates due to the Earth's revolution around the sun can change by 25%. This factor seems to be quite independent of the other parameters of the theory. Event rates up to 80 $Kg^{-1}y^{-1}$ can be obtained in the parameter space considered.

1 Introduction

There is ample evidence that about 90% of the matter in the universe is non-luminous and non-baryonic of unknown nature [1]-[3]. Furthermore, in order to accommodate large scale structure of the universe, one is forced to assume the existence of two kinds of dark matter [3]. One kind is composed of

[1] Presented by J. D. Vergados

particles which were relativistic at the time of the structure formation. This is called Hot Dark Matter (HDM). The other kind is composed of particles which were non-relativistic at the time of stucture formation. These constitute the Cold Dark Matter (CDM) component of the universe. The COBE data [4] by examining the inisotropy on background radiation suggest that the ratio of CDM to HDM most likely is 2:1. Since about 10% of the matter of the universe is known to be baryonic, we know that we have 60% CDM, 30% HDM and 10% baryonic matter.

The most natural candidates for HDM are the neutrinos provided that they have a mass greater than $1eV/c^2$. The situation is less clear in the case of CDM. The most appealing possibility, linked closely with Supersymmetry (SUSY), is the LSP i.e. the Lightest Supersymmetric Particle [5]-[7] (see ref. [7] for a recent review).

In recent years, the phenomenological implications of Supersymmetry are being taken very seriously [5,6]. More or less, accurate predictions at low energies are now feasible in terms of few input parameters in the context of SUSY models. Such predictions do not appear to depend on arbitrary choices of the relevant parameters or untested assumptions. In any case the SUSY parameter space is somewhat restricted [5]-[7].

In such theories derived from Supergravity the LSP is expected to be a neutral Majorana fermion with mass in the $10-500GeV/c^2$ region travelling with non-relativistic velocities ($<\beta>\approx 10^{-3}$), i.e. with energies in the KeV region. In practice, however, one expects a velocity distribution which is supposed to be Maxwellian (see sect. 4). In the absence of R-parity violating interactions this particle is absolutely stable. But, even in the presence of R-parity violation, it may live long enough to be a CDM candidate.

The detection of the LSP, which is going to be denoted by χ_1, is extremely difficult, since this particle interacts with matter extremely weakly. One possibility is the detection of secondary high energy neutrinos which are produced by pair annihilation in the sun where this particle is trapped. Such high energy neutrinos can be detected via neutrino telescopes.

The other possibility, to be examined in the present work, is the detection of the energy of the recoiling nucleus in the reaction

$$\chi_1 + (A, Z) \rightarrow \chi_1 + (A, Z)^* \tag{1}$$

This energy can be converted into phonon energy and detected by a temperature rise in cryostatic detector with sufficiently high Debye temperature [3,8,9]. The detector should be large enough to allow a sufficient number of counts but not too large to permit anticoincidence shielding to reduce background. A compromise of about $1Kg$ is achieved.

Another possibility is the use of superconducting granules suspended in a magnetic field. The heat produced will destroy the superconductor and one can detect the resulting magnetic flux. Again a target of about 1Kg is favored.

There are many targets which can be employed. The most popular ones contain the nuclei ${}^{3}_{2}He$, ${}^{19}_{9}F$, ${}^{23}_{11}Na$, ${}^{29}_{14}Si$, ${}^{40}_{20}Ca$, ${}^{73,74}_{32}Ge$, ${}^{75}_{33}As$, ${}^{127}_{53}I$, ${}^{134}_{54}Xe$ and ${}^{207}_{82}Pb$.

In order to be able to calculate the event rate for the process (1) the following ingredients are necessary.

1) One must be able to construct the effective Lagrangian at the elementary particle level in the framework of Supersymmetry [10]-[16]. We will follow the procedure adopted in ref. [16]. For the readers convenience we will provide the important elements in sect. 2.

2) One must make the transition from the quark to the nucleon level [17]-[23]. This is not straightforward for the scalar couplings, which dominate the coherent part of the cross-section, and the isoscalar axial current which is important for the incoherent cross section for odd targets.

3) One must properly treat the nucleus. Admittedly, the uncertainties here are smaller than those of even the most restricted SUSY parameter space. One, however, would like to put as accurate nuclear physics input as possible in order to constrain the SUSY parameters as much as possible when the data become available.

For the coherent production, if one ignores the momentum transfer dependence, the procedure is straightforward. The spin matrix element, however, is another story. For its evaluation practically every known nuclear model has been employed.

At first, the Independent Single Particle Shell Model (ISPSM) had been employed [10,12,24,25]. Subsequent calculations using the Odd Group Method (OGM) and the Extended Odd Group Method (EOGM), utilizing magnetic moments and mirror β-decays, by Engel and Vogel [26], showed that the ISPSM was inadequate (see also ref. [12]). Eventually, however, by performing shell model calculations [27], this model was also found lacking (see ref. [28]). Iachello, Krauss and Maino [29] employed the Interacting Boson Fermion Model (IBFM) and Nikolaev and Klapdor- Kleingrothaus [13,30] the finite fermion theory in order to reliably evaluate the spin matrix elements.

One additional complication arise from the fact that the LSP appears to be quite massive, perhaps heavier than $100 GeV$. For such heavy LSP and sufficiently heavy nuclei, the dependence of the nuclear matrix elements on the momentum transfer cannot be ignored even if the LSP has energies as low as $100 KeV$) [11,24,25]. This affects both the coherent and the spin matrix elements. For the coherent mode the essential new features can be absorbed in the nuclear form factor. The evaluation of the spin matrix elements is quite a bit more complicated. Quite a number of high multipoles can now contribute, some of them getting contributions from components of the wave function which do not contribute in the static limit (i.e. at q=0, see sect. 3). Thus, in general, sophisticated Shell Model calculations are needed to account both for the observed retardation of the static spin matrix element and its correct q-dependence. For the experimentally interesting nuclear systems ${}^{29}_{14}Si$ and ${}^{73}_{32}Ge$ very elaborate calculations have been performed by Ressell *et al.* [31]. In the case of ${}^{73}_{32}Ge$ a further improved calculation by Dimitrov, Engel and Pittel has recently been performed [32] by suitably mixing variationally determined tri-axial Slatter determinants. Indeed, for this complex nucleus many multipoles contribute and the needed calculations involve techniques which are extremely sophisticated.

In the present paper we will report the results of LSP-nucleus scattering cross section using some representative input in the restricted SUSY parameter space as outlined above. The coherent matrix elements will be computed throughout the periodic table. The needed form factors were obtained using the method of ref. [33], which are in good agreement with experiment. For the spin matrix elements we have chosen ${}^{207}_{82}Pb$ as target. This target, in addition to its experimental qualifications, has the advantage of simple nuclear structure [34] (see sect. 3). Thus, only two multipoles can contribute. We may be able to draw some conclusions this way, which may reveal some physics obscured in the complexities of the calculations involving ${}^{73}_{32}Ge$. Finally, the obtained counting rate will be convoluted with a reasonable velocity distribution [7] (see sect. 4).

2 Brief description of the operators

It has recently been shown that process (1) can be described by a four fermion interaction [10]-[15] of the type [16]

$$L_{eff} = -\frac{G_F}{\sqrt{2}} \left[J_\lambda \bar{\chi}_1 \gamma^\lambda \gamma^5 \chi_1 \; + \; J \bar{\chi}_1 \chi_1 \right] \tag{2}$$

where

$$J_\lambda = \bar{N} \gamma_\lambda [f_V^0 + f_V^1 \tau_3 + (f_A^0 + f_A^1 \tau_3) \gamma_5] N \tag{3}$$

and

$$J = \bar{N} (f_S^0 + f_S^1 \tau_3) N \tag{4}$$

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, due to the Majorana nature of the LSP, $\bar{\chi}_1\gamma^\lambda\chi_1 = 0$ (identically). The vector and axial vector form factors can arise out of Z-exchange and s-quark exchange [10]-[16] (s-quarks are the SUSY partners of quarks with spin zero). They have uncertainties in them (for three choices in the allowed parameter space of ref. [5] see ref. [16]). In our choice of the parameters the LSP is mostly a gaugino. Thus, the Z- contribution is small. It may become dominant in models in which the LSP happens to be primarily a Higgsino. Such a possibility will be examined elsewhere. The transition from the quark to the nucleon level is pretty straightforward in this case. This is in general the case of vector current contribution. We will see later that, due to the Majorana nature of the LSP, the contribution of the vector current, which can lead to a coherent effect of all nucleons, is suppressed [10-16]. The vector current is effectively multiplied by a factor of $\beta = v/c$, v is the velocity of LSP (see table 1). Thus, the axial current, especially in the case of light and medium mass nuclei, cannot be ignored.

