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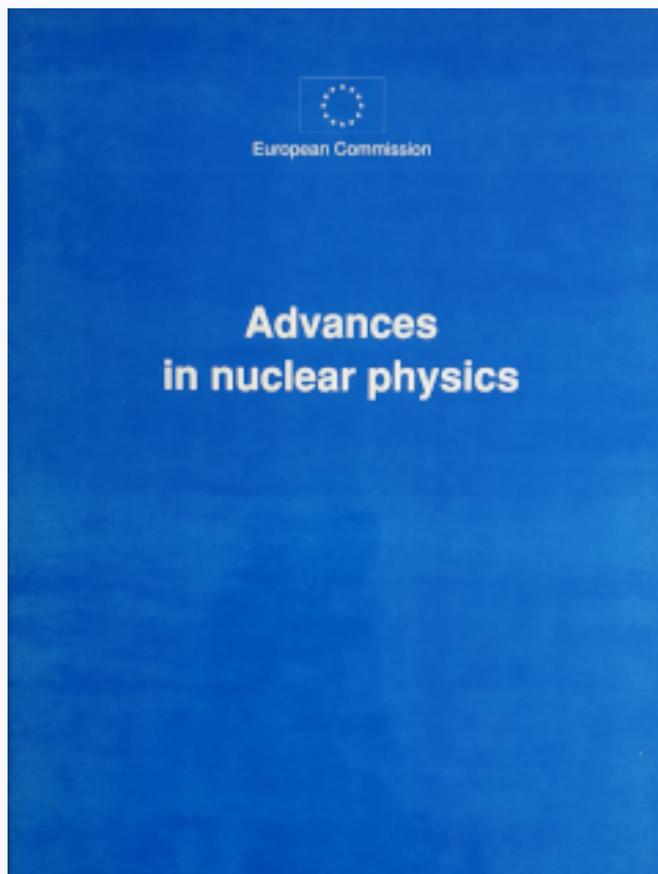
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### The Time Concept in Atonic and Sub-Atonic Systems - Reconciliation of the Time-Reversal-Invariance and the Macroscopic Arrow of Time

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# THE TIME CONCEPT IN ATOMIC AND SUB-ATOMIC SYSTEMS - RECONCILIATION OF THE TIME-REVERSAL-INVARIANCE AND THE MACROSCOPIC ARROW OF TIME

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## Summary

A new conception of time is presented in the framework of the quantum generalized stochastic and infinitely divisible fields. A non-unitary evolution operator lacking the continuous group property is derived from a time-reversal-invariant field theory in Minkowski's space. It describes the arrow of time on the quantum level. By quantizing the field action integral the usual evolution operator is obtained as a particular case. Quantum processes violating the T-symmetry are possible in the present theory. It is also explained why Born's interpretation of the wave function is necessary. The Feynman path integral is obtained as the limit of a series of similar integrals with finitely additive measures. This form of the Feynman integral does not conflict on the quantum level Heisenberg's Uncertainty Principle.

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## 1 INTRODUCTION

"Time is money" is probably not only a commercial wisdom. It may equally be a fundamental physical principle, if it is true, in a sense, that time is intimately related to energy changes.

Macroscopically, time is perfectly defined since ancient epochs as a continuously flowing quantity on the basis of directly or indirectly observable changes in the motion of material bodies. On the quantum level, however, time cannot be of the same structure as in the macroscopic scale phenomena.

The purpose of the present paper is to introduce a new concept of the time and to apply it to derive a new evolution operator in the framework of the quantum field theory which will be a realization of the unified arrow of time on the quantum level and in the macrocosmos.

No matter what kind of a mechanism is used to implement the time flow - the clepsydra or the pendulum or the Earth or the pulsars - an energy change invariably takes place.

In the clepsydra the minimum time duration generated is determined by the total water quantity, by the hole diameter of the container and by the acceleration of the gravity,  $g$ , among other things. Larger time neighbourhoods are realized by adding any number of the minimum time neighbourhoods in each case. The accompanying energy change consists in the change of the potential energy of the water during flowing.

In the pendulum case the minimum time neighbourhood, the semi-period, is determined by the pendulum moment of inertia and by  $g$ , the acceleration of the gravity. Again, larger time neighbourhoods are constructed by taking any number of semi-periods. The relevant energy change in this case is connected with the change of the potential (or, equivalently, the kinetic) energy of the pendulum mass.

If the Earth rotation around the Sun is used to define the time, then the minimum time neighbourhood is defined as the year, and larger intervals are obtained in the same way as above. Here, the accompanying energy change is that of the potential energy of the Earth in Sun's field of gravity, but to the humans' perception of the time flow serve other accompanying phenomena of

shorter duration as night-day (or seasons) succession.

Similarly, in the chemical clock or in the atomic clock there exists in each case a minimum time from which larger intervals can be constructed. In the case of the atomic clock the minimum time is determined by the inverse frequency of the emitted light quanta during transitions between certain states of the Cs atom.

