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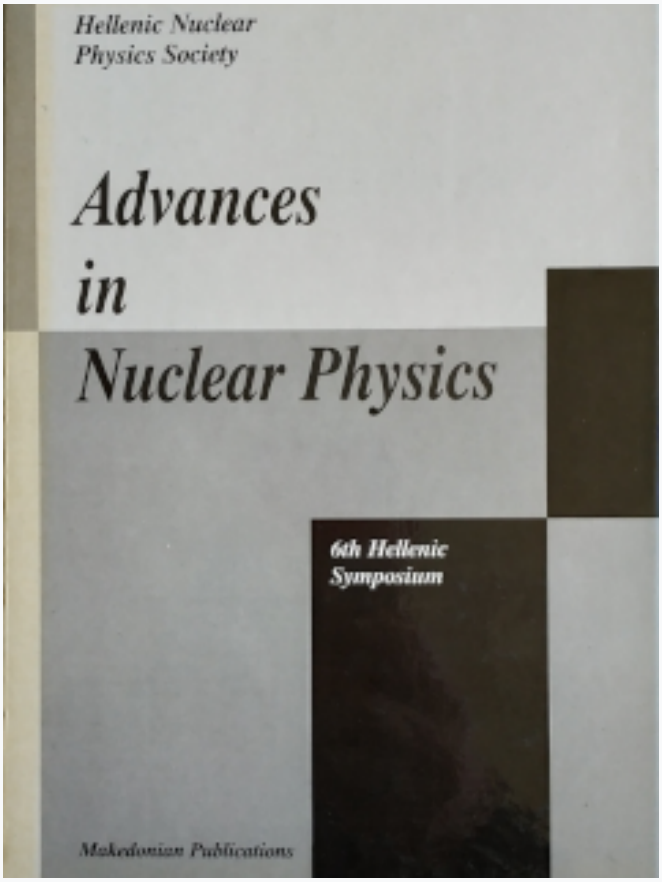

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Binding energies of hypernuclei and hypernuclear interactions

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Abstract

In part 1 the effect of nuclear core dynamics on the binding energies of Λ hypernuclei is discussed in the framework of variational correlated wave functions. In particular, we discuss a new rearrangement energy contribution and its effect on the core polarization. In part 2 we consider the interpretation of the Λ single-particle energy in terms of basic Λ-nuclear interactions using a local density approximation based on a Fermi hypernetted chain calculation of the Λ binding to nuclear matter. To account for the data strongly repulsive 3-body ΛNN forces are required. Also in this framework we discuss core polarization for medium and heavier hypernuclei.

1 Nuclear Core Dynamics and Rearrangement Energy

1.1 Introduction

To introduce the problem we consider the simple and very old effective-interaction model of the binding of a Λ to its core nucleus (consisting of A-1 nucleons) to make a hypernucleus (HN) of baryon number A. To be specific we focus on the hypernucleus ${}^{5}_{\Lambda}He \equiv {}^{4}He + \Lambda$ for which the experimental Λ-separation energy is 3.12 MeV. For. simplicity, in Part 1 we consider mostly a central 2-body ΛN potential $V_{\Lambda N}$ with a repulsive core, although for a realistic description 3-body ΛNN forces are required. From $V_{\Lambda N}$ one may obtain an effective interaction $\tilde{V}_{\Lambda N}$ which is smooth and (approximately) takes account of ΛN correlations (mostly due to the repulsive core). $\tilde{V}_{\Lambda N}$ can be obtained in various ways. Thus $\tilde{V}_{\Lambda N} = t_{\Lambda N}$ where $t_{\Lambda N}$ is some appropriate reaction matrix,

or more appropriately for a variational approach we can use

$$\tilde{V}_{\Lambda N} = f_{\Lambda N}^2[V_{\Lambda N} - \frac{\hbar^2}{4\mu_{\Lambda N}}\nabla^2 ln f_{\Lambda N}], \tag{1}$$

where $\mu_{\Lambda N}$ is the ΛN reduced mass and $f_{\Lambda N}$ is an appropriate correlation function, obtained e.g. from a nuclear matter calculation. The second term, the so-called induced kinetic energy arises from the action of the K.E. operator on $f_{\Lambda N}$ which depends on the nuclear coordinates. The Λ-core nucleus potential is then obtained from

$$V_{\Lambda c}(r) = \int \tilde{V}_{\Lambda N}(|\vec{x} - \vec{r}|)\rho_c(r; R)d^3x, \tag{2}$$

where ρ_c is the core density which is obtained from the core wave function Ψ. For brevity we represent this dependence on Ψ through a generic variational parameter R which emphasizes the importance of the core radius. The resulting single-particle Λ energy $\epsilon_\Lambda(R)$ is then obtained from the Λ-core Schrödinger equation for the potential $V_{\Lambda c}$. If distortion of the core by the Λ is neglected, and if the isolated core corresponds to the value R_0, then the Λ separation energy is given by

$$B_\Lambda = -\epsilon_\Lambda(R_o). \tag{3}$$

Thus the single-particle energy $\epsilon_\Lambda(R)$ gives the Λ binding for a fixed core of radius R; ϵ_Λ is expected to decrease slowly and smoothly as R decreases.

To include the response of the core to the Λ (core polarization) we write, plausibly, for the total energy of the HN,

$${}^A_\Lambda E(R) = \epsilon_\Lambda(R) + {}^{A-1}E(R) \tag{4}$$

where

$${}^{A-1}E(R) = < \Psi|H_N|\Psi > / < \Psi|\Psi > \tag{5}$$

is energy of the core whose Hamiltonian is H_N. The minimum E_m of ${}^{A-1}E(R)$ at R_o corresponds to the ground state of the isolated core nucleus. In the presence of the Λ this minimum is shifted to ${}^{A-1}E(R_\Lambda) > {}^{A-1}E(R_o)$, where R_Λ represents the core polarized by the Λ. Since $\epsilon'_\Lambda \equiv d\epsilon_\Lambda/dR > 0$, the HN energy will decrease as the core contracts and thus $\Delta R = R_\Lambda - R_o < 0$; physically: the Λ drives the core to a smaller size. Thus with a quadratic approximation for ${}^{A-1}E$:

$${}^{A-1}E(R) \simeq {}^{A-1}E(R_o) + \frac{1}{2}C(R - R_o)^2 \tag{6}$$

one obtains

$$\Delta R \equiv R_\Lambda - R_o \simeq -\epsilon'_\Lambda(R_o)/C \tag{7}$$

$$\Delta E \equiv {}^{A-1}E(R_\Lambda) - {}^{A-1}E(R_o) \simeq \frac{1}{2}[\epsilon'_\Lambda(R_o)]^2/C \tag{8}$$

where ΔE is the core polarization energy. C is a measure of the nuclear response (and for a heavy nucleus is related to the incompressibility constant K through $C \simeq 2AK/R_o^2$). The Λ separation energy is then

$$-B_\Lambda = {}^{A}_{\Lambda}E(R_\Lambda) - {}^{A-1}E(R_o) = \epsilon_\Lambda - \Delta E < \epsilon_\Lambda. \tag{9}$$

Thus B_Λ increases by ΔE as a result of core polarization. Estimates for ${}^{5}_{\Lambda}He$ give $\Delta E \simeq 0.1MeV$, $\Delta R \simeq -0.05fm$. We will show that this intuitive approach to the polarization energy is essentially correct in the absence of ΛN correlations, i.e. if $\tilde{V}_{\Lambda N} \simeq V_{\Lambda N}$. However, with correlations there is a rearrangement-energy contribution E_R involving the core Hamiltonian H_N, and which depends on the ΛN correlations and on the difference between Ψ and the exact isolated core wave function Φ_N. The presence of E_R leads to qualitative changes in the core polarization mechanism.