For the isovector axial current one is pretty confident about how to go from the quark to the nucleon level. We know from ordinary weak decays that the coupling merely gets renormalized from $g_A = 1$ to $g_A = 1.24$. For the isoscalar axial current the situation is not completely clear. The naive quark model (NQM) would give a renormalization parameter of unity (the same as the isovector vector current). This point of view has, however, changed in recent years due to the so-called spin crisis, i.e. the fact that in the EMC data [17] it appears that only a small fraction of the proton spin arises from the quarks. Thus, one may have to renormalize f_A^0 by $g_A^0 = 0.28$, for u and d quarks, and $g_A^0 = -0.16$ for the strange quarks [18], i.e. a total factor of .12. These two possibilities, labeled as NQM and EMC, are listed in table 1. One cannot completely rule out the possibility that the actual value maybe anywhere in the above mentioned region [19].

The scalar form factors arise out of the Higgs exchange or via s-quark exchange when there is mixing between s-quarks $\tilde{q}_L$ and $\tilde{q}_R$ [10]-[12] (the partners of the left-handed and right-handed quarks). They have two types of uncertainties in them. One, which is the most important, at the quark level due to the uncertainties in the Higgs sector. The other in going from the quark to the nucleon level [14,15]. Such couplings are proportional to the quark masses, and hence sensitive to the small admixtures of $q\bar{q}$ (q other than u and d) present in the nucleon. Again values of f_S^0 and f_S^1 in the allowed SUSY parameter space are considered [16].

The actual values of the parameters f_S^0 and f_S^1 used here, arising mainly from Higgs exchange, were obtained by considering 1-loop corrections in the Higgs sector. As a result, the lightest Higgs mass is now a bit higher, i.e. more massive than the value of the Z-boson [20,21]. The thus obtained values of

Table 1 : The parameters $\beta f_V^0, f_S^0, f_A^0, f_A^1$ and $f_V^1/f_V^0, f_S^1/f_S^0$ for three SUSY solutions (see ref. [16]). The value of $\beta = 10^{-3}$ was used. For the definition of f_S^0 and f_S^1 (models A, B and C) see ref. [16] and for the values of m_h, m_H and m_A employed see ref. [5].

Quantity	Solution #1	Solution #2	Solution #3
$\beta f_V^0(Z)$	0.475×10^{-5}	1.916×10^{-5}	0.966×10^{-5}
$f_V^1(Z)/f_V^0(Z)$	-1.153	-1.153	-1.153
$\beta f_V^0(\tilde{q})$	1.271×10^{-5}	0.798×10^{-5}	1.898×10^{-5}
$f_V^1(\tilde{q})/f_V^0(\tilde{q})$	0.222	2.727	0.217
$f_s^0(\tilde{q})/\beta f_V^0(\tilde{q})(model A)$	6.3×10^{-3}	3.6×10^{-3}	2.4×10^{-3}
$f_s^0(\tilde{q})/\beta f_V^0(\tilde{q})(model B)$	0.140	3.5×10^{-2}	5.8×10^{-2}
βf_V^0	1.746×10^{-5}	2.617×10^{-5}	2.864×10^{-5}
f_V^1/f_V^0	-0.153	-0.113	-0.251
$LSP\ mass(GeV)$	126	27	102
$tan2a$	0.245	6.265	0.528
m_h, m_H, m_A	116,346,345	110,327,305	113,326,324
$f_S^0(H)$ (model A)	1.31×10^{-5}	1.30×10^{-4}	1.38×10^{-5}
f_S^1/f_S^0 (model A)	-0.275	-0.107	-0.246
$f_S^0(H)$ (model B)	5.29×10^{-4}	7.84×10^{-3}	6.28×10^{-4}
$f_S^0(H)$ (model C)	7.57×10^{-4}	7.44×10^{-3}	7.94×10^{-4}
$f_A^0(Z)$	-	-	-
$f_A^1(Z)$	1.27×10^{-2}	5.17×10^{-2}	2.58×10^{-2}
$f_A^0(\tilde{q})(NQM)$	0.510×10^{-2}	3.55×10^{-2}	$.704 \times 10^{-2}$
$f_A^0(\tilde{q})(EMC)$	0.612×10^{-3}	0.426×10^{-2}	$.844 \times 10^{-3}$
$f_A^1(\tilde{q})$	0.277×10^{-2}	0.144×10^{-2}	0.423×10^{-2}
$f_A^0(NQM)$	0.510×10^{-2}	3.55×10^{-2}	$.704 \times 10^{-2}$
$f_A^0(EMC)$	0.612×10^{-3}	0.426×10^{-2}	$.844 \times 10^{-3}$
f_A^1	1.55×10^{-2}	5.31×10^{-2}	3.00×10^{-2}

the parameters f_S^0 and f_S^1 are smaller than those of ref. [16] (see table 1). The next source of ambiquities involves the step of going from the quark to the nucleon level for the scalar and isoscalar couplings. Here we adopt the procedure described in ref. [16] as a result of the analysis of ref. [14,22,23].

3 Total cross section

The invariant amplitude in the case of non-relativistic LSP takes the form [16]

$$|m|^2 = \frac{E_f E_i - m_1^2 + \mathbf{p}_i \cdot \mathbf{p}_f}{m_1^2} |J_0|^2 + |\mathbf{J}|^2 + |J|^2$$
$$\simeq \beta^2 |J_0|^2 + |\mathbf{J}|^2 + |J|^2 \tag{5}$$

where $|J_0|$ and $|\mathbf{J}|$ indicate the matrix elements of the time and space components of the current J_λ of eq. (3), respectively, and J represents the matrix element of the scalar current J of eq. (4). Notice that $|J_0|^2$ is multiplied by β^2 (the suppression due to the Majorana nature of LSP mentioned above). It is straightforward to show that

$$|J_0|^2 = A^2 |F(\mathbf{q}^2)|^2 \left(f_V^0 - f_V^1 \frac{A-2Z}{A} \right)^2 \tag{6}$$

$$J^2 = A^2 |F(\mathbf{q}^2)|^2 \left(f_S^0 - f_S^1 \frac{A-2Z}{A} \right)^2 \tag{7}$$

$$|\mathbf{J}|^2 = \frac{1}{2J_i+1} | < J_i || \, [f_A^0 \Omega_0(\mathbf{q}) \; + \; f_A^1 \Omega_1(\mathbf{q})] \, || J_i > |^2 \tag{8}$$

with $F(\mathbf{q}^2)$ the nuclear form factor and

$$\Omega_0(\mathbf{q}) = \sum_{j=1}^{A} \sigma(j) e^{-i\mathbf{q}\cdot\mathbf{x}_j}, \qquad \Omega_1(\mathbf{q}) = \sum_{j=1}^{A} \sigma(j) \tau_3(j) e^{-i\mathbf{q}\cdot\mathbf{x}_j} \tag{9}$$

where $\sigma(j)$, $\tau_3(j)$, $\mathbf{x}_j$ are the spin, third component of isospin ($\tau_3|p>=|p>$) and coordinate of the j-th nucleon and $\mathbf{q}$ is the momentum transferred to the nucleus.

The differential cross section in the laboratory frame takes the form [16]

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{\pi} \left(\frac{m_1}{m_p}\right)^2 \frac{1}{(1+\eta)^2} \xi \{ \beta^2 |J_0|^2 [1 - \frac{2\eta+1}{(1+\eta)^2} \xi^2] + |\mathbf{J}|^2 + |J|^2 \} \tag{10}$$

where $\eta = m_1/m_p A$ (m_p = proton mass), m_1 is the mass of LSP, $\xi = \hat{\mathbf{p}}_i \cdot \hat{\mathbf{q}} \geq 0$ (forward scattering) and

$$\sigma_0 = \frac{1}{2\pi} (G_F m_p)^2 \simeq 0.77 \times 10^{-38} cm^2 \tag{11}$$

The momentum transfer $\mathbf{q}$ is given by

$$|\mathbf{q}| = q_0 \xi, \qquad q_0 = \beta \frac{2 m_1 c}{1+\eta} \tag{12}$$

Table 2 : The quantity q_0 (forward momentum transfer) in units of fm^{-1} for three values of m_1 and three typical nuclei.

	q_0 (MeV/c^2)		
Nucleus	$m_1 = 30GeV$	$m_1 = 100GeV$	$m_1 = 150GeV$
${}^{40}_{20}Ca$	.174	.290	.321
${}^{72}_{32}Ge$	.215	.425	.494
${}^{208}_{82}Pb$	.267	.685 .	.885

Some values of q_0 (forward momentum transfer) for some characteristic values of m_1 and representative nuclear systems (light, intermediate and heavy) are given in table 2. It is clear that the momentum transfer can be sizable for large m_1 and heavy nuclei.