For us, humans, the continuous time flow feeling is perceived also independently from clepsydra's observation etc. by the immense number of internal (biological) and external (physical) stimulations acting on our five senses.

These facts let appear the idea of the continuity of time as not more justified than the non-continuity, although macroscopically one can imagine physical processes giving minimum continuous time neighbourhoods long enough for all practical purposes.

On the macroscopic level the time is, as it will become clear, in fact always continuous. The overwhelming majority of the differential equations of Physics are based on the idea of a continuous time variable and everything is working perfectly.

On atomic and sub-atomic scales, however, the discontinuity of the time variable is more evident. Despite this fact the time variable enters the differential equations of Quantum Physics<sup>1</sup> in the same way as in Classical Physics<sup>2</sup>. The consequences of this fact will be also discussed.

The consequences of using, instead of the *universal time* the interaction *proper time neighbourhood* of every process do not appear in practical issues of the macrocosmos. However, when atomic or sub-atomic processes are handled which eventually are expressed by the evolution operator or by the *S*-matrix<sup>3</sup>, then the use of the interaction proper time neighbourhood is unavoidable in the time integrations of the above mentioned operators.

This paper is organized in 6 sections. In section 2 the necessity is discussed for a relationship between time and energy. In section 3 a procedure is presented showing that the time is since long recognized as a non-continuous (physical?) quantity.

An other very intriguing question related to the structure of time is the statistical nature of the wave function in Quantum Theory as it has been ingeniously postulated by M. Born.

It is one of the most surprising facts in Physics that, while the Schrödinger equation does not contain<sup>1</sup> any random, stochastic or statistical term, derivative or factor, its solution is of statistical character.

It seems that the way for understanding the Born interpretation of the wave function goes via the consequent recognition of the fact that the time used for the description of the observable phenomena in nature is composed, in a set theoretic sense, from the time neighbourhoods of the realm of the microscopic phenomena to which are due the microscopic or the macroscopic observable changes. This will be the subject of section 4.

The relationship between the many times of the microscopic processes and the time of the macroscopic phenomena-it is believed-is the deeper reason for which the quantum fields are in fact *generalized and infinitely divisible stochastic fields*.

An evolution operator is derived in section 5 making use of the stochasticity of the quantum fields. The questions of *reversibility-irreversibility* and of *conservation dissipation* are discussed. It is shown that the *arrow of time* and *dissipation* are spontaneously generated by the *stochasticity of the quantum fields* and by rejecting the property of the continuous group of the evolution operator. *It is interesting to note that the famous Feynman path integral appears as a particular case in the framework of the generalized and infinitely divisible stochastic fields..*

Finally, in section 6 the obtained results are discussed and some conclusions are presented.

## 2 THE GENERATION OF TIME IDEA AND ITS RELATIONSHIP TO ENERGY

### 2a The origine of the time idea.

If the entire Universe consisted of one single, structureless particle, e.g., an electron, then the idea of the time would be for a "foreign" observer neither definable nor useful. Motion would be, on the basis of our familiar physical criteria, unobservable and meaningless. The particle would be describable by its intrinsic characteristics, mass, spin, charge, etc., and no change whatsoever would be possible. In particular no change of the particle energy would be possible.

If the entire Universe consisted of two non-interacting structureless particles, then the idea of time would again be undefinable and the motion, if any, would be unobservable by an observer in the frame of reference of either particle (due to the lack of interaction).

If the two particles do interact, then messages between them conveying physical characteristics exist, and a new parameter is required for the description of their evolution: This parameter is the interaction proper time neighbourhood.

However, interaction means change of physical characteristics and exchange of parts of them between the particles. Moreover, transfer of physical characteristics implies in any case energy changes inside the Universe of the two particles. Consequently, it appears that associated with any time laps is an energy change. This association has not the character of a causal relationship. This becomes clear from the fact that if no description of the phenomenon is desired, then there is no time variable.

Conversely, it is empirically clear that no time laps is observed, if no energy change - and more generally no change whatsoever - takes place.

### 2b The relationship "energy-time".

The idea that time is related to the energy is not new. Already Schrödinger<sup>4</sup> and Pauli<sup>5</sup> considered the relation of the time with the energy as direct consequence of the commutation relations

$$[x_m, p_n] = i\hbar \delta_{mn} \quad m, n=1, 2, 3. \quad 2.1$$

However, it was not sufficiently emphasized that the position coordinate  $x$  and the conjugate momentum  $p_x$  are related, despite their independence in the sense of the Mathematical Analysis, not just by the commutation relations. There is still an other reciprocal physical relationship: The change of the position variable,  $x$ , of a particle generates its momentum  $p_x$ . The converse is also true. The change of the momentum  $p_x$  of a particle necessarily implies change of its position  $x$ . This mutual relationship has not been sufficiently emphasized although its reality is quite evident.