1.2 Λ Separation Energy and Rearrangement Energy

We shall now sketch a rigorous treatment, in a variational context, of the effect of the core dynamics on B_Λ. The total Hamiltonian of the HN is

$$H = H_\Lambda + H_N \tag{10}$$

where

$$H_\Lambda = T_\Lambda + \sum_{i=1}^{A-1} V_{\Lambda N_i} \tag{11}$$

is the Λ Hamiltonian in an obvious notation. The nuclear Hamiltonian is

$$H_N = T_N + V_N \tag{12}$$

$$T_N = \sum_{i=1}^{A-1} T_i\,, \quad V_N = \sum_{i<j} V_{N_i N_j}. \tag{13}$$

The variational wave function is of the Jastrow type

$${}^{A}_{\Lambda}\Psi = F\Psi \tag{14}$$

$$F = \prod_{i=1}^{A-1} f_{\Lambda N_i}\,, \quad f_{\Lambda N_i} = f_{\Lambda N}(|\vec{x}_\Lambda - \vec{x}_i|), \tag{15}$$

where we ignore spin dependent and other correlations which are inessential for the present discussion. Ψ is the core-nucleus wave function which we do

not need to specify further at this point. For the exact (ground state) core wave function Φ_N and the exact ground state energy E_N we have

$$H_N \Phi_N = E_N \Phi_N . \tag{16}$$

The total HN energy is

$$^{A}_{\Lambda}E[\Psi] = < F\Psi | H_\Lambda + (T_N + V_N) | F\Psi > / < F\Psi | F\Psi >, \tag{17}$$

where $[\Psi]$ emphasizes the functional dependence on the core wave function. Now $T_N F \neq F T_N$ because $f_{\Lambda N}$ has a dependence on the nuclear coordinate. Then by partial integrations

$$< F\Psi | T_N | F\Psi > = < F\Psi | T_N^{(\Lambda)} | F\Psi > + < F^2\Psi | T_N | \Psi > . \tag{18}$$

Here $T_N^{(\Lambda)}$ is an "induced" (by the Λ) nucleon kinetic energy due to the ΛN correlations.

$$^{A}_{\Lambda}E = \epsilon_\Lambda + < F^2\Psi | H_N | \Psi > / < F\Psi | F\Psi >, \tag{19}$$

where

$$\epsilon_\Lambda = < F\Psi | H_\Lambda + T_N^{(\Lambda)} | F\Psi > / < F\Psi | F\Psi > \tag{20}$$

is an exact expression (for a wave function of the form of Eq. (14)) for the effective single-particle Λ energy considered in the introduction. (In the lowest order of the cluster expansion one obtains Eq. (1)). We write

$$\Psi = \Phi_N + \delta\Phi_N . \tag{21}$$

Thus $\delta\Phi_N$ is the difference between Ψ and the exact core wave function. After some manipulations which makes use of Eq. (16) we obtain

$$^{A}_{\Lambda}E = \epsilon_\Lambda + {}^{A-1}E + E_R \tag{22}$$

where

$$^{A-1}E = < \Psi | H_N | \Psi > / < \Psi | \Psi > \tag{23}$$

is the core energy corresponding to Ψ. The rearrangement energy E_R is

$$\begin{aligned} E_R = & < F^2\Phi_N | (H_N - E_N) | \delta\Phi_N > / < F\Psi | F\Psi > + \\ & < F^2\delta\Phi_N | (H_N - E_N) | \delta\Phi_N > / < F\Psi | F\Psi > - \\ & < \delta\Phi_N | (H_N - E_N) | \delta\Phi_N > / < \Psi | \Psi > . \end{aligned} \tag{24}$$

Thus the exact expression Eq. (22) for $^{A}_{\Lambda}E$ differs from Eq. (4) by the additional term E_R.

It is convenient to define an effective single-particle E_Λ with respect to the core energy for a core wave function Ψ:

$$E_\Lambda[\Psi] \;=\; {}^{A}_{\Lambda}E[\Psi] \;-\; {}^{A-1}E[\Psi] \;=\; \epsilon_\Lambda + E_R. \tag{25}$$

The total HN and the Λ separation energies are then (the Ψ dependence is from now on implicit)

$${}^{A}_{\Lambda}E \;=\; E_\Lambda \;+\; {}^{A-1}E \tag{26}$$

$$-B_\Lambda \;=\; {}^{A}_{\Lambda}E - E_N \;=\; E_\Lambda + \Delta E_N \;>\; E_\Lambda \tag{27}$$

$$\;=\; \epsilon_\Lambda \;+\; < F^2\Phi_N|(H_N - E_N)|\delta\Phi_N > / < F\Psi|F\Psi >, \tag{28}$$

where

$$\Delta E_N \;=\; {}^{A-1}E[\Psi] - E_N \tag{29}$$

is the core polarization energy (for Ψ). The separation energy $\tilde{B}_\Lambda$ with respect to the variational minimum

$$E_m \;= < \Psi_m|H_N|\Psi_m > / < \Psi_m|\Psi_m > \;> E_N \tag{30}$$

is

$$\tilde{B}_\Lambda \;=\; B_\Lambda + (E_m - E_N) \;\geq B_\Lambda \tag{31}$$

thus trivially $B_\Lambda \leq \tilde{B}_\Lambda$.

If we compare Eq. (22) with Eq. (4) we see from the discussion following the latter, that the driving effect of the Λ in distorting the core is in fact measured by $E'_\Lambda = \epsilon'_\Lambda + E'_R$ where $'$ denotes differentiation with respect to the appropriate variational parameter (denoted generically by R in the introduction) at the core wave function optimum.

We summarize some general properties of E_R obtained from Eq. (24). Numerical results will be discussed in Section 1.3.

1. $E_R \simeq 0$ if $F = \phi_\Lambda(\vec{r})$ corresponds to a single-particle Λ-core wave function, where $\vec{r}$ is the Λ coordinate with respect to the center-of-mass of the core. Thus if there are no ΛN correlations and if the core is held fixed, then as can be seen from Eq. (24) by intergrating over $\vec{x}_\Lambda$ one obtains $E_R = 0$. Recoil of the core nucleus could give some small value for E_R. A single-particle approximation for the Λ, represented by ϕ_Λ, thus brings us back - not surprisingly - to the situation represented by Eq. (4) and discussed in 1.1.

2. $E_R = 0$ if $\delta\Phi_N = 0$, i.e. if $\Psi = \Phi_N$ is the exact ground-state wave function of the core. This is obvious from Eq. (24).

3. Equation (24) shows that for "small" $\delta\Phi_N$ one expects $E_R \propto \delta\Phi_N$. Thus we expect E_R to change sign as $\delta\Phi_N$ changes sign.

4. Even with both ΛN correlations and $\delta\Phi_N \neq 0$ one can have $E_R = 0$ for some particular value of $\delta\Phi_N$ because of the nonlinear terms in E_R.

5. For infinite (translationally invariant) nuclear matter, it is straightforward to show that $E_R \equiv 0$. This arises because the Λ does not cause any change in the nucleonic part of the wave function.

Thus in general $E_R \neq 0$ only if there are ΛN correlations ($F \neq \phi_\Lambda$) and if $\delta\Phi_N \neq 0$. We can have $\delta\Phi_N \neq 0$, and therefore $E_R \neq 0$ with a correlated F, either if the optimum core wave function Ψ is in error and/or if some external agency, such as the Λ distorts the core wave function from its exact ground-state value Φ_N. This latter change is just core polarization by the Λ and will be illustrated by our numerical results. Also property 5 implies that because $E_R = 0$ for nuclear matter, E_R is expected to become relatively less important for large A and thus that the finite nuclear size is an essential ingredient for $E_R \neq 0$.