The total cross section can be cast in the form

$$\sigma = \sigma_0(\frac{m_1}{m_p})^2 \frac{1}{(1+\eta)^2} \{A^2 [[\beta^2 (f_V^0 - f_V^1 \frac{A-2Z}{A})^2 + (f_S^0 - f_S^1 \frac{A-2Z}{A})^2] I_0(q_0^2) - \frac{\beta^2}{2} \frac{2\eta+1}{(1+\eta)^2} (f_V^0 - f_V^1 \frac{A-2Z}{A})^2 I_1(q_0^2)] + (f_A^0 \Omega_0(0))^2 I_{00}(q_0^2) + 2 f_A^0 f_A^1 \Omega_0(0) \Omega_1(0) I_{01}(q_0^2) + (f_A^1 \Omega_1(0))^2 I_{11}(q_0^2) \} \tag{13}$$

where

$$I_\rho(q_0^2) = 2(\rho+1) \int_0^1 \xi^{1+2\rho} \, |F(q_0^2 \xi^2)|^2 \, d\xi, \qquad \rho = 0,1 \tag{14}$$

In evaluating the integrals $I_{00}(q_0^2)$, $I_{01}(q_0^2)$ and $I_{11}(q_0^2)$, which contain the momentum dependence of the spin matrix elements, we follow the standard procedure of the multipole expansion of the $e^{-i\mathbf{q}\cdot\mathbf{r}}$. Thus, we can write

$$I_{\rho\rho'}(q_0^2) = 2 \int_0^1 \xi \, d\xi \sum_{\lambda,\kappa} \frac{\Omega_\rho^{(\lambda,\kappa)}(q_0^2\xi^2)}{\Omega_\rho(0)} \frac{\Omega_{\rho'}^{(\lambda,\kappa)}(q_0^2\xi^2)}{\Omega_{\rho'}(0)}, \qquad \rho, \rho' = 0,1 \tag{15}$$

We warn the reader that our normalization is different than that found in the previous literature. Our integrals are normalized to unity as q becomes zero. We have also made the identification

$$\Omega_\rho^{(0,1)} = \Omega_\rho(q_0^2\xi^2) = (2J_i+1)^{-\frac{1}{2}} < J_f || \sum_{j=1}^{A} j_0(q_0 \xi r_j) \omega_\rho(j) \sigma(j) || J_i > \tag{16}$$

where $\rho = 0, 1$ and $\omega_0(j) = 1$ and $\omega_1(j) = \tau_3(j)$. In general,

$$\Omega_\rho^{(\lambda,\kappa)} = (2J_i+1)^{-\frac{1}{2}}\sqrt{4\pi} < J_f || \sum_{j=1}^{A} \left[\mathbf{Y}^\lambda(\hat{\mathbf{r}}_j)\otimes\sigma(j)\right]^\kappa j_\lambda(q_0\xi r_j)\omega_\rho(j) || J_i > \quad (17)$$

where $\rho = 0, 1$. With the above expressions we have managed to separate the elementary parameters f_A^0 and f_A^1 from the nuclear parameters. The latter were so parametrized that in the static limit ($q_0 = 0$) $I_{\rho\rho'}(0) = 1$. The static spin matrix element then takes the simple expression

$$|\mathbf{J}|^2 = \left|f_A^0\Omega_0(0) + f_A^1\Omega_1(0)\right|^2$$

In a previous paper [33] we have shown that the nuclear form factor $F(q^2)$ can be adequately described within the harmonic oscillator model as follows

$$F(q^2) = [\frac{Z}{A}\Phi(qb, Z) + \frac{N}{A}\Phi(qb, N)]\, e^{-q^2b^2/4} \quad (18)$$

where Φ is a polynomial of the form

$$\Phi(qb, \alpha) = \sum_{\lambda=0}^{N_{max}(\alpha)} \theta_\lambda^{(\alpha)}(qb)^{2\lambda}, \qquad \alpha = Z, N \quad (19)$$

$N_{max}(Z)$ and $N_{max}(N)$ depend on the major harmonic oscillator shell occupied by protons and neutrons [33], respectively. The integral $I_\rho(q_0^2)$ can be written as

$$I_\rho(q_0^2) \to I_\rho(u) = (1+\rho)u^{-(1+\rho)} \int_0^u x^{1+\rho}\, |F(2x/b^2)|^2\, dx, \quad (20)$$

where

$$u = q_0^2b^2/2, \qquad b = 1.0A^{1/3}\, fm \quad (21)$$

With the use of eqs. (14), (20) we obtain

$$I_\rho(u) = \frac{1}{A^2}\{ Z^2 I_{ZZ}^{(\rho)}(u) + 2NZ I_{NZ}^{(\rho)}(u) + N^2 I_{NN}^{(\rho)}(u)\} \quad (22)$$

where

$$I_{\alpha\beta}^{(\rho)}(u) = \sum_{\lambda=0}^{N_{max}(\alpha)} \sum_{\nu=0}^{N_{max}(\beta)} \frac{\theta_\lambda^{(\alpha)}}{\alpha}\frac{\theta_\nu^{(\beta)}}{\beta}\frac{2^{\lambda+\nu+\rho}\,(\lambda+\nu+\rho)!}{u^{1+\rho}}\left[1 - e^{-u}\sum_{\kappa=0}^{\lambda+\nu+\rho}\frac{u^\kappa}{\kappa!}\right] \quad (23)$$

with $\alpha, \beta = N, Z$. The coefficients $\theta_\lambda^{(\alpha)}$ for light and medium nuclei have been computed in ref. [33]. In table 3 we present them by including in addition

Table 3. The coefficients θ_λ determining the proton and neutron form factors for all closed (sub)shell nuclei. In a harmonic oscillator basis they are rational numbers. The coeficients for $\lambda = 0$ are equal to Z (or N).

nlj	$\lambda=0$	$\lambda=1$	$\lambda=2$	$\lambda=3$	$\lambda=4$	$\lambda=5$	$\lambda=6$
$0s_{1/2}$	2						
$0p_{3/2}$	6	-2/3					
$0p_{1/2}$	8	-1					
$0d_{5/2}$	14	-3	1/10				
$1s_{1/2}$	16	-11/3	11/60				
$0d_{3/2}$	20	-5	1/4				
$0f_{7/2}$	28	-9	13/20	-1/105			
$1p_{3/2}$	32	-11	61/60	-11/420			
$0f_{5/2}$	38	-14	79/60	-1/30			
$1p_{1/2}$	40	-15	3/2	-1/24			
$0g_{9/2}$	50	-65/3	5/2	-5/56	1/1512		
$0g_{7/2}$	58	-27	33/10	-107/840	1/840		
$1d_{5/2}$	64	-31	17/4	-173/840	1/336		
$1d_{3/2}$	68	-101/3	293/60	-31/120	1/240		
$2s_{1/2}$	70	-35	21/4	-7/24	1/192		
$0h_{11/2}$	82	-45	29/4	-73/168	37/4032	-1/27720	
$0h_{9/2}$	92	-160/3	107/12	-31/56	151/12096	-1/15120	
$1f_{7/2}$	100	-60	217/20	-653/840	449/20160	-1/5040	
$1f_{5/2}$	106	-65	123/10	-397/420	199/6720	-1/3360	
$2p_{3/2}$	110	-205/3	403/30	-153/140	253/6720	-1/2240	
$2p_{1/2}$	112	-70	14	-7/6	1/24	-1/1920	
$0i_{13/2}$	126	-84	35/2	-3/2	1/18	-49/63360	1/617760

those for heavy nuclei. The integrals $I_\rho(u)$ for three typical nuclei ($^{40}_{20}Ca$, $^{72}_{32}Ge$ and $^{208}_{82}Pb$) are presented in fig. 1 as a function of m_1. We see that, for light nuclei the modification of the cross section by the inclusion of the form factor is small. For heavy nuclei and massive m_1 the form factor has a dramatic effect on the cross section and may decrease it by a factor of about five. The integral $I_1(u)$ is even more suppressed but it is less important.