In quite a similar way, the change of the energy of any particle generates the time laps which is appropriate for the description of the history of this particular event.

The commutation relation between the energy operator  $H$ , the Hamiltonian, and the time operator  $t$  is

$$[t, H] = i\hbar. \quad 2.2$$

Here, as well as in 2.1, the "generating" relationship between energy change and time change is apparent from 2.2 and it follows from physical considerations analogous to those valid for 2.1.

A further analogy between 2.1 and 2.2 of great importance for the understanding of the nature of the time is the following:

The result of applying 2.1 on a wave function is to describe the generation of a quantum pertaining to the particle having the momentum  $p$ .

Similarly, the application of 2.2 on a wave function generates a quantum pertaining to the particle having the energy  $E$ .

At this point it is appropriate to emphasize that each time neighbourhood pertains to the particle subject to the corresponding interaction and only to that. It refers to its rest frame. It would not be in agreement with Relativity, if the same time neighbourhood would be used universally to describe the evolution of other particles at different points of the space (events).

By mixing the time and the space variables as it is done in the transformations of Relativity<sup>6</sup> we do not yet fully eliminate the classical, absolute character of the time. Such could be achieved to a greater extent by attaching to every one act of

interaction its own time variable, which takes values exactly as long as the interaction lasts.

Considering that in a many particle system each particle's history is described by its own set of time neighbourhoods - each one starting and ending with the starting and the ending of the corresponding interaction (causing the associated changes in the energy) of the respective particle-it is not obvious at first sight, which one of the many pieces of time (which, by the way, may, clearly, or may not overlap, in the sense of simultaneity of Relativity, partially or entirely) would be appropriate to describe the whole set of particles as a physical system.

### 2c Microscopic and macroscopic universal time.

These considerations make clear that any time variable defined to describe a macroscopic system will be conceived as a superposition of time neighbourhoods generated during the numerous interactions of the atomic or sub-atomic constituents of the system under observation.

It becomes, therefore, clear that every system of many particles has its own macroscopic time. If two different particle systems have equal numbers of identical particles which interact via identical interactions they may or they may not have identical microscopic times. The macroscopic time variables,  $\tau$ , however, of the two systems of  $N$  particles will be with very high probability the same and take values on the union<sup>7</sup> of the microscopic interaction proper time neighbourhoods  $\{t_{\text{micr.}}^{(a,\beta)}\}$  pertaining to the interactions  $\{a\}$  between the particles  $\{\beta\}$  of the system (fig.1):

Microscopic universal time =

$$\tau := \bigcup_a \bigcup_{\beta} t_{\text{micr.}}^{(a,\beta)}$$

= Union of all factual interaction proper-time

neighbourhoods. 2.3

Despite the possible leaks between the particular interaction proper time neighbourhoods for a small number of particles in the system there is no practical difficulty in describing the macrocosmos by a continuous time variable, because the number of

the interacting particles in other particle systems perceived simultaneously by the observer and the corresponding number of interaction proper time neighbourhoods is so large that the mathematical (and the psychological) continuity (in the sense of 2.3) is assured.

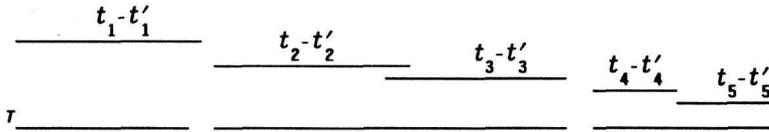


Fig.1 Interaction proper time neighbourhoods ( $t_n-t'_n$ ) of particle pairs in a system with 5 interactions. ( $t_1-t'_1$ ) has no common point ( $t_2-t'_2$ ) which partially overlaps with ( $t_3-t'_3$ ). The end point of ( $t_4-t'_4$ ) coincides with the initial point of ( $t_5-t'_5$ ). The union of the above interaction proper time neighbourhoods constitute the universal time of the given system of particles which is sectionally continuous. For very large numbers of interacting particles  $r$  becomes equivalent to  $R^1$ .

Hence, in case  $N$  is very large,  $r$  consists entirely of partially overlapping interaction proper time neighbourhoods, and a continuous macroscopic time variable may emerge, which would be identical with the macroscopic or psychological time.

**2d Time and entropy**

Whenever the scientific discussion is about the evolution in time one invariably recalls entropy. This quantity introduced by R.E.Clausius as a thermodynamic state variable, has been later related by L.E.Boltzmann to the probability of the thermodynamic state of a gas.

Since every particle system in a non-equilibrium thermodynamic state tends to an equilibrium state, and since, during this tendency, the probability of the state increases with entropy, and since entropy increases, or at least does not diminish, during every evolution of an isolated system, there automatically emerged impression of a relationship between time and entropy.

This relationship provided the entropy with the reputation of a fundamental quantity for the evolution generally in Nature. To

which extent is this reputation justified?