1.3 Numerical Studies for ${}^{5}_{\Lambda}He$

We have studied and illustrated our general considerations by calculations for ${}^{5}_{\Lambda}He$. For our NN potential, we mostly used a Mafliet-Tjon (M-T) central spin-independent potential[4] which we used extensively in earlier work[5]. We also made calculations for the central and spin-independent Afnan-Tang (A-T) S3 NN potential[6] which gives a somewhat better value for the ${}^4He$ energy and radius. We use a central spin-independent ΛN potential $V(r)$ with a repulsive core V_c and a two-pion attractive part $V_{2\pi} = -V_o T_\pi^2$ where T_π is the OPE tensor shape factor (Refs 5,7). For most of our (illustrative) calculations we used $V_o = 6.2 MeV$ which gives agreement with the ΛN scattering. For the core wave function (appropriate for a central spin-independent V_{NN}) we use a

Jastrow function

$$\Psi = \prod_{i<j}^{4} f_{N_i N_j}(r_{ij}). \tag{32}$$

For a more realistic V_{NN} the appropriate correlation operators must be included. The variational HN wave function is then given by Eq. (14) with F given by Eq. (15). The specific variational Monte Carlo (VMC) method as applied to the s-shell HN is described in detail in Ref. [5]. The correlation functions f_{NN} and $f_{\Lambda N}$ each depend on a number of variational parameters (through effective potentials in Schrödinger-type equations for the fs). We concetrate on the two most significant parameters (for central V_{NN}). These are κ_N, κ_Λ and s_N, s_Λ respectively for f_{NN}, $f_{\Lambda N}$. Here κ^{-1} is a length parameter determining the asymptotic behavior of $f(\sim e^{-\kappa r})$ and s is a potential strength parameter multiplying the attractive parts of V_{NN}, $V_{\Lambda N}$ in the effective Schrödinger equation for f. Since our emphasis is on the core wave function we focus mostly on κ_N, s_N. Optimization with respect to κ_Λ, s_Λ is implied.

Our variational results (VMC) for ${}^4He$ give ${}^4E = -31.40 \pm .03 MeV$ (M-T) and $-27.30 \pm .03 MeV$ (A-T). This is to be compared with the exact Green function MC (GFMC) results: $-31.36 \pm .01 MeV$ (M-T) and $-27.35 \pm .01 MeV$ (A-T). Within errors our VMC results agree with the GFMC results indicating that our VMC wave function is quite close to the exact wave function: $\Psi \simeq \Phi_N$ for both M-T and A-T potentials.

For ${}^5_\Lambda He$ a "brute force" VMC calculation of B_Λ requires calculation of the minimum of ${}^5_\Lambda E$ by optimization with respect to the parameters of both $f_{\Lambda N}$ and f_{NN}. Then $-B_\Lambda = {}^5_\Lambda E_{min} - {}^4E_{min}$ where ${}^4E_{min}$ are the above VMC values.

The VMC values of B_Λ are $7.05 \pm .05 MeV$ (M-T) and $6.70 \pm .05 MeV$ (A-T). The GFMC value (only for M-T) is $7.20 \pm .015 MeV$. We note the expected overbinding obtained for ΛN and NN potentials, implying the need for repulsive ΛNN forces (Refs. 5,9). The smaller B_Λ obtained for the A-T potential we attribute to the somewhat larger ${}^4He$ size obtained with this potential.

We now discuss the dependence of our results on κ_N and s_N; for each set of values we have optimized with respect to κ_Λ, s_Λ. We show the energies: ${}^4E$ (Eq. (23)), ϵ_Λ (Eq. (20)), $E_\Lambda = {}^4_\Lambda E - {}^4E$, $E_R = E_\Lambda - \epsilon_\Lambda$; ${}^4_\Lambda E$ is obtained from Eq. (17) as just discussed.

Figure 1 for the M-T potential shows the dependence on κ_N for $s_N = 1$ (increasing κ_N corresponds to decreasing core size). We see a nice confirmation of the general properties discussed previously. ${}^4E$ has the expected quadratic dependence near the minimum ($-31.4 MeV$) which occurs at $\kappa_N = 0.323 fm^{-1}$. At the minimum: $\epsilon_\Lambda = E_\Lambda = -B_\Lambda$ and thus $E_R = 0$, providing confirmation

of the goodness of our variational core wave function: $\Psi \simeq \Phi_N$. As expected ϵ_Λ decreases slowly with κ_N, i.e. with contraction of the core, and $\epsilon'_\Lambda > 0$. However, E_R increases with κ_N and at a rate $|E'_R| > |\epsilon'_\Lambda|$ such that $E_\Lambda = E_R + \epsilon_\Lambda$ also increases with κ_N. Since it is E_Λ which drives the core, this has the unexpected result that the core for the HN has expanded relative to the isolated core - contrary to the intuitive consideration of the introduction. Thus, the optimum $B_\Lambda = 7.05 MeV$ for the HN occurs at $\kappa_N = 0.30 fm^{-1}$ corresponding to a larger core size than for the isolated core. The increase of B_Λ from its value at the core optimum is $\Delta B_\Lambda = 7.05 - 6.70 \simeq 0.35 MeV$ which is just the core polarization energy ΔE_N and is quite small. The results for the A-T potential, not shown, are quite similar with a polarization energy $\Delta E_N = 0.4 MeV$. The values of E_R for the optimum B_Λ are $-0.75 MeV$ (M-T) and $-1.0 MeV$ (A-T).

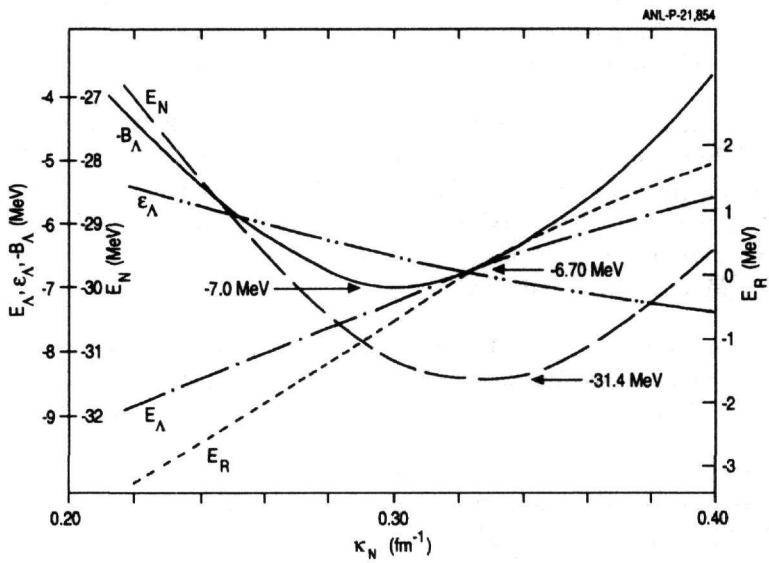

Fig. 1: Core and hypernuclear energies for the M-T potential vs. the core variational parameter κ_N for $s_N = 1$.

Our results clearly exhibit the dynamics of core polarization which because of the presence of E_R is qualitatively different from the intuitive picture considered in the introduction where the core-polarization driving force is only the single-particle energy ϵ_Λ. Although complete and reliable calculations for realistic V_{NN} are required, we do not expect the core polarization energy ΔE_N to be very different, at least in order of magnitude, from the value $\simeq 0.4 MeV$

obtained for the M-T and A-T potentials. This has very recently been confirmed by Murali and Usmani[12]. They obtain $\Delta E_N \simeq 0.2-0.6 MeV$ depending on V_{NN}. Calculations which give much larger values for ΔE_N would have to be considered suspect.