The spin matrix elements unfortunately depend, in general, rather sensitively on the details of the nuclear structure. As we mentioned in the introduction, the first attempt for quantitative description was based on the Odd Group Method [26]. Subsequent shell model calculations demonstrated that the OG-M was not adequate and showed that more elaborate calculations were needed [31,32]. Furthermore, since the matrix elements at $q = 0$ are often quenched, the momentum dependence of the matrix elements was more important than it was naively expected. As a matter of fact, one has to include a lot of configurations to accommodate all multipoles, which in complex nuclei like ^{29}Si and ^{73}Ge, result in very large Hilbert spaces. It will be therefore, a very hard task to substantially improve those calculations.

Among the targets which have been considered for LSP detection, ^{207}Pb stands out as an important candidate. The spin matrix element of this nucleus has not been evaluated, since one expects the relative importance of the spin versus the coherent mode to be more important on light nuclei. But, as we have mentioned, the spin matrix element in the light systems is quenched. On the other hand, the spin matrix element of ^{207}Pb, especially the isoscalar one, does not suffer from unusually large quenching, as is known from the study of the magnetic moment. Thus, we view it as a good theoretical laboratory since: i) It is believed to have simple structure, one $2p1/2$ neutron hole outside the doubly magic nucleus ^{208}Pb. ii) Because of its low angular momentum, only two multipoles $\lambda = 0$ and $\lambda = 2$ with a J-rank of $k = 1$ can contribute even at large momentum transfers. One can thus view the information obtained from this nucleus as complementary to that of ^{73}Ge.

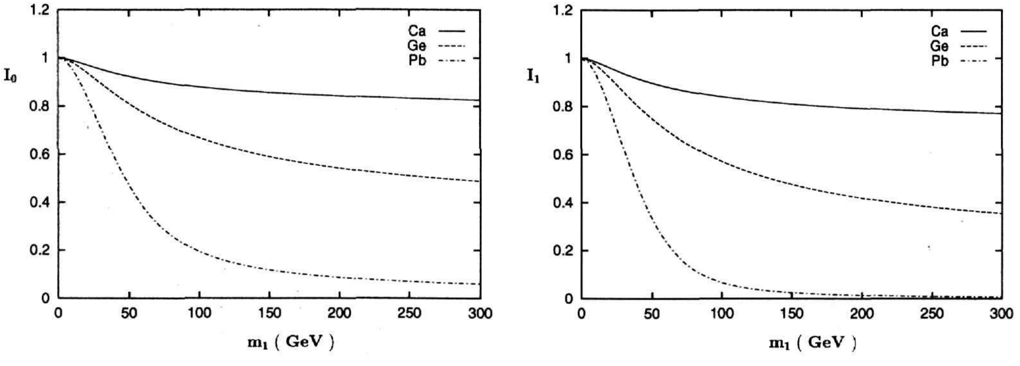

Figure 1: The integrals I_0 and I_1, which describe the coherent contribution of the total cross section as a function of the LSP mass (m_1), for three typical nuclei: $^{40}_{20}Ca$, $^{72}_{32}Ge$ and $^{208}_{82}Pb$.

To a good approximation [34] the ground state of the $^{207}_{82}Pb$ nucleus can be described as a $2p_{1/2}$ neutron hole in the $^{208}_{82}Pb$ closed shell. Then for $\lambda = 0$ one finds

$$\Omega_0(\mathbf{q}) = -(1/\sqrt{3})F_{2p}(\mathbf{q}^2), \qquad \Omega_1(\mathbf{q}) = (1/\sqrt{3})F_{2p}(\mathbf{q}^2) \tag{24}$$

and

$$I_{00} = I_{01} = I_{11} = 2\int_0^1 \xi\,[F_{2p}(q^2)]^2\,d\xi \tag{25}$$

Even though the probability of finding a pure $2p_{1/2}$ neutron hole in the $\frac{1}{2}^-$ ground state of $^{207}_{82}Pb$ is greater than 95%, the ground state magnetic moment is quenched due to the 1^+ p-h excitation involving the spin orbit partners. Hence, we expect a similar suppression of the isovector spin matrix elements. Thus we write

$$\begin{aligned} |(1/2)^- >_{gs} = C_0|(2p_{1/2})^{-1} > \Big[1 + C_1|[0i_{11/2}(n)(0i_{13/2})^{-1}(n)]1^+ > \\ + C_2|[0h_{9/2}(p)(0h_{11/2})^{-1}(p)]1^+ > +...\Big] \end{aligned} \tag{26}$$

Due to angular momentum and parity selection rules, we have $k = 1$ and $\lambda = 0, 2$. Retaining terms which are at most linear in the coefficients C_1, C_2 we obtain

i) $\lambda = 0$

$$\Omega_0(\mathbf{q}) = C_0^2\,\{F_{2p}(q^2)/\sqrt{3} - 8\,[(7/13)^{1/2}C_1F_{0i}(q^2) + (5/11)^{1/2}C_2F_{0h}(q^2)]\,\} \tag{27}$$

$$\Omega_1(\mathbf{q}) = -\,C_0^2\,\{F_{2p}(q^2)/\sqrt{3} - 8\,[(7/13)^{1/2}C_1F_{0i}(q^2) - (5/11)^{1/2}C_2F_{0h}(q^2)]\,\} \tag{28}$$

where

$$F_{nl}(q^2) = e^{-q^2b^2/4}\sum_{\mu=0}^{N_{max}} \gamma_\mu^{(nl)}(qb)^{2\mu} \tag{29}$$

The coefficients $\gamma_\mu^{(nl)}$ are given in table 4.

The coefficients C_0, C_1 and C_2 were obtained by diagonalizing the Kuo-Brown G-matrix [35,36] in a model space of 2h-1p configurations. Thus, we find

$$C_0 = 0.973350, \qquad C_1 = 0.005295, \qquad C_2 = -0.006984$$

We also find

$$\Omega_0(0) = -(1/\sqrt{3})(0.95659), \qquad (small\ quenching) \tag{30}$$

$$\Omega_1(0) = (1/\sqrt{3})(0.83296), \qquad (sizable\ quenching) \tag{31}$$

Table 4. The coefficients $\gamma_\mu^{(nl)}$, entering the polynomial of eq. (29) describing the form factor of a single particle harmonic oscillator wave function up to $6\hbar\omega$, i.e. throughout the periodic table.

$n\ l$	$\mu=0$	$\mu=1$	$\mu=2$	$\mu=3$	$\mu=4$	$\mu=5$	$\mu=6$
0 0	1						
0 1	1	-1/6					
1 0	1	-1/3	1/24				
0 2	1	-1/3	1/60				
1 1	1	-1/2	11/120	-1/240			
0 3	1	-1/2	1/20	-1/840			
2 0	1	-2/3	11/60	-1/60	1/1920		
1 2	1	-2/3	19/120	-11/840	1/3360		
0 4	1	-2/3	1/10	-1/210	1/15120		
2 1	1	-5/6	17/60	-31/840	9/4480	-1/26880	
1 3	1	-5/6	29/120	-47/1680	37/30240	-1/60480	
0 5	1	-5/6	1/6	-1/84	1/3024	-1/332640	
3 0	1	-1	17/40	-31/420	27/4480	-1/4480	1/322560
2 2	1	-1	2/5	-1/15	41/8064	-1/5760	1/483840
1 4	1	-1	41/120	-1/20	1/315	-1/11880	1/1330560
0 6	1	-1	1/4	-1/42	1/1008	-1/55440	1/8648640

The amount of retardation of the total matrix element depends on the values of f_A^0 and f_A^1.

ii) $\lambda = 2$.