First it is recalled that entropy, defined by

$$S = \int dq/T,$$

depends on a heat quantity and on the temperature of the heat source. No one of the quantities  $Q$  or  $T$  is fundamental. The fundamental physical fact here is that, if heat flows spontaneously from one source with temperature  $T'$  to a heat sink of temperature  $T'' < T'$ , it will never flow spontaneously in the opposite way.

To investigate this question one should at least analyze in detail the various situations, in which entropy changes with time.

The simplest question to ask: Is the relationship "entropy - time" linear? Even this question is incomplete, because we left unspecified which time we are talking about.

Since Thermodynamics is concerned with a large number of particles the system time defined by 2.3 coincides with the universal time. We can, therefore, assume that time continuously flowing. If the thermodynamic transformation of the system proceeds irreversibly, then the most reasonable assumption seems to be that entropy increases linearly with the time.

This assumption, however, may be wrong. The reason is that our question is still incomplete. We have not yet specified, whether the heat sources involved in our irreversible thermodynamic transformation are of variable or of constant temperatures.

Suppose, then, that the source temperatures are constant. In this case the relationship "entropy - time" is linear, if among other things the heat transferred between the heat sources is linear in time.

Hence, under the stated conditions, the entropy increases with constant speed and is, in a sense, a measure for the time flow.

We postpone, for a moment, other pressing questions concerning the linearity and we ask, whether this speed is universal, i.e., valid or not for all thermodynamic systems in our universe. The answer is, of course, "No". Because, other temperature differences of heat sources imply other speeds for the entropy increase in irreversible transformations.

The other pressing questions regard the degree of irreversibility in the transformations of our universe. Clearly, while the ti-

me defined by 2.3 for the system under discussion flows, during reversible transformations, entropy stops to increase. This destroys the linearity of the relationship.

In short: The increase of the entropy in our universe does not, with high probability, proceed linearly with the time; it depends on the temperature differences and, consequently, entropy is an average measure for the total change in our universe.

Let us next suppose that there exists in our universe a space neighbourhood from which not all physical changes in our universe are observable. The observable physical changes from the neighbourhood in question allow to define a macroscopic time by means of 2.3.

If the physical changes in other similar neighbourhoods of the universe do not proceed in the same way (for example, the temperature differences are quite differently distributed, entropy changes in different ways in different particle systems and they do not define the same time), then the macroscopic time defined there may in principle differ from that given in the first neighbourhood. This opens the question of the possibility for a *constructive* definition of a universal time on the basis of entropy. The most likely answer is that there cannot constructively defined a universal time for all causally separated regions of the universe. If this is so, the question might be of some interest to Cosmologists.

### 3 TIME CHANGING BY STEPS AND LATTICE STRUCTURE OF SPACE-TIME

#### 3a The Liouville elementary solution.

In all physical processes the elementary interaction time is finite and, therefore, the corresponding energy change is also finite (quantum exchange). From this fact it becomes clear that the time variable can in general be only a sectionally continuous variable. This can be seen also in a more formal way.

We consider a system of  $N$  particles interacting via forces  $\{F^{(n)}\}$  and moving in the phase space  $Q^{(3)} \otimes P^{(3)}$  according to the Liouville equation

$$L g(\mathbf{q}, \mathbf{p}, t) = 0. \quad 3.1$$

Here the operator  $L$  is defined by

$$L = \partial_t + \sum_n (\mathbf{p}^{(n)} \cdot \nabla^{(\mathbf{q}_n)} + F^{(n)} \cdot \nabla^{(\mathbf{p}_n)}) \quad 3.2$$

and  $\nabla^{(\mathbf{q}_n)}$ ,  $\nabla^{(\mathbf{p}_n)}$ , are the gradient with respect to the position vector  $\mathbf{q}^{(n)}$  of the  $n$ -th particle, the gradient with respect to the momentum  $\mathbf{p}^{(n)}$  of the  $n$ -th particle.  $g(\mathbf{q}, \mathbf{p}, t)$  is an elementary solution of the Liouville equation.

For simplicity it shall be assumed that the forces are independent of  $\mathbf{q}^{(n)}$ ,  $\mathbf{p}^{(n)}$ ,  $t$ .

According to the above definitions an elementary solution of 3.1 is given by<sup>8</sup>

$$g(\mathbf{q}^{(n)}, \mathbf{p}^{(n)}, t) = \int \frac{1}{h} [\lambda \varepsilon_n t - \mu_n F^{(n)} \cdot \mathbf{q}^{(n)} + \frac{1}{2} \mu_n (\mathbf{p}^{(n)} - \nu_n F^{(n)})^2] + \theta \quad 3.3$$

in which  $\theta$  is a constant to be conveniently determined. This solution of the Liouville equation can be used to construct more general distribution functions  $f(g)$  for the system of particles under consideration.