The linear change of E_R, or equivalently of $E_\Lambda - \epsilon_\Lambda$ about the optimum core wave function, with $E_R = 0$ at this optimum, suggests an intriguing use of the HN as a probe of the core wave function - in this case improving the accuracy of the optimum Ψ. Thus, calculation of E_Λ and ϵ_Λ for a few (2-3) value of each significant variational parameter could significantly improve the accuracy of the optimum values of these parameters by identifying the point $\epsilon_\Lambda = E_\Lambda$.

The dynamics of core polarization just discussed illustrates how a distortion of the core from its exact wave function Φ_N is associated wth a non-zero value of E_R. To discuss the effects of an error in the variational core wave function ($\Psi_m \neq \Phi_N$), we have simulated such an error by "detuning" our optimum Ψ by changing one of the parameters κ_N, s_N from its optimum and then calculating the various energies as a function of the other parameter. Figure 2 shows the case where s_N is varied with fixed $\kappa_N = 0.39 fm^{-1}$ (optimum at $0.234 fm^{-1}$). The minimum of ${}^4E$ is now $E_m = -30.45 MeV$ at $s_N = 0.9$. Now $E_R = 0$ for $s_N = 0.75$ with $\epsilon_\Lambda = E_\Lambda = -6.7 MeV$ but with $\delta\Phi_N \neq 0$ as a consequence of the nonlinear terms. However, there is now no intersection of ϵ_Λ and $\tilde{B}_\Lambda$ (the Λ separation energy with respect to E_m). The point $E_R = 0$ is quite far from E_m and there is no meaningful core response at $E_R = 0$. In this case $\tilde{B}_\Lambda = 6.2 MeV$ is well below the absolute minimum of $7.05 MeV$ and $B_\Lambda = 5.3 MeV$ (Eq. (31)) is even smaller. The plot of $\tilde{B}_\Lambda$ vs. s_N also shows that even quite small errors in the optimum $s_N(=0.9)$ can lead to quite large errors in $\tilde{B}_\Lambda$. This mostly reflects the effect of the core energy ${}^4E$ which depends strongly on s_N.

Figure 3 shows results vs. κ_N when the core wave function is "detuned" to $s_N = 1.2$ from its optimum $s_N = 1$. Although $E_m = -29.05 MeV$, at $\kappa_N \simeq .265 fm^{-1}$, is now $2.3 MeV$ greater than $E_N = -31.4 MeV$, the overall picture strongly resembles that of Fig. 1 for the optimum Ψ. The point $E_\Lambda = \epsilon_\Lambda$ occurs close to E_m but now for $\delta\Phi_N \neq 0$. For $E_R = 0$: $\epsilon_\Lambda = E_\Lambda = -6.65 MeV$. The maximum of $\tilde{B}_\Lambda = 7.0 MeV$ is again shifted to smaller κ_N (larger core) and is quite close to the optimum B_Λ; the core polarization dynamics is quite similar to that of Fig. 1.

In all the cases we considered for the M-T potential: $E_R = 0$ occurs for $E_\Lambda = \epsilon_\Lambda \simeq -(6.65 - 6.70) MeV$. Furthermore, E_R corresponds to the intersection of the rather slowly varying quantities ϵ_Λ and E_Λ which are less subject to appreciable statistical errors than ${}^{A-1}E$ and $\tilde{B}_\Lambda$.

For a more realistic V_{NN} the optimum Ψ could still differ appreciably from Φ_N because of structural inadequacies which could lead to errors even for the optimum Ψ. Perhaps as, or more important, could be statistical errors in the optimum Ψ. Our results suggest that for such more complicated wave functions, it may be better to use a less direct procedure based on E_Λ and ϵ_Λ rather than a "brute-force" optimization of ${}^A_\Lambda E$.

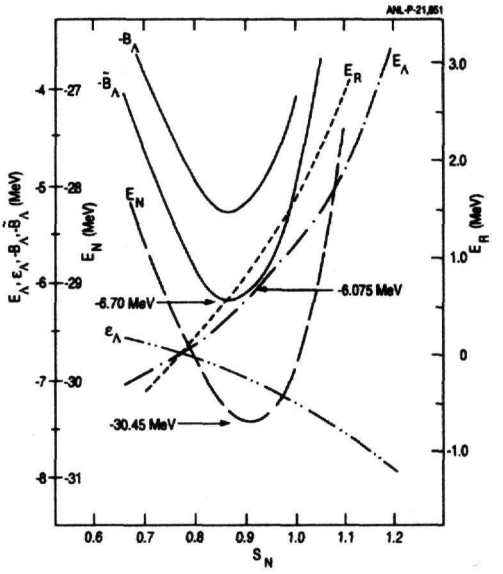

Fig. 2: Core and hypernuclear energies for the M-T potential vs. s_N for $\kappa_N = 0.39 fm^{-1}$.

Very recently calculations for more realistic NN potentials have been made: Ref. [10] for ${}^{17}O$ and Ref. [11] for ${}^5_\Lambda He$ both for Argonne v6 + U7. Reference [12] is for ${}^5_\Lambda He$ with Argonne v14 + U7/U8 (Ref. 13) and Argonne v6 + U7/U8. Here v6 is a truncated version of Argonne v14 which ignores the (last 8) L^2 and LS terms in the full Argonne v14 potential and where U7 and U8 are different 3-body NNN potentials. For ${}^5_\Lambda He$, a calculation with Argonne v6 + U8 and with the same $V_{\Lambda N}$ as used in this paper (but with $V_o = 6.16 MeV$ in $V_{2\pi}$) gives $B_\Lambda \cong 5.8 MeV$, $E_R \simeq 0.6 MeV$, $\Delta E \simeq 0.2 MeV$. These values are quite close to those found in the present paper for the much simpler M-T and A-T potentials. With a realistic V_{NN} which includes L^2 and LS terms there will be induced terms arising from these momentum dependent terms, in addition to the induced kinetic energy. Thus with Argonne v14 + U8, Murali and Usmani[12], and for the same $V_{\Lambda N}$ just quoted, obtain $B_\Lambda \simeq 5.2 MeV$,

$E_R \simeq 1.9 MeV$, $\Delta E_N \simeq 0.6 MeV$, thus giving a significantly smaller B_Λ and a significantly larger E_R than without the momentum dependent terms.

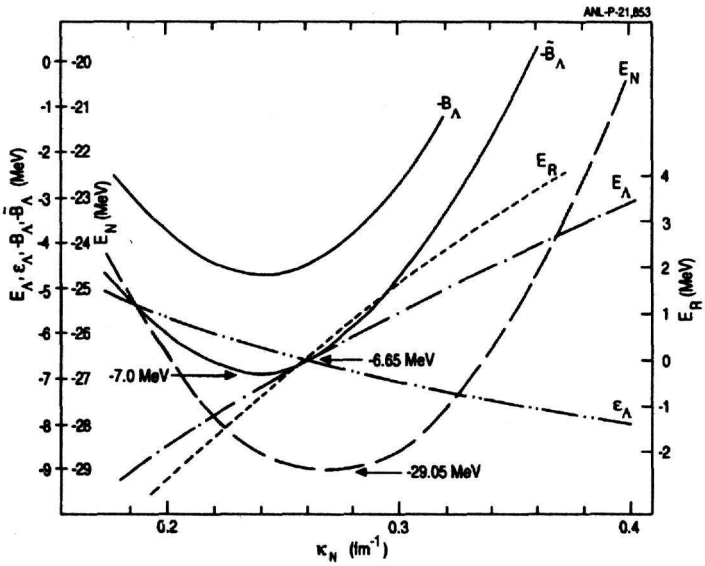

Fig. 3: Core and hypernuclear energies for the M-T potential vs. κ_N for $s_N = 1.2$.