In this case, in addition to the leading $(2p1/2)^{-1}$ configuration, the first leading correction to the nuclear matrix element is linear in the mixing coefficients $C_{j_1 j_2}$ appearing in the expression:

$$|(\frac{1}{2})^{-1}>= C_0\Big\{|(2p_{1/2})^{-1}(n)> + \sum_{j_1 j_2} C_{j_1 j_2}|(2p_{1/2})^{-1}(n);(j_1^{-1}j_2)J_{12}=1;\frac{1}{2}>\Big\} \quad (32)$$

i.e.

$$\Omega_\rho^{(2,1)} = \frac{C_0^2}{\sqrt{2J_i+1}}\{2\sum_{j_1 j_2} C_{j_1 j_2} G(j_1,j_2,\rho) < n_1 l_1 | j_2(q_0\xi r)| n_2 l_2 > \quad (33)$$
$$+(-1)^\rho < (2p_{1/2})^{-1} || T^k || (2p_{1/2})^{-1} >\}$$

Table 5. The coefficients $\varepsilon_\kappa(n_1 l_1, n_2 l_2, 2)$, which describe the matrix elements $< n_1 l_1 | j_2(qr) | n_2 l_2 >$ (see apendix), for harmonic oscillator wave function.

$n_1 l_1$	$n_2 l_2$	$\kappa = 0$	$\kappa = 1$	$\kappa = 2$	$\kappa = 3$	$\kappa = 4$	$\kappa = 5$
2 1	2 1	$\frac{26}{15}$	$-\frac{64}{21}$	$\frac{40}{21}$	$-\frac{16}{35}$	$\frac{4}{105}$	
0 5	0 5	$\frac{26}{15}$	$-\frac{208}{105}$	$\frac{208}{315}$	$-\frac{832}{10395}$	$\frac{32}{10395}$	
0 6	0 6	2	$-\frac{20}{7}$	$\frac{80}{63}$	$-\frac{160}{693}$	$\frac{160}{9009}$	$-\frac{64}{135135}$
0 6	0 4	$\frac{2}{15}\sqrt{143}$	$-\frac{16}{105}\sqrt{143}$	$\frac{16}{315}\sqrt{143}$	$-\frac{64}{945}\sqrt{\frac{13}{11}}$	$\frac{32}{945}\sqrt{\frac{1}{143}}$	
0 7	0 5	$2\sqrt{\frac{13}{15}}$	$-\frac{4}{7}\sqrt{\frac{65}{3}}$	$\frac{16}{63}\sqrt{\frac{65}{3}}$	$-\frac{32}{693}\sqrt{\frac{65}{3}}$	$\frac{32}{693}\sqrt{\frac{5}{39}}$	$-\frac{64}{10395}\sqrt{\frac{1}{195}}$

where $G(j_1, j_2, \rho)$ are isospin dependent geometrical factors which can be evaluated by standard techniques. Notice, however, that unlike the $\lambda = 0$ case, many amplitudes can contribute if the quadrupole modes coupled to the single hole wavefunction are admixed in the ground state of the system.

In the simple model of ref. [34], in addition to the C_1, C_2 encountered above, one needs the amplitudes of the two additional configurations, $0j_{13/2}0j_{11/2}(n)$ and $0i_{11/2}0g_{9/2}(p)$, which are $C_3 = 0.000239$ and $C_4 = -0.000642$. Obviously, this is a simplification, since one should consider the spin Giant Quadrupole Resonance (GQR), which may have a small admixture in the ground state of the nucleus but a very large transition matrix element. Such a detailed calculation including all $2\hbar\omega$ excitations is in progress and it will be reported elsewhere.

The radial integrals $< n_1 l_1 | j_2(q_0 \xi r) | n_2 l_2 >$ can be cast in the form of eq. (29), but they depend on two single particle quantum numbers (see appendix). The relevant coefficients needed for the present work are listed in table 5.

Using eqs. (15), (27),(28) and (33), we can evaluate the integrals I_{00}, I_{01} and I_{00}. The results are presented in fig. 2. We see that for a heavy nucleus and high LSP mass the momentum transfer dependence of the spin matrix elements cannot be ignored.

For orientation purposes we have estimated the spin matrix element of the light nucleus ${}^{19}_{9}F$. Assuming that its ground state wave function is a pure $SU(3)$ state with the largest symmetry, i.e $f = [3], (\lambda\mu) = (60)$, which may be a very crude approximation, we obtain [37,38] the expression

$$\frac{\Omega_1(\mathbf{q})}{\Omega_1(0)} = \frac{4}{9} F_{2s}(q^2) + \frac{5}{9} F_{0d}(q^2) \tag{34}$$

There is only isovector component contribution (the isoscalar matrix element vanishes). As expected that the effect of the nuclear form factor on the cross section for light nuclei is insignificant [39].

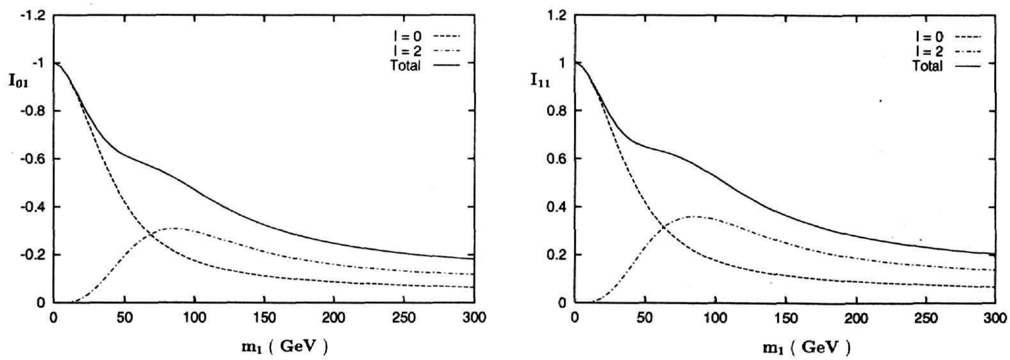

Figure 2: The integrals I_{01} and I_{11} which give the spin contribution to the cross section for ${}^{207}_{82}Pb$ in the model described in sect. 3. For their definition see eqs. (20) and (22) in the text.

4 Convolution of the cross section with the velocity distribution

Let us assume that the LSP is moving with velocity v_z with respect to the detecting apparatus. Then the detection rate is

$$\frac{dN}{dt} = \frac{\rho(0)}{m_1} v_z \sigma(v), \qquad v_z \geq 0 \tag{35}$$

where $\rho(0)$ the LSP density in our vicinity. This density has to be consistent with the LSP velocity distribution. Such a consistent choise can be a Maxwell distribution

$$f(v) = (\sqrt{\pi} v_0)^{-3} e^{-v^2/v_0^2} \tag{36}$$

provided that [30]

$$v_0 = \sqrt{(2/3) < v^2 >} = 220Km/s, \qquad \rho(0) = 0.3GeV/cm^3$$

For our purposes it is convenient to express the above distribution in the laboratory frame, i.e.

$$f(\mathbf{v}, \mathbf{v}_E) = (\sqrt{\pi}v_0)^{-3} e^{-(\mathbf{v}-\mathbf{v}_E)^2/v_0^2} \tag{37}$$

where $\mathbf{v}_E$ is the velocity of the earth with respect to the center of the distribution. Thus,

$$\left\langle \frac{dN}{dt} \right\rangle = \frac{\rho(0)}{m_1} \int f(\mathbf{v}, \mathbf{v}_E) v_z \Theta(v_z) \sigma(|\mathbf{v}|) d^3\mathbf{v} \tag{38}$$

or

$$\left\langle \frac{dN}{dt} \right\rangle = \frac{\rho(0)}{m_1} \sqrt{< v^2 >} J(\mathbf{v}_E, \mathbf{v}_0) \tag{39}$$

where

$$J(\mathbf{v}_E, \mathbf{v}_0) = \int \frac{v_z \Theta(v_z)}{\sqrt{< v^2 >}} f(\mathbf{v}, \mathbf{v}_E) \sigma(|\mathbf{v}|) d^3\mathbf{v}$$

and

$$\Theta(v_z) = \begin{cases} 1, & v_z > 0 \\ 0, & v_z < 0 \end{cases}$$

or

$$J(\mathbf{v}_E, \mathbf{v}_0) = (\sqrt{\pi}v_0)^{-3} \int_0^\infty \frac{v^3}{\sqrt{< v^2 >}} e^{-(v^2 + v_E^2)/v_0^2} \sigma(v) dv \tag{40}$$

$$\times \int_0^{\pi/2} cos\theta sin\theta d\theta \int_0^{2\pi} e^{2\mathbf{v} \cdot \mathbf{v}_E / v_0^2} d\phi$$

In order to perform the angular integration, we expand the exponential in terms of spherical harmonics in a fashion which is familiar from the plane wave expansion, i.e.