#### 3b An example for time discontinuity

To do this with the above form 3.3 we shall postulate two principles: The reality of  $f(g)$ :

$$\text{Im } f(g) = 0, \quad (\text{reality}) \quad 3.4$$

and the additivity of two solutions corresponding to two systems

of particles:

$$f(g_1) \cdot f(g_2) = f(g_1 + g_2) \quad (\text{additivity}). \quad 3.5$$

The total energy of the system is equal to the sum  $E = \sum \epsilon_n$  and it is assumed that the total energy is conserved in the sense that

$$L E = 0. \quad 3.6$$

The simplest form  $f(g)$  satisfying 3.4 and 3.5 is  $f(g) = C \cdot e^{-g}$ , where  $C$  is a normalization constant.  $f(g)$  satisfies the Liouville equation, if, besides 3.4 to 3.6 and some relationships concerning the linear momenta of the particles, it satisfies also the condition

$$\lambda \epsilon_n t = 2i\pi k_n, \quad 3.7$$

where  $k_n = 0, 1, 2, \dots$

It is seen from other considerations that  $\lambda^{-1} = -i\hbar$ . Hence, 3.7 allows the following conclusions:

- i) Time cannot change continuously.
- ii) For given energy  $\epsilon_n$  of the  $n$ -th particle the least time step for this particle is  $2\pi\hbar/\epsilon_n$ .
- iii) A quantum process is more rapid the higher the energy change.

These conclusions, if taken with consequence, modify our picture of the time in atomic and sub-atomic Physics.

The initial question "why time changes by steps" is justified in classical non relativistic theories such as the theory of the Liouville equation. The answer is that such a theory describes observable phenomena only if the distribution function satisfies the reality condition, and if the condition that the sum of two similar systems must be described by added similar distribution functions. These two conditions are satisfied if, among other things, the time associated with elementary events changes step-wise.

### 3c The space quantisation

In 3b conditions have been described under which time cannot change continuously for a system of particles of given constant energy interacting via external forces.

It is, therefore, seen that the most important condition for the

noncontinuity of the time is the additivity of the solutions for two systems. In the present example this has been obtained by requiring the particles to interact at most via external forces.

However, this does not constitute a proof for the assertion that the solutions lose the additivity property as defined in 2b if there exist interaction forces between the particles whose they are the distribution functions.

Since the solutions as given by 3.3 are directly related to energy, the above discussion is equivalent to the discussion about the additivity of the particle energies. Hence, if a representation can be found in which the energy of each particle in each system is independent of the number of the system's particles, then the solutions become strictly additive.

It is, of course, well-known that the above requirement is met in the thermodynamic limit, where the ratio surface/volume of the system tends to zero with the number of particles tending to infinity.

It follows, therefore, that at least under the above restrictive conditions space too must be quantized. A full discussion of these matters will be presented elsewhere.

Based on the above results and observations Born's statistical interpretation of the wave function will be analysed from the interaction proper time point of view.

## 4 ON THE ORIGIN OF THE STATISTICAL NATURE OF THE WAVE FUNCTION

### 4a The wave function of the many particle beam

Interactions are the reason for any change in Nature. The description of any state is implemented in the microcosmos by means of a wave function satisfying a quantum equation (Schrödinger, Dirac, Klein-Gordon, etc.). These equations are partial differential equations with a derivative with respect to time. No one of these fundamental physical equations is of statistical character. Despite this fact, one of the most surprising discoveries in Physics is that of Max Born:

Although the Schrödinger equation does not contain any random, stochastic or statistical term, factor or derivative, its solution, the wave function, must be statistically interpreted.

The mathematical structure of the time used in the above mentioned differential quantum equations to describe the elementary particles' behavior and their motions, is usually identical to that of the time with the help of which we rule our daily affairs. It is the time which we put, e.g., into classical equations to solve engineering problems. It has nothing to do with the particle interaction proper time required by relativity..

The tacit assumption that these two time structures are identical is certainly well done in many problems as long as no elementary interaction takes place. Our question is whether this can or cannot be done in problems of atomic or sub-atomic scale, where interactions occur and the proper time of each particle in the sense of Relativity plays its own part. Is the time structure in conjunction with the interaction processes which we wish to describe adequately taken into account?

The possibility of a continuous time variable to be the proper time of a series of events is not related to the velocities of that series events being comparable or not to the velocity of light,  $c$ . It is rather related to the time, which is *created simultaneously* in its neighbourhood with each particular interaction event on the quantum level. The proper time neighbourhood does not exist prior to its creation by the elementary interaction process.

There is no hope to discover experimentally the absence of the space-time anywhere. Because an observation for this purpose pre-requires a series of elementary physical interaction events which themselves create the space-time observed. Vacuous space-time on the quantum level is not observable. Only interaction processes and their space-time positions are observable.

Our preliminary answer to the question whether we can or cannot identify a macroscopic time variable with the proper time variable of an elementary process in problems of atomic or sub-atomic scale is: Probably we cannot do that. Because the classical universal time variable (as a point set) may have at most a non-empty intersection with the intervals of the proper times pertaining to interactions of an atomic or sub-atomic system.