2 Λ Single Particle Energies

2.1 Introduction

In this part, we consider the Λ single-particle (s.p.) energies obtained from the $\pi^+ + {}^A Z \rightarrow K^+ + {}^A_\Lambda Z$ reaction for a wide range of HN with baryon numbers $A \leq 81$ and for orbital angular momenta $l_\Lambda \leq 4$, Ref [14], and in particular their interpretation in terms of the basic Λ-nuclear interactions. These generate a Λ-nucleus potential which roughly follows the density distribution $\rho_c(r)$ of the core nucleus, with an approximately constant calue D_Λ in the interior. This well depth D_Λ is then identified with the Λ binding in nuclear matter at normal nuclear density ρ_o. Then the Λ separation energy is $B_\Lambda \approx D_\Lambda - T_\Lambda$ where the Λ kinetic energy $T_\Lambda \sim A^{-2/3}$. Figure 4 shows the experimental B_Λ vs. $A^{-2/3}$, in particular the s.p. energies. Extrapolation to

$A \to \infty$, i.e. $A^{-2/3} \to \infty$, in particular for the s_Λ (i.e. $l_\Lambda = 0$) states gives $D_\Lambda \approx 30 \pm 3MeV$, a value which has been known for a long time.

For the heavier hypernuclei adequate variational techniques are in general not yet feasible. We therefore use the local density approximation whose central element is the Fermi hypernetted chain (FHNC) calculation of the Λ binding to nuclear matter $D(\rho, k_\Lambda)$ as a function of the nuclear matter density ρ and of the Λ momentum k_Λ, with the identification $D_\Lambda = D(\rho, k_\Lambda = 0)$. The depth $D(\rho, k_\Lambda)$ is then used to generate a Λ core-nucleus potential $U_\Lambda(r)$ and an effective mass $m^*_\Lambda(r)$ which are used in a Schrödinger equation for the Λ-nucleus wave function to calculate B_Λ.

Previous calculations of the single-particle B_Λ have used various approaches. Closest to ours is that of Millener *et al.*[15] who in addition to a purely phenomenological analysis also considered a local density approximation but one based on a purely phenomenological zero-range Skyrme force. Other approaches use a self-consistent Hartree-Fock approach and a relativistic mean-field theory approach.

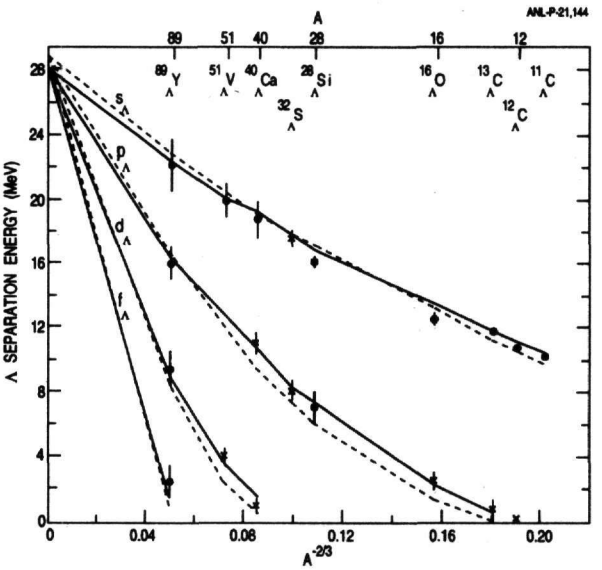

Fig. 4: The experimental B_Λ are shown with errors. The curves depict calculated B_Λ. The solid curve is for two interactions with both dispersive and $2\pi\Lambda NN$ potentials ($V_0 = 6.16MeV, W = 0.01MeV, C_p = 2MeV, \epsilon = 0.34$ and $V_0 = 6.2MeV, W = 0.013MeV, C_p = 2MeV, \epsilon = 0.32$). The dashed curve is for a purely dispersive ΛNN potential ($V_0 = 6.2MeV, W = 0.016MeV, C_p = 0, \epsilon = 0.32$).

Our interactions are in large part phenomenological but are generally consistent with and suggested by meson-exchange models, and are such that they can be used in few- and many-body calculations.

ΛN potential. Since the Λ has isospin $I = 0$ there is no (strong) $\Lambda\Lambda\pi$ vertex, and hence no OPE potential. However, isospin allows a $\Lambda\Sigma\pi$ vertex. Since the Σ is only about 80 MeV heavier than the Λ, the two-pion-exchange (TPE) potential is a dominant part of the ΛN potential being in turn dominated by the strong tensor OPE component acting twice. There will also be K, K^* exchange potentials which will, in particular, contribute to the space-exchange and the ΛN tensor potentials. The latter is of quite short range because there is no long range OPE and furthermore is also quite weak because K and K^* tensor contributions are of opposite sign. Also there will be short-range contributions from ω, quark-gluon exchange, etc. which we represent with a short-range Saxon-Wood repulsive potential which we take to be the same as for the NN potential.

We then use an Urbana-type central potential with space exchange and a TPE attractive tail which is consistent with Λp scattering. This has been discussed in section 1.3. However, we now also include a space exchange component:

$$V_{\Lambda N}(r) = V(r) + V_x \ , \quad V_x = -\epsilon V(r)(1 - P_x) \ , \tag{33}$$

where P_x is the ΛN space exchange operator; V_x is the space-exchange potential with ϵ its strength relative to the direct potential $V(r)$. The strength of $V_{2\pi}$ which is consistent with Λp scattering is $V_o = 6.15 \pm 0.05 MeV$. The space-exchange parameter is quite poorly determined from the Λp forward-backward asymmetry: $\epsilon \approx 0.1 - 0.38$. For our fits to the s.p. data we take ϵ to be a free parameter. ϵ determines the odd-state potential, in particular the p-state potential to be $V_p = (1 - 2\epsilon)V(r)$.

With only a ΛN potential fitted to Λp scattering, and even with rather large space exchange, the HN for $A > 5$ are strongly overbound relative to the experimental values. Furthermore, our results for the s.p. data show that these do not permit a fit with a ΛN potential alone even if the requirement that this fit the scattering data is dispensed with. This overbinding implies that many-body effects are large.

ΛNN potentials. Many-body effects can arise for a central $V_{\Lambda N}$ through changes in the ΛN correlation function $g_{\Lambda N}$ due to the presence of other nucleons. However, for our potentials such effects are quite small in the absence of ΛNN forces. Related effects are modifications (suppression) by other nucleons of an effective interaction due to e.g. a tensor force which must act at

least twice. Such tensor-force suppression of the NN force is a very important contributor to nuclear saturation. However, a ΛN tensor force is suppressed much less because of its short range and weakness. For the TPE ΛN potential $V_{2\pi}$ a closely related suppression effect arises from the modifications of the propagation of the intermediate Σ or N by other nucleons (Fig. 5). We represent such suppression effects by a phenomenological (repulsive) "dispersive" ΛNN potential of the form

$$V^D_{\Lambda NN} = WT^2_\pi(r_{1\Lambda})T^2_\pi(r_{2\Lambda}), \tag{34}$$

where $r_{i\Lambda}$ are the Λ-nucleon separations. The other type of three-body ΛNN force (Fig. 5) arises from TPE, appropriate to a p-wave pion interaction of the Λ with two nucleons (1 and 2), and has the form

$$V^{2\pi}_{\Lambda NN} = -(C_p/6)(\vec{\tau}_1 \cdot \vec{\tau}_2)$$
$$\{[(\vec{\sigma}_1 \cdot \vec{\sigma}_\Lambda)Y(r_{1\Lambda}) + S_{1\Lambda}T(r_{1\Lambda})], [(\vec{\sigma}_2 \cdot \vec{\sigma}_\Lambda)Y(r_{2\Lambda}) + S_{2\Lambda}T(r_{2\Lambda})]\} \tag{35}$$

where $\{A, B\} = AB + BA, Y(x) = exp(-x)(1 - exp(-cr^2)/x$ and $T(x)$ is OPE tensor potential shape with a cut off. S_{ij} is the tensor operator for particles i, j and $\vec{\sigma}_i$ and $\vec{\tau}_i$ are the spin and isospin Pauli operators for particle i. Theoretical estimates give $C_p \approx 1 - 2MeV$. Thus our ΛNN potential is $V_{\Lambda NN} = V^D_{\Lambda NN} + V^{2\pi}_{\Lambda NN}$ and involves the two strengths W and C_p.