$$e^{2\mathbf{v} \cdot \mathbf{v}_E / v_0^2} = 4\pi \sum_{l=0}^{\infty} i_l(2vv_E/v_0^2) \sum_{m=-l}^{l} Y_m^l(\theta, \phi) Y_m^{l\,*}(\alpha, \gamma) \tag{41}$$

where (α, γ) are the two angles needed to specify the earth's velocity. $i_l(x)$ is the modified spherical Bessel function, which is finite at the origin, given by

$$i_l(x) = x^l \left(\frac{1}{x}\frac{d}{dx}\right)^l \frac{sinhx}{x} \tag{42}$$

Then, the ϕ angular integration is trivial due to axial symmetry (only the m=0 contributes, i.e. the result is independent of the angle γ). Furthermore, it is ea^sy to show that

$$\tilde{f}_l \equiv \int_0^1 P_l(\xi)\xi d\xi = \begin{cases} \delta_{l1}/3, & l = odd \\ f_{2n}, & l = 2n = even \end{cases} \tag{43}$$

with

$$f_{2n} = \frac{1}{2^{2n}} \sum_{\kappa=0}^{n} (-)^{\kappa} \frac{(4n-2\kappa)!}{\kappa!(2n-\kappa)!(2n-2\kappa)!(2n-2\kappa+2)} \tag{44}$$

We thus get

$$J(\mathbf{v}_E, \mathbf{v}_0) = 2\pi(\sqrt{\pi}v_0)^{-3} \int_0^{\infty} dv \frac{v^3}{\sqrt{< v^2 >}} e^{-(v^2+v_E^2)/v_0^2} \sigma(v) \tag{45}$$
$$\times \sum_{l=0}^{\infty} (2l+1) i_l(2vv_E/v_0^2) \tilde{f}_l P_l(cos\alpha)$$

Introducing

$$\delta = \frac{2v_E}{v_0}, \qquad \psi = \frac{v}{v_0} \tag{46}$$

and noting that the quantity u entering the nuclear form factors eq. (20) can be written as

$$u = u_0 \psi^2 \tag{47}$$

with

$$u_0 = \frac{1}{2}\left(\frac{2\beta_0 m_1 c^2}{(1+\eta)} \frac{b}{\hbar c}\right)^2, \qquad \beta_0 = \frac{v_0}{c} \tag{48}$$

we see that the integral $J(\mathbf{v}_E, \mathbf{v}_0)$ depends on the parameters $\delta, cos\alpha$ and u_0, i.e.

$$J(\mathbf{v}_E, \mathbf{v}_0) \rightarrow J(\delta, cos\alpha, u_0) = \frac{1}{\sqrt{6\pi}} \sum_{l=0}^{\infty} (2l+1) 2\tilde{f}_l P_l(cos\alpha) K_l(\delta, u_0) \tag{49}$$

with

$$K_l(\delta, u_0) = 2e^{-\delta^2/4} \int_0^\infty \psi^3 e^{-\psi^2} i_l(\delta\psi)\sigma(u_0\psi^2)d\psi \tag{50}$$

The dependence on the nuclear parameters is implicit in the cross section $\sigma(u_0\psi^2)$. The factor $1/\sqrt{6\pi}$ is a consequence of the fact that

$$< v_z\Theta(v_z) >= \frac{1}{\sqrt{6\pi}}\sqrt{< v^2 >} \tag{51}$$

It has been customary to use $\sqrt{< v^2 >}$ in the expression for the flux (see eq. (39)). With this factor absent the counting rate has been exaggerated. To proceed further, we must disentangle the nuclear physics dependence in $\sigma(u_0\psi^2)$. We begin with eq. (13) and take note of the extra velocity dependence of the coherent vector contribution. Then, using eqs. (12) and (49), we can write the counting rate with a target of mass m as follows:

$$\Big\langle\frac{dN}{dt}\Big\rangle = \frac{\rho(0)}{m_1}\frac{m}{Am_p}\sqrt{< v^2 >} < \Sigma >$$

with

$$\begin{aligned} < \Sigma > = & \Big(\frac{m_1}{m_p}\Big)^2 \frac{\sigma_0}{(1+\eta)^2} \\ & \Big\{A^2\Big[< \beta^2 > \Big(f_V^0 - f_V^1\frac{A-2Z}{A}\Big)^2\Big(J_0 - \frac{2\eta+1}{2(1+\eta)^2}J_1\Big) + \\ & (f_S^0 - f_S^1\frac{A-2Z}{A})^2\tilde{J}_0\Big] + \\ & \Big(f_A^0\Omega_0(0)\Big)^2 J_{00} + 2f_A^0 f_A^1\Omega_0(0)\Omega_1(0)J_{01} + \Big(f_A^1\Omega_1(0)\Big)^2 J_{11}\Big\} \end{aligned} \tag{52}$$

The quantities $\tilde{J}_0$, J_ρ, $J_{\rho\sigma}$, with $\rho, \sigma = 0, 1$, depend on u_0, δ and $cos\alpha$. If $J_{00} = J_{01} = J_{11}$, as seems to be the case for ^{207}Pb, the spin dependent expression is reduced to the familiar expression $\big[f_A^0\Omega_0(0) + f_A^1\Omega_1(0)\big]^2 J_{00}$, where the quantity in the bracket represents the spin matrix element at $q = 0$

The integrals $\tilde{J}_0$, J_ρ are obtained from the corresponding $\tilde{J}^{(0)}_{\alpha\beta}$, $J^{(\rho)}_{\alpha\beta}$ via eq. (22) for $\alpha, \beta = N, Z$. The integrals $\tilde{J}^{(0)}_{\alpha\beta}$, $J^{(\rho)}_{\alpha\beta}$ and $J_{\rho\sigma}$ are given by eqs. (15), (49) with the obvious modification of K_l, namely

$$\tilde{K}_l^{(0)} = 2e^{-\delta^2/4} \int_0^\infty \psi^3 e^{-\psi^2} i_l(\delta\psi) I_0(u_0\psi^2)d\psi \tag{53}$$

$$K_l^{(\rho)} = 2e^{-\delta^2/4} \frac{\beta_0^2}{< \beta^2 >} \int_0^\infty \psi^5 e^{-\psi^2} i_l(\delta\psi) I_\rho(u_0\psi^2)d\psi \qquad \rho = 0, 1 \tag{54}$$

$$K_{l,\rho\sigma} = 2e^{-\delta^2/4} \int_0^\infty \psi^3 e^{-\psi^2} i_l(\delta\psi) I_{\rho\sigma}(u_0\psi^2) d\psi, \qquad \rho,\sigma = 0,1 \tag{55}$$

The parameters I_ρ and $I_{\rho\sigma}$ have been defined in the previous section. The integrals (53)-(55) can only be done numerically. Since, however, the velocity of the earth around the sun is small, $v_E \approx 30Km/s$, the parameter δ is small ($\delta \approx 0.27$). Thus, to a good approximation

$$i_l(\delta\psi) \approx \frac{\delta^l \psi^l}{(2l+1)!!} \tag{56}$$

and one can limit oneself to the first two leading multipoles, $l = 0$ and $l = 1$. The integrals of eqs. (53) and (54) for these multipoles are shown in fig. 3. For the integrals of eq. (54) see ref. [39]. In order to have some idea of the dependence of the counting rate on the earth's position, we will consider cases in which the dependence of the matrix elements on the nuclear form factor can be neglected, i.e. let $I_\rho = I_{\rho\sigma} = 1$ and utilize eq. (56) to get

$$\tilde{K}_0^{(0)} = K_{0,\rho\sigma} = 1, \quad K_0^{(\rho)} = 2\frac{\beta_0^2}{<\beta^2>}, \qquad for \;\; l = 0 \tag{57}$$

$$\tilde{K}_1^{(0)} = K_{1,\rho\sigma} = \frac{\sqrt{\pi}}{4}\delta K_1^{(\rho)} = \frac{5\sqrt{\pi}}{8}\frac{\beta_0^2}{<\beta^2>}\delta, \qquad for \;\; l = 1 \tag{58}$$

Furthermore, since

$$2\tilde{f}_0 = 1, \qquad 2\tilde{f}_1 = 2/3 \tag{59}$$

we obtain

$$\begin{aligned} J_0 = J_1 &= 2\frac{\beta_0^2}{<\beta^2>}(1 + \frac{5\sqrt{\pi}}{8}\delta\cos\alpha)/\sqrt{6\pi} \\ &= \frac{4}{3}(1 + \frac{5\sqrt{\pi}}{8}\delta\cos\alpha)/\sqrt{6\pi} \end{aligned} \tag{60}$$

$$\tilde{J}_0 = J_{00} = J_{01} = J_{11} = (1 + \frac{\sqrt{\pi}}{2}\delta\cos\alpha)/\sqrt{6\pi} \tag{61}$$

We notice that, the amplitude of the oscillatory term of the leading contribution (eq. (60) is

$$amplitude \;\; of \;\; oscillation = \frac{\sqrt{\pi}}{2}\delta \approx 0.24$$

i.e.

$$\left\langle\frac{dN}{dt}\right\rangle = \left(\frac{dN}{dt}\right)_{pr}(1 + 0.24cos\alpha)/\sqrt{6\pi}$$

where $(dN/dt)_{pr}$ represents the previous estimates for the counting rate (ignoring the form factor dependence).