When the number of particles is very large, the situation changes radically, because the union of a large number of proper time neighbourhoods may, if they pairwise partially overlap, acquire the structure of  $R^1$ .

Whereas the elementary physical interactions which create time are of finite extent, we tacitly consider the physical interactions as continuous functions in time neighbourhoods beyond the proper interaction time neighbourhood. In so doing we tacitly identify the macroscopic time in the quantum equation describing the particular process with the set of proper interaction time neighbourhoods of other, macroscopic processes. This cannot be done, because interactions are identical to exchange of field quanta, and these are per definition non-continuous entities.

It was no less a person than P.A.M. Dirac who pointed out early<sup>9</sup> in the development of the Quantum Theory the necessity to introduce the many-times theory in Quantum Physics. Despite this proposal, the universal time methods continued to be applied since the beginnings of the Quantum Theory. For example, the integrations in the  $\Sigma$ -matrix or in the evolution operator expression are done without regard to the interaction proper time neighbourhood of the particle reaction. The many times variables which then must be chronologically ordered are introduced by the way of the repeated integrations of the iteration theory.

Contrary to this practice regarding the time, in the case of

space integrations (e.g., in integrating the mass space density distribution of a particle system) physical importance is attached to a space point,  $r$ , only, if there exists a particle mass at  $r$ . In time integrations one considers as physically important time points,  $t$ , at which no interaction takes place (e.g., in the  $S$ -matrix,  $t \pm \infty$ ).

This is generally done, e.g., in integrating the interactions in the chronological products, despite the  $\epsilon\chi\pi\epsilon\rho\tau\mu\epsilon\nu\tau\alpha\lambda$  knowledge that interactions are of finite time duration.

As a matter of fact, in particle reaction experiments the particles are free of interaction at distances of a few wave lengths apart from the target. If the duration of the interactions were infinite, no experimental measurements on the nuclear and particles reactions would in fact be possible.

On the other hand, according to Relativity, space and time coordinates are mixed in the Lorentz transformation formulas and, therefore, they always must be handled in an equivalent way.

Consequently, a distinction in sub-atomic processes between the physical interpretations of the space coordinates, on the one hand, and the time coordinates, on the other hand, implies an nonsymmetrical and non-covariant handling of these coordinates.

Let us consider two particles at rest. If they do not interact, the macroscopic time flows, but their proper times in their respective rest frames do not flow.

Suppose the contrary: Then, since the proper time is always connected with some change in the neighbourhood of a particle, and since the neighbourhood of the particle in the absence of interaction is the particle itself, at least one observable of the particles should change. This conclusion is, however, false because, according to the assumption there is no interaction to cause the change.

Consequently, the proper times of the particles do not flow without interaction in their respective rest systems of reference. As soon as their interactions start various changes of their physical observables may take place and the proper time starts flowing for the two particles.

When the interaction ceases, the time stops again for the two

particles, because in the rest frames of the two particles there again happens nothing to cause any change of their physical properties. Meanwhile, macroscopic time continues flowing for us, the observers.

The situation would be similar for us too, if the motions of the stars, the planets, the biological phenomena, the atoms and all elementary particles stopped in the whole universe. Then, a super-observer might consider his own time as still flowing, while for the humans time would not exist.

This procedure will be applied to show that the statistical character of the wave function according to Max Born is a consequence of two facts:

- i) Every elementary process has its own proper time neighbourhood during interaction.
- ii) The particle beam in a reaction experiment contains (many) non-interacting particles at various distances between them.

To demonstrate the Born hypothesis we consider the interaction of a particle beam with a target particle. Contrary to the usual procedure of the reaction theory (attaching to all beam particles one and the same space-time variable) in the wave function, we assume with Dirac that the beam wave function of the  $N$  particles has the form  $\psi_0(\mathbf{r}'_1, t'_1; \dots \mathbf{r}'_N, t'_N)$ . This will be done for the following reasons: (i) Each of the beam particles has its own position in the beam. (ii) Different beam particle interaction processes with the target have in general different proper time durations.

This form  $\psi_0(\mathbf{r}'_1, t'_1; \dots \mathbf{r}'_N, t'_N)$  of the initial beam wave function is necessary and meaningful in view of the sequential production by the particle source and the subsequent observation in the reaction experiment. If all beam particles were produced at the same space-time point by the source, then, obviously, a representation by an initial wave function of the form  $\psi_0(\mathbf{r}'_1, t'_1; \dots \mathbf{r}'_N, t'_N)$  would not be correct.

#### **4b The role of the time structure in perturbation**

In preparing the discussion of the interaction "beam-target" we introduce the evolution operator  $U(t, t')$  which implements the

transition of the beam particle from the initial state to the final one in the time neighbourhood  $(t', t)$ . Since this operator describes certainly one single interacting particle within proper interaction time neighbourhood  $(t', t)$ , it cannot describe the interactions of other particles having interaction proper time neighbourhoods  $\{(t'_j, t_j), j=1, 2, \dots\}$ . This deprives the evolution operator of the continuous group property.