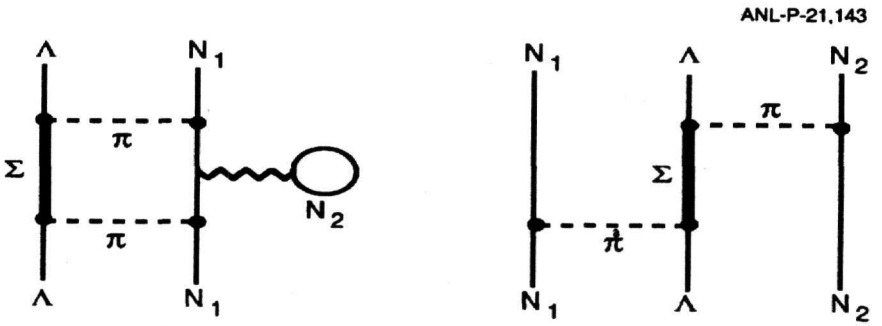

Fig. 5: Diagrams for dispersive and TPE ΛNN potentials.

For our earlier calculations of the s-shell HN: ${}^3_\Lambda H$, ${}^4_\Lambda H$ and ${}^4_\Lambda He$ (both $J =$ 0 and 1), and ${}^5_\Lambda He$ we used a M-T central NN potential. The calculations included ΛN, NN and ΛNN correlations: $f_{\Lambda N}$, f_{NN}, $f_{\Lambda NN} = f^D_{\Lambda NN} f^{2\pi}_{\Lambda NN}$. The most pertinent result for our present work is that $V^{2\pi}_{\Lambda NN}$ alone gives at best only a small repulsive contribution. Consequently a strongly repulsive ΛNN dispersive potential $V^D_{\Lambda NN}$ is required if ${}^5_\Lambda He$ is appreciably overbound. This is

because the ΛNN correlations $f^{2\pi}_{\Lambda NN}$ reduce the contribution of $V^{2\pi}_{\Lambda NN}$ from one which is appreciably repulsive to one which is only slightly repulsive or even attractive, whereas the effect of correlations on the repulsive contribution of $V^{D}_{\Lambda NN}$ is much less. For our calculations of the s.p. energies we have considered a ΛN potential only and several families of $\Lambda N + \Lambda NN$ potentials of the type discussed above, some of which in particular are constrained to give the experimental value of $B_\Lambda(^5_\Lambda He)$.

2.3 Calculation of the s.p. Energies

The s.p. energies B_Λ are obtained from a Schrödinger equation with a Λ-nucleus potential U_Λ and an effective mass m^*_Λ which are obtained in the local density approximation using the FHNC method. This is used to calculate the Λ binding $D(\rho, k_\Lambda)$ for nuclear matter of density ρ and for a Λ momentum of k_Λ. The FHNC method is based on a variational wave function of the form of Eq. (14) where $\Psi = \Phi^{A-1}$ (with $A \to \infty$) is now the uncorrelated Fermi gas wave function for nuclear matter of density ρ. The correlation factor F now includes both ΛN and ΛNN correlations. Details of the correlation factors $f_{\Lambda N}$, f_{NN} and $f_{\Lambda NN}$ as well as of the calculational method are given in Ref. [5]. The effective mass $m^*_\Lambda(\rho)$ is obtained from the quadratic term in k_Λ in $D(\rho, k_\Lambda)$. We also allow approximately for a "fringing field" (FF) due to the finite range of the ΛN and ΛNN potentials by a folding procedure. For a zero-range $V_{\Lambda N}$, i.e. without a FF, one has $U_\Lambda(r) = D(\rho_c(r), k_\Lambda = 0)$. The densities ρ_c are obtained from the electron-scattering data. The effective mass as a function of r is given by $m^*_\Lambda(r) = m^*_\Lambda(\rho_c(r))$. Finally, B_Λ is obtained as the lowest eigenvalue of the appropriate radial Schrödinger equation for an orbital angular momentum l_Λ.

We briefly discuss the expressions for $D(\rho)$ and $m^*_\Lambda(\rho)$ obtained with the FHNC method. We define (always for a given density ρ of nuclear matter)

$$D \equiv D(k_\Lambda = 0) = D^{\Lambda N} + D^{\Lambda NN}, \tag{36}$$

where $D^{\Lambda N}$ and $D^{\Lambda NN}$ are the ΛN and ΛNN contributions respectively. Further

$$D^{\Lambda N} = D^{\Lambda N}_0 + D^{\Lambda N}_x, \tag{37}$$

where $D^{\Lambda N}_0$ is the direct ΛN contribution and $D^{\Lambda N}_x$ is the exchange ΛN contribution. $D^{\Lambda N}_0$ is proportional to ρ to a good approximation, reflecting the small depedence of $f_{\Lambda N}$ on ρ. The exchange contribution $D^{\Lambda N}_x$ is proportional to the exchange strength ϵ: $D^{\Lambda N}_x = \epsilon\Delta$, where

$$\Delta = -\frac{3}{10}\left(\frac{3\pi^2}{2}\right)^{2/3} b_0 \rho^{5/3} F_1. \tag{38}$$

The form factor F_1 represents finite-range effects for $D_x^{\Lambda N}$; b_0 is given by

$$b_0 \;=\; b(\rho=0) \;=\; \frac{1}{3}\int g_{\Lambda d} V_{\Lambda N} r^2 d^3x, \tag{39}$$

where $g_{\Lambda d}$ is a correlation factor. The effective mass m_Λ^* is given by

$$\chi \;\equiv\; \frac{m_\Lambda}{m_\Lambda^*} - 1 \;=\; \frac{-m_\Lambda}{\hbar^2}\epsilon b \rho, \tag{40}$$

where b is given by the expression for b_0 but with an additional factor $D_F(k_F r)$ in the integrand and such that $b_0 = b(\rho = 0)$. Finite-range effects occur through b and are represented by the form factor $F_2 = b/b_0$.

Exchange contributes both through $D_x^{\Lambda N}$, and therefore through the Λ-nucleus potential U_Λ, as well as through m_Λ^*. For $\epsilon > 0$, $D_x^{\Lambda N}$ is repulsive (odd state potential less attractive than even state) and $m_\Lambda^* < m_\Lambda$ thus giving a larger kinetic energy relative to that for m_Λ and therefore also an effective repulsion. $F_1(\rho)$, $F_2(\rho)$ are form factors which represent finite-range effects of $V_{\Lambda N}$: F_1, $F_2 = 1$ for $\rho = 0$, or equivalently for a zero-range $V_{\Lambda N}$. F_1, F_2 differ by less than 2% between different interactions and together with b_0 then determine $\Delta = D_x^{\Lambda N}/\epsilon$ and χ/ϵ. Finite range effects (F_1, $F_2 < 1$) are much more important for χ, i.e. for m_Λ^*, than for Δ: thus $F_1 \approx 0.82, F_2 \approx 0.5$ at ρ_0.