The corresponding amplitude of oscillation in eq. (61) is a bit bigger ($\approx$ 0.30). However, this coherent vector contribution is suppressed due to the Majorana nature of LSP (through the factor $< \beta^2 >$). The folding improves this contribution by a factor of 4/3.

The folding procedure can be applied in the differential rate (by convoluting the relevant expression before the angular integration of the differential cross section $d\sigma/d\Omega$) . The resulting expressions are, however, a bit more complicated and they will not be given here.

5 Results and discussion

In this paper we have calculated the event rate for LSP-nucleus scattering for a number of experimentally interesting spin zero nuclear targets (coherent scattering) as well as the spin such contribution in the case of ^{207}Pb. The coherent scattering depends on the isoscalar scalar, f_S^0, and vector, f_V^0, patameters. The latter is effectively multiplied by the average velocity $\left\langle \beta^2 \right\rangle^{1/2}$ of the LSP due to its Majorana nature. These parameters were evaluated in the allowed SUSY paremeter space of Kane et al. [5]. The construction of the scalar parameters suffers from additional uncertainties, which involve the step of going from the quark to the nucleon level. In other words, the results are very sensitive to the presence of quarks other than u and d in the nucleon. Three such choices indicated by A, B, C are presented in table 1.

The second ingredient is the nuclear structure input. The coherent scattering does not depend on the details of the nuclear wave function. It does, however, depend on the nuclear density, i.e. the assumed form factor, for fairly massive LSP and correspondingly heavy nuclei. This is because the momentum transfer in such cases can be quite high (by nuclear standards) even though the energy transfer is small. The used form factors were as realistic as possible (see tables 3 and 4). The inclusion of the form factor results in sizable retardation of the cross-section which can be up to 40% for Ge and 85% for Pb. The above results were folded with the velocity distribution, which was assumed to be Maxwellian relative to its center. This folding tends to reduce the form factor retardation. We find that

$$\left\langle \frac{dN}{dt} \right\rangle = \left\langle \frac{dN}{dt} \right\rangle_0 (1 + hcos\alpha) \qquad (62)$$

where the angle α describes the Earth's annual motion. The parameters $\left\langle \frac{dN}{dt} \right\rangle_0$

Table 6 : The quantity $\frac{dN}{dt}$ in $y^{-1}Kg$ and the parameter h (oscillation due to the earth's motion around the sun) for the coherent vector and scalar contributions. For the definition of A, B, C, see text. NFM and WNFM stand for Nuclear Form Factor and Without Nuclear Form Factor respectively.

	Case	Vector		Scalar			
		$\frac{dN}{dt}$	h	$\frac{dN}{dt}$			h
	Model			A	B	C	
Pb	$\#1(NFM)$	0.958×10^{-3}	0.30	0.597×10^{-3}	0.880	1.802	0.24
	$\#1(WNFM)$	0.747×10^{-4}	0.24	0.817×10^{-4}	0.121	0.247	0.18
	$\#2(NFM)$	0.804×10^{-3}	0.30	0.238×10^{-1}	82.9	74.6	0.24
	$\#2(WNFM)$	0.497×10^{-3}	0.27	0.143×10^{-1}	44.9	40.4	0.25
	$\#3(NFM)$	0.244×10^{-2}	0.30	0.556×10^{-3}	12.1	19.1	0.24
	$\#3(WNFM)$	0.245×10^{-4}	0.24	0.978×10^{-4}	2.30	3.36	0.18
Ge	$\#1(NFM)$	0.115×10^{-3}	0.30	0.715×10^{-4}	0.106	0.216	0.24
	$\#1(WNFM)$	0.417×10^{-4}	0.28	0.350×10^{-4}	0.052	0.106	0.20
	$\#2(NFM)$	0.192×10^{-2}	0.30	0.570×10^{-2}	19.9	17.9	0.24
	$\#2(WNFM)$	0.148×10^{-3}	0.27	0.470×10^{-2}	16.5	14.9	0.23
	$\#3(NFM)$	0.342×10^{-3}	0.30	0.818×10^{-4}	1.69	2.67	0.24
	$\#3(WNFM)$	0.126×10^{-3}	0.32	0.437×10^{-4}	0.900	1.43	0.21

and h are shown in table 6 for two typical and experimentally interesting nuclei, ^{72}Ge and ^{207}Pb. From this table we see that, the event rate is highest when the LSP is lightest ($m_1 = 27GeV$, case #2). We notice that, even within the allowed parameter space, the event rate may vary by two orders of magnitude. We also notice that, with the possible exception of the not so realistic model A, the vector contribution is negligible, i.e. the Higgs contribution becomes dominant. This is even more so in models where quarks other than u and d are present in the nucleus with appreciable probabilities, due to their large masses. In the most favorable cases, one may have more than 80 events per year per kilogram of target. We notice that, the amplitude due to the Earth's annual motion can change by 20-25%. Finally, we should mention that, for cases #1 and #3 (massive LSP), the rate due to the nuclear form factor can be reduced by a factor of approximately 6. Because the form factor dependence is more pronounced in heavy nuclei, there is no advantage in going to the heavier targets if the LSP turns out to be massive.

Next we come to the discussion of the spin matrix contribution. As we have mentioned in the introduction, the relative importance of the spin matrix element is expected to decrease as one goes to the heavier systems vis a vis the coherent scattering. We have seen, however, that the increase due to the mass number in the coherent scattering is offset by the decline due to form factor. Similarly, ^{207}Pb can be adequately described as a single neutron hole in the doubly magic closed shell ^{208}Pb nucleus. Thus, the retardation of the static spin matrix element may be less dramatic than in the light systems. Furthermore, because of the fact that more than one multipole may contribute, the retardation due to the form factor may be less dramatic.

We find that indeed the ^{207}Pb ground state, in a 1h and 2h-1p model space is more than 95% a $2p_{1/2}$ neutron configuration. We have evaluated the spin matrix elements up to terms linear in the small amplitudes. Out of the 250 components, only two of them contribute to the $\lambda = 0$ multipole while two more contribute to the $\lambda = 2$ multipole. Thus, we find that, the isoscalar static matrix element suffers from very little retardation, while the isovector matrix element is reduced by 17%. Since, in the parameter space we are using, the isovector coupling f_A^1 is larger this results in a similar retardation of the total matrix element. The isoscalar axial coupling f_A^0 has more uncertainties. In passing from the quark to the nucleon level the amplitude must be multiplied by a factor g_A which ranges from 1, in the Naive Quark Model (NQM), to .12, if extracted from the EMC data. In our calculations we considered both of these extremes. Also, since the dominant configuration is of the neutron variety, the isoscalar tends to subtract from the isovector, but this is not so dramatic in our case since the isoscalar is smaller in absolute value, especially in the EMC case.

The momentum dependence can be described in terms of three integrals I_{00}, I_{01}, I_{11}, where the subscripts indicate the isospin channels. In our case, these integrals receive contributions from two multipoles, $\lambda = 0$ (spin monopole) and $\lambda = 2$ (spin-quadrupole). They were judiciously normalized to their static value (unity). When so normalized these integrals are approximately equal (see fig. 3 and also ref. [39]). We notice that, the monopole contribution falls with momentum transfer, as expected, quite fast in fact for large LSP mass. On the contrary, the quadrupole contribution starts out at zero and keeps increasing up to about 90 GeV. As a result its contribution in this mass regime is crucial since it tends to partly compensate for the supression of the monopole term. We expect these trends to persist even in the most elaborate calculations which include the Giant Quadrupole Resonance (GQR) and are currently under way.