It follows, therefore, that for each one interaction the operator  $U(t, t')$  must act once on the wave function.

The evolution operator obeys the equation:

$$U(t, t') = 1 - \frac{i}{\hbar} \int_{t'}^t dt H_I(t) U(t, t'). \quad 4.1$$

Here great attention must be paid to the fact that  $U(t, t')$  has not the property of a continuous group

$$U(t, a)U(b, t') \neq U(t, t') \text{ for } a \neq b$$

This happens, because the interaction proper time neighbourhoods  $(t-a)$  and  $(b-t')$  may not have common the points  $a$  and  $b$ . In either case  $a > b$  or  $b > a$  is the group property lost. In the case of beam particles interacting with a target there most frequently holds  $b < a$ . In rare cases there holds  $b = a$ , i.e., the next particle's reaction starts at the moment the previous one stops (see fig.1).

The iterative procedure for the series solution of 4.1 introduces an infinite set  $\{t_n | n=1, 2, \dots\}$  of time variables which are assigned the meaning of artificial interaction proper time neighbourhoods  $(t, t')$ ,

$$U(t, t') = \sum_{n=0}^{\infty} (i\hbar)^{-n} \int_{t'}^t dt_1 \int_{t'}^{t_1} dt_2 \dots \int_{t'}^{t_{n-1}} dt_n [H_I(t_1)H_I(t_2)\dots H_I(t_n)] \quad 4.2$$

or the equivalent form with the chronological ordering operator  $\Pi$

$$U(t, t') = \sum_{n=0}^{\infty} \frac{(i\hbar)^{-n}}{n!} \int_{t'}^t dt_1 \int_{t'}^{t_1} dt_2 \dots \int_{t'}^{t_{n-1}} dt_n \Pi [H_I(t_1)H_I(t_2)\dots H_I(t_n)]. \quad 4.2'$$

In the first form 4.2 there holds  $(t', t_n) \subseteq (t', t_{n+1})$  for  $n=1, 2, \dots$ . The relationships  $\{(t', t_n) \subseteq (t', t_{n+1}), \forall n \in \mathbb{N}\}$  indicate

that the integration intervals overlap, and the indices  $\{n\}$  indicate merely different mathematical variables and not different physical interaction proper times neighbourhoods.

The question is whether the time variables  $\{t_n | n=1,2,\dots\}$  represent proper times of any physical events, since there are no such events in the end product of the experiment or they are simply a mathematical device to solve the equation 4.1 .

For the sake of completeness let us suppose the unlikely event that the interactions represented by the factors of the products  $\{H_I(t_1)H_I(t_2)\dots H_I(t_n) | n = 1,2,\dots\}$  were sequential in the time neighbourhood  $(t', t)$  and let us scrutinize their time structure within  $(t', t)$ . Then:

(i) If the integration interval,  $(t', t)$ , is taken larger than the interaction proper time neighbourhood,  $(t'_r, t_r)$ , of the reaction at hand, then, of course, the interaction Hamiltonian  $H(t)$  must be put equal to zero outside  $(t'_r, t_r) \subseteq (t', t)$ .

(ii) If the  $(n+1)$ -th interaction strictly precedes the  $n$ -th, for all  $n$ , then the interaction proper time neighbourhoods  $(t'_n, t_n)$  have the sum

$$T_N = \sum_{n=1}^N [t_n - t'_n]$$

tends to infinity for  $N \rightarrow \infty$ . This result would be not compatible with the finiteness of the integration time neighbourhood  $(t', t)$ . In other words, the evolution would not consistently be definable for finite time neighbourhoods.

(iii) If the integration time neighbourhood  $(t', t)$  is taken to be equal to the fastest process interaction proper time neighbourhood, and the product

$$\{H_I(t_1)H_I(t_2)\dots H_I(t_n) | n = 1,2,\dots\} \quad 4.3$$

represents  $M(n)$  sequential interactions, then the sum of the interaction proper times of these interaction processes would exceed the integration time interval, although it is supposed to be a sub-set of it.

(iv) Associated to the product 4.3 of  $n$  factors there is a number  $F(n)$  of sequential Feynman diagrams. If  $n \rightarrow \infty$ , then the number of internal lines tends also to infinity in some of the cases. In order that the propagation along the internal lines in question be

realized within a finite integration time neighborhood,  $(t, t + \Delta t)$ , the propagation would have - according to the perturbation theory - to proceed with infinite velocity, which is contrary to the Relativity.

The physically incorrect conclusions (i-iv) were reached supposing that the products 4.3 represented real sequential particle interactions. The integrations are indeed sequential and help to eliminate the artificially introduced integration time variables by means of the iteration solution method of 4.1 .