The ΛNN contribution to the Λ binding is

$$D^{\Lambda NN} \;=\; t_3 \rho^2 F_{\Lambda NN}, \tag{41}$$

where $F_{\Lambda NN}(\rho)$ is a form factor such that $F_{\Lambda NN} = 1$ for $\rho = 0$ or equivalently for a zero-range ΛNN potential. $F_{\Lambda NN}$ depends on various correlation functions. The maximum variation of $F_{\Lambda NN}$ for all the interactions we considered is less than a factor of two over the range of ρ considered: $0 \le \rho \le 0.25 fm^{-3}$. The dominant ρ variation therefore comes from the ρ^2 factor. We also define $D_0 \equiv D(\epsilon = 0) = D_0^{\Lambda N} + D^{\Lambda NN}$, with the total Λ binding $D = D_0 + D_x^{\Lambda N}$.

2.4 Fits to the s.p. Energies

In our fits to the s.p. data only the exchange parameter ϵ is varied for a given interaction. The well depth is given by $D_\Lambda = D(\rho_0)$ where $\rho_0 = 0.165 fm^{-3}$ is the density of normal nuclear matter: $D_\Lambda = B_\Lambda(A = \infty)$ for all l_Λ. For a satisfactory fit to the $s_\Lambda(l_\Lambda = 0)$ data, $D(\rho)$ must satisfy $D_\Lambda = D(\rho_0) \approx 30 MeV$ in order to allow a satisfactory fit for large A. On the other hand for a fit for lighter nuclei which have relatively more surface, $D(\rho)/\rho$ must be larger for $\rho < \rho_0$. This implies a "saturation" behavior of $D(\rho)$ with a maximum at ρ_{max} which is not very different from ρ_0. These features are illustrated in Fig. 6,

in particular for an interaction ($V_0 = 6.16MeV, W = 0.01MeV, C_p = 2MeV$) with $\epsilon = 0.32$, which gives a satisfactory fit to the s.p. energies. The saturation features were previously emphasized by Millener *et al.*[15] in their analysis of the s.p. energies. To give the empirical differences between the B_Λ for different l_Λ requires quite generally that $m^*_\Lambda \approx 0.7 m_\Lambda$, which in our approach requires $\epsilon \approx 0.3 - 0.5$.

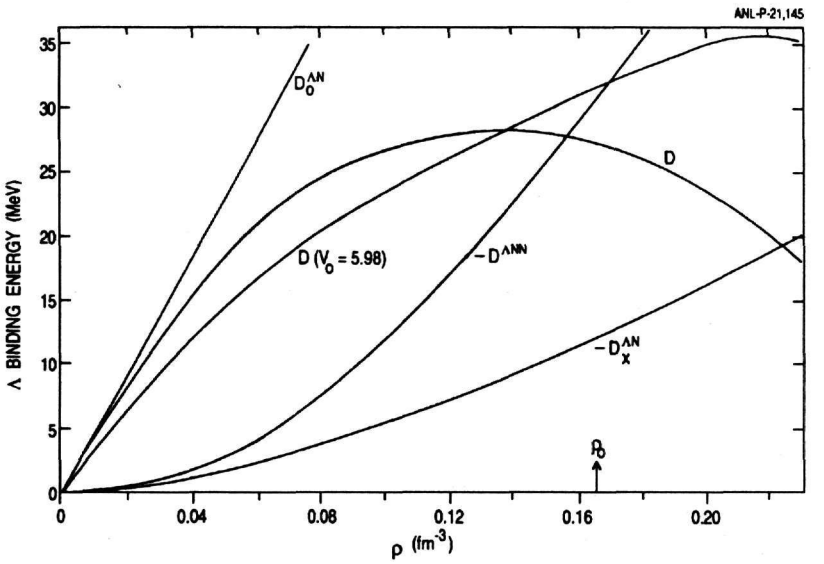

Fig. 6: The Λ binding $D(\rho)$ and its components vs. ρ for an interaction with $V_0 = 6.16MeV, W = 0.01MeV, C_p = 2MeV, \epsilon = 0.34$ for which the s.p. energies are shown in Fig. 4. Also shown is $D(\rho)$ for only $V_{\Lambda N}$ ($V_0 = 5.18MeV$) with $\epsilon = 0.88$.

Direct ΛN potential only. For this $\epsilon = 0$ and thus $m^*_\Lambda = m_\Lambda$. As Fig. 6 shows $D^{\Lambda N} = D_0^{\Lambda N} \approx 400\rho MeV$ for a $V_{\Lambda N}$ which fits the s-wave scattering; any nonlinearity with ρ being quite small. Thus, $D_\Lambda = D_0^{\Lambda N}(\rho_0) \approx 70MeV$. All the s.p. states and in particular the s_Λ states are then much too strongly bound, even for quite small A. If alternatively the strength V_0 were adjusted (without any justification) to give $D_\Lambda \approx 30MeV$ so as to fit the B_Λ for the heaviest HN, then conversely the B_Λ for even medium heavy HN would be much too small (and the ΛN scattering would be very much too small). These considerations clearly indicate the need for the "saturation" features of $D(\rho)$ if a fit to B_Λ is to be obtained for both small and large A. Thus a direct ΛN potential cannot fit the s.p. energies.

ΛN potential with space exchange. Now $D^{\Lambda N} = D_0^{\Lambda N} + D_x^{\Lambda N}$. To obtain $D_{\Lambda N}(\rho_0) \approx 30MeV$ requires a large and repulsive exchange contribution $D_0^{\Lambda N} \approx -30MeV$ which is obtained for $\epsilon \approx 0.88$. The Λ binding $D(\rho)$, shown in Fig. 6, then has a maximum $\approx 35MeV$ at $\rho_{max} \approx 0.215fm^{-3}$. However the large value of ϵ implies a correspondingly small value of $m_\Lambda^*/m_\Lambda \approx 0.48$ at ρ_0. The results for the s_Λ states are then reasonable for large A as expected, but the large Λ kinetic energy (small m_Λ^*) gives too small B_Λ for smaller A and also much too small B_Λ for the $l_\Lambda > 0$ states. In fact, no even tolerable fit to the s.p. data can be obtained with a ΛN potential with space exchange. Thus a central ΛN potential with and without exchange is ruled out by the s.p. data.

ΛN tensor force. This will make a nonlinear contribution in ρ to $D_0^{\Lambda N}$. $V_{\Lambda N}^T$ will arise mostly from K and K^* exchange, the associated range being quite small. Further, the K and K^* contributions are of opposite sign giving a small net $V_{\Lambda N}^T$. The short range implies predominantly quite high momentum components which are only slightly modified in nuclear matter. This results in only quite weakly ρ-dependent ΛN correlations, or equivalently an effective central interaction $\sim (V_{\Lambda N}^T)^2$ which is only slightly more repulsive in nuclear matter than the free interaction. Thus, with a reasonable ΛN tensor force, the direct ΛN contribution $D_0^{\Lambda N}$ will be approximately linear with ρ, and the repulsive (nonlinear in ρ) contributions can be ascribed almost entirely to $D^{\Lambda NN} + D_x^{\Lambda N}$, with only a small contribution from $D_0^{\Lambda N}$.