Table 7 : The quantity $\frac{dN}{dt}$ in $y^{-1}Kg$ and the parameter h (oscillation due to the earth's motion around the sun) for the spin contribution in the LSP nucleus scattering in ^{207}Pb. NFM and WNFM stand for nuclear Form Factor and Without Nuclear Form Factor respectively.

Case	$\frac{dN}{dt}$	h	$\frac{dN}{dt}$	h
	EMC	EMC	NQM	NQM
$\#1(NFM)$	0.365×10^{-2}	0.24	0.158×10^{-2}	0.24
$\#1(WNFM)$	0.120×10^{-2}	0.27	0.523×10^{-3}	0.19
$\#2(NFM)$	0.168×10^{-1}	0.24	0.110×10^{-2}	0.24
$\#2(WNFM)$	0.728×10^{-1}	0.22	0.800×10^{-3}	0.23
$\#3(NFM)$	0.135×10^{-1}	0.24	0.646×10^{-2}	0.24
$\#3(WNFM)$	0.540×10^{-3}	0.20	0.253×10^{-2}	0.21

ref. [5], barring unforseen lack of retardation of the spin matrix element in a light nucleus, the spin induced LSP-nucleus scattering may not be detectable.

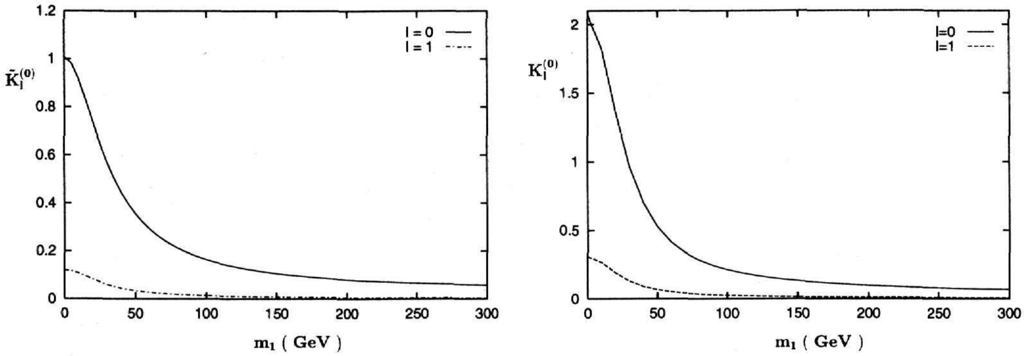

Figure 3: The quantities $\tilde{K}_l^{(0)}$ and $K_l^{(\rho)}$ (for l=0 and l=1) entering the event rates due to earth's revolution around the sun. For their definition see sect. 4, eqs. (53) and (54), in the text.

6 Conclusions

In the present work we have performed calculations of the event rate for LSP-nucleus scattering for two typical experimentally interesting nuclei, i.e. Pb and Ge. The three basic ingredients of our calculation were the input SUSY parameters, a quark model for the nucleon and the structure of the nuclei involved. The input SUSY parameters were calculated in a phenomenologically allowed parameter space (cases #1, #2, #3 of table 1) as explained in the text. In going from the quark to the nucleon level the quark structure of the nucleon was essential. In particular its content in quarks other than u and d. For the scalar interaction we considered three models (labeled A, B, C in table 1) as described in the text. For the isovector axial coupling one encounters the so-called nucleon spin crisis. Again we considered two possibilities depending on the assumed portion of the nucleon spin which is due to the quarks (indicated by EMC and NQM in table 1) as described in the text. As regards nuclear stucture we employed as detailed as feasible nuclear wave functions. For the coherent part (scalar and vector) we used realistic nuclear form factors. For the ^{207}Pb system we also computed the spin matrix element. The ground state wave function was obtained by a diagonalizing the nuclear Hamiltonian in a 2h-1p space which is standard for this doubly magic nucleus. The momentum dependence of the matrix elements was taken into account and all relevant multipoles were retained (in this system one encounters only two, $l = 0$ and $l = 2$ due to selection rules). Finally, the obtained results were convoluted with a suitable Maxwell-Boltzmann velocity distribution of the LSP's. This convolution was necessary to partially neutralize the form factor retardation, but mainly to compute the modulation of the event rate due to the Earth's motion. We find that in almost all cases the event rate due to the Earth's revolution around the sun can change from -25% to +25% around its average value. Given enough counts this is a significant effect which can be used to discriminate against background. The event rates thus obtained are listed in tables 6 and 7. Unfortunately, the obtained results are sensitive to the input parameters. The inclusion of the nuclear form factor significantly retards the event rates for heavy nuclei ($A > 100$) and fairly massive LSP ($m_1 > 100GeV$). However, this retardation does not outweight completely the advantages of using a heavy target. For the spin matrix elements the form factor retardation of the usual $l = 0$ multipole is partially neutralized by the higher multipoles. From the data of tables 6 and 7 we see that it is possible to have detectable rates ($>$ 20 per kilogramme per year) for case #2 and the realistic nucleon models B and C, resulting from the scalar Higgs exchange term. In all other cases, including the spin contribution, the calculated event rates are too small.

Acknowledgements: This work was partially supported by the grants PENED91 and PENED95 of the Greek Secretariat for Research. The hospitality of the University of Cyprus to one of us (J.D.V.) is also happily acknowledged.

Appendix

In the case when $\lambda = 2$, the operator is written as

$$T^{\kappa} = \sqrt{4\pi} j_{\lambda}(qr) \left[Y^{\lambda}(\hat{\mathbf{q}}) \otimes \sigma \right]^{\kappa} \tag{63}$$

For ^{207}Pb we are interested in calculated the matrix elements of this operator in the case $\kappa = 1$ and $\lambda = 0, 2$ for the states

$$|(2p_{1/2})^{-1} >, \qquad C_{j_1 j_2} \left| \left[2p_{1/2})^{-1} j_1^{-1} \right] J; j_2; 1/2 > \tag{64}$$

or the matrix elements

$$\begin{aligned} ME = C_0^2 \Big[& \pm < (2p_{1/2})^{-1} || T^{\kappa} || (2p_{1/2})^{-1} > + \\ & \pm 2 \sum_{j_1 j_2} C_{j_1 j_2} < (2p_{1/2})^{-1} || T^{\kappa} || (2p_{1/2})^{-1} [j_1^{-1} j_2]\kappa; 1/2 > \end{aligned} \tag{65}$$

where the $+$ sign is for isoscalar and the $-$ for isovector matrix elements.

The reduced matrix elements $< j_1 || T^{\kappa} || j_2 >$ are given in ref. [40]. The relevant radial matrix elements $< n_1 l_1 | j_l(qr) | n_2 l_2 >$ for harmonic oscillator basis can be witten in the compact way

$$< n_1 l_1 | j_l(qr) | n_2 l_2 > = \chi^{l/2} e^{-\chi} \sum_{\kappa=0}^{\kappa_{max}} \varepsilon_{\kappa} \chi^{\kappa}, \qquad \chi = (qb)^2/4 \tag{66}$$

where

$$\kappa_{max} = n_1 + n_2 + m, \qquad m = (l_1 + l_2 - l)/2$$

The coefficients $\varepsilon_{\kappa}(n_1 l_1, n_2 l_2, l)$ are given by

$$\varepsilon_{\kappa} = \left[\frac{\pi\, n_1! n_2!}{4\, \Gamma(n_1 + l_1 + \frac{3}{2}) \Gamma(n_2 + l_2 + \frac{3}{2})} \right]^{\frac{1}{2}} \sum_{\kappa_1 = \phi}^{n_1} \sum_{\kappa_2 = \sigma}^{n_2} n!\, \Lambda_{\kappa_1}(n_1 l_1) \Lambda_{\kappa_2}(n_2 l_2) \Lambda_{\kappa}(nl) \tag{67}$$

where $n = \kappa_1 + \kappa_2 + m$ and

$$\phi = \begin{cases} 0, & \kappa - m - n_2 \leq 0 \\ \kappa - m - n_2, & \kappa - m - n_2 > 0 \end{cases}$$

$$\sigma = \begin{cases} 0, & \kappa - m - \kappa_1 \leq 0 \\ \kappa - m - \kappa_1, & \kappa - m - \kappa_1 > 0 \end{cases}$$

For some spacial cases used in the present work see table 5.

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