ΛN + dispersive ΛNN forces. To obtain an adequate fit to the s.p. energies and to ${}^5_\Lambda He$ the latter must be overbound by a ΛN potential (fit to the ΛN scattering) by $3MeV$ or more, since otherwise $V_{\Lambda NN}^D$ (fitted to $B_\Lambda({}^5_\Lambda He)$) is insufficiently repulsive for heavy HN. This insufficient repulsiveness cannot be compensated by a larger exchange parameter ϵ (which would give a more repulsive $D_x^{\Lambda N}$) since then m_Λ^*/m_Λ would be too small leading to a mediocre to poor fit for $l_\Lambda > 0$ as depicted in Fig. 4 for a situation where ${}^5_\Lambda He$ is overbound by $2.5MeV$. In fact it is quite likely that ${}^5_\Lambda He$ is even less overbound than this in which case a purely dispersive ΛNN force is even more strongly excluded.

ΛN + dispersive ΛNN + TPE ΛNN forces. For such interactions we obtain excellent fits to all the s.p. data as illustrated in Fig. 4. Figure 6 shows $D(\rho)$ and its components vs. ρ for one of these interactions. The depth $D(\rho)$ has the characteristic saturation features needed for a fit to the s.p. data: $D_\Lambda = D(\rho_0) \approx 27MeV$ required to fit B_Λ for large A, and a maximum $\approx 28MeV$ at $\rho_{max} \approx 0.14fm^{-3}$. A combination $D_{\Lambda NN}^D + V_{\Lambda NN}^{2\pi}$ permits a fit to the s.p. data and to ${}^5_\Lambda He$ because the $2\pi\Lambda NN$ correlations $f_{\Lambda NN}^{2\pi}$ in ${}^5_\Lambda He$ strongly reduce the repulsion due to $V_{\Lambda NN}^{2\pi}$ and can even give attraction, whereas this is not so for nuclear matter, i.e. for D. Thus a sizeable $V_{\Lambda NN}^{2\pi}$ which gives a small repulsive or even attractive contribution in ${}^5_\Lambda He$ can give a large repulsive contribution in nuclear matter. This, together with the repulsion from $V_{\Lambda NN}^D$ (which is

required for ${}^{5}_{\Lambda}He$ if this is sufficiently overbound and for which there is no such dramatic change between $A = 5$ to $A = \infty$) provides sufficient overall repulsion $D^{\Lambda NN}(\rho_0) \approx -30MeV$ needed for the s.p. data. More generally it seems clear that what is required for our family of interactions is that the effect of correlations for $V^{D}_{\Lambda NN}$ does not change too much with A, whereas for $V^{2\pi}_{\Lambda NN}$ the effect of $f^{2\pi}_{\Lambda NN}$ should depend quite strongly on A in such a way as to give relatively more attraction for small A.

As an average for the interactions which fit the s.p. data we obtain $\epsilon \approx 0.32 \pm 0.02$ and $m^{*}_{\Lambda}(\rho)/m_{\Lambda} \approx 0.72 \pm 0.02$, which implies an exchange contribution to D_{Λ} of $D^{\Lambda N}_{x} \approx -12MeV$. For the ratio of the p to the s-state potentials we obtain $V_p/V_s = 0.35 \pm 0.05$. For D_{Λ} we then obtain $D_{\Lambda} = 27 \pm 1MeV$.

2.5 Core Polarization for Medium and Heavy Hypernuclei

We have extended our local density calculations of the s.p. B_{Λ} to include polarization by the Λ. In view of the contribution of the rearrangement energy E_R discussed in section 1, such estimates must be considered valid only for medium and heavy HN.

As in 1.1 the total HN energy is assumed to be

$$ {}^{A}_{\Lambda}E[\rho] \;=\; {}^{A-1}E[\rho] \;+\; \epsilon_{\Lambda}[\rho], \tag{42} $$

where ρ is the density of the core nucleus and $[\rho]$ emphasizes the functional dependence on ρ; $\epsilon_{\Lambda} = -B_{\Lambda}$ is the s.p. energy as obtained in 2.3, and ${}^{A-1}E$ is the core energy. We now consider $\rho(r) \sim [1 + e^{-(r-R)/t}]^{-1}$ as a variational trial density dependent on radius (R) and surface thickness (t) parameters. The core energy ${}^{A-1}E[\rho]$ is obtained as an integral of an energy density $\epsilon(\rho) + C(\nabla\rho)^2/\rho$ plus the Coulomb energy and a fixed asymmetry term. The gradient term represents finite-range effects of the nuclear forces and is required to obtain a nuclear surface of finite width. The energy per particle $\epsilon(\rho)/\rho$ is chosen to give the correct saturation values ($-16MeV$ at $\rho_0 = 0.165fm^{-3}$) and involves the incompressibility constant K and two additional parameters. These and C (for given $K = 200$ and $300MeV$) are chosen to fit the experimental nuclear binding energies by calculating the minimum of ${}^{A-1}E[\rho] = {}^{A-1}E(R,t)$ for any given set of parameters in the energy density and then choosing that set which gives the best fit to the experimental energies. We obtain an excellent fit ${}^{A-1}E$ to these experimental energies for densities ${}^{A-1}\rho$ which give quite good agreement with the empirical densities.

To obtain the total HN energy we calculate $\epsilon_{\Lambda}[\rho] = -B_{\Lambda}$ as discussed in 2.3, but now for the trial core density ρ rather than for the fixed empirical electron-scattering density. The trial density is then varied to give the minimum of the

total energy ${}^{A}_{\Lambda}E[\rho]$ for our best fit parameters for the nuclear energy density. This minimization gives a changed density ${}^{A-1}\hat{\rho}$ and core energy ${}^{A-1}\hat{E}$ appropriate for the HN. The core polarization energy $\Delta E = {}^{A-1}\hat{E} - {}^{A-1}E$ is also the energy by which the total HN energy is lowered due to core polarization.

Core polarization results. Both the polarization energy δE and the change in rms radius $\delta < r^2 >^{1/2}$ decrease in magnitude with A and for larger K, and are generally quite small. For a ΛN potential only, Fig. 6, shows that $D_0^{\Lambda N}$ increases with ρ (this corresponds to the decrease of ϵ_Λ with R discussed in section 1.1). In this case the Λ binding prefers larger ρ and hence a smaller core radius, i.e. one expects $\delta < r^2 >^{1/2}$ to decrease due to the presence of the Λ (quite similar to the discussion of 1.1). The response of the core nucleus to the Λ is determined roughly by AK (which is effectively the coefficent of the quadratic term $\sim (\rho - \rho_0)^2$ in $\epsilon(\rho)/\rho$) and thus the polarizing effect of the Λ is approximately proportional to $(KA)^{-\alpha}$ with $\alpha \simeq 1$. Numerically we obtain (for a ΛN force only): $\Delta E \approx 0.12MeV(A = 50)$, $0.06MeV(A = 150)$ for $K = 200MeV$. With ΛNN forces the situation is qualitatively changed since $D(\rho)$ now has a maximum at ρ_{max} not very different from the situation density ρ_0. In fact for the $D(\rho)$ of Fig. 6 (with $\rho_{max} \simeq 0.15fm^{-3} < \rho_0$) we obtain a small expansion of the core-nucleus surface. The "driving force" due to the Λ is now much less than for $D_0^{\Lambda N}$ because of the much smaller variation of D with ρ near ρ_0. Numerically we now obtain $\Delta E \approx 0.05MeV(A = 50)$, $0.02MeV(A = 150)$ for $K = 200MeV$, much smaller than for $D_0^{\Lambda N}$; as expected δE is smaller for larger K, e.g. $\Delta E \approx 0.03MeV(A = 50)$ for $K = 300MeV$.

We conclude that polarization effects are small and can generally be neglected, especially for a "saturating" $D(\rho)$ which is required to fit the s.p. energies.

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