These interaction operator products do not represent any observable particle reactions which would require definite proper times with a total sum which is not available to the actual reaction. This conclusion is supported also by the corresponding Feynman diagrams in whose external lines do not appear the particles involved in all diagrams, but appear only the particles in fact observed.

These interaction operator products 4.3 do not represent simultaneous reactions either for the following reasons:

- a) If the initial state contains only one elementary particle, it would have to interact at the same time in as many different ways,  $F(n)$ , as the Feynman diagrams are. This, however, is equivalent to the statement that the *ελεμενταριου* *ποστιθλας* after the initial state is *distributed* over the  $F(n)$  reaction modes. This contradicts the currently accepted concept of an elementary particle.
- b) If the initial state contains many particles, the reactions would necessarily be at least partially sequential, because the beam particles are at least partially sequentially produced. The event of sequential reactions, however, has been rejected by the conclusions (i-vi) above.

Consequently, the  $n-1$  integrations are effected sequentially and represent a kind of elimination of the integration variables.

The general conclusion so far which regards the interpretation of the wave function is:

Each action of the evolution operator on the wave function describes one single reaction of one beam particle with the field or with the target of the experimental setting.

#### 4c The interactions and the evolution of the wave function.

Now we turn our attention to the wave function. The action of  $U(t, t')$  in the expression

$$\psi(t) = U(t, t')\psi(t')$$

is to bring  $\psi(t')$  from time  $t'$  to time  $t$ .

Next we observe that, if the beam consisted of one single particle the beam transition due to the interaction with the target would be realized by means of a single action of the evolution operator  $U(t, t')$  on the one-particle beam wave function  $\psi(\mathbf{r}, t)$ . Now the actual particle number in the beam is  $N$ , and it is clear that the beam transition due to the interaction with the target will be correctly realized by means of  $N$  distinct actions of the evolution operator on the initial wave function  $\psi_0(\mathbf{r}'_1, t'_1; \dots; \mathbf{r}'_N, t'_N)$ . Because it is not possible that one and the same evolution operator implements a one particle interaction transition and a many particles interaction transition.

The integration in the evolution operator is done in each one of its actions over the interaction proper time neighbourhood  $(t_j - t'_j)$  of the corresponding  $j$ -th beam particle with the target whose space-time coordinates  $(\mathbf{r}'_j, t'_j)$  are acted on.

Hence, we let  $U(t, t')$  act  $N$  times on the initial state wave function of the beam, i.e., we let act the product

$U(t_N, t'_N)U(t_{N-1}, t'_{N-1}) \dots U(t_1, t'_1)$  on  $\psi_0(\mathbf{r}'_1, t'_1; \dots; \mathbf{r}'_N, t'_N)$  to obtain the scattering wave:

$$\begin{aligned} & \psi(\mathbf{r}_1, t_1; \dots; \mathbf{r}_N, t_N) \\ &= U(t_N, t'_N)U(t_{N-1}, t'_{N-1}) \dots U(t_1, t'_1)\psi_0(\mathbf{r}'_1, t'_1; \dots; \mathbf{r}'_N, t'_N), \\ &= U(t_N, t'_N)U(t_{N-1}, t'_{N-1}) \dots U(t_2, t'_2)\psi_0(\mathbf{r}_1, t_1; \mathbf{r}'_2, t'_2; \dots; \mathbf{r}'_N, t'_N), \\ &= U(t_N, t'_N)U(t_{N-1}, t'_{N-1}) \dots U(t_3, t'_3)\psi_0(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2; \mathbf{r}'_3, t'_3; \dots; \mathbf{r}'_N, t'_N) \\ &= U(t_N, t'_N)U(t_{N-1}, t'_{N-1})\psi_0(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2; \dots; \mathbf{r}_{N-1}, t_{N-1}; \mathbf{r}'_{N-1}, t'_{N-1}; \mathbf{r}'_N, t'_N), \end{aligned}$$

$$= U(t_N, t'_N) \psi_0(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2; \mathbf{r}_3, t_3; \dots; \mathbf{r}_{N-1}, t_{N-1}; \mathbf{r}'_N, t'_N) \quad 4.4$$

Now, we observe that the wave function after the reaction depends on  $N$  random quantities,  $(t_i - t'_i)$  the interactions proper times of the beam particles.

The ordering of the factors  $U(t_j, t'_j)$  in the product  $U(t_N, t'_N)U(t_{N-1}, t'_{N-1}) \dots U(t_1, t'_1)$  is not arbitrary, because the arrival of the beam particles on the target is time ordered in the target rest frame of reference. To calculate the cross section for the reaction the effective practice is to have a one-particle wave function<sup>11</sup>.

In order to obtain a one-particle beam wave function,  $\psi(\mathbf{r}, t)$ , representing the ensemble of the beam particles after their interactions with the target, two operations are required which impose the statistical character onto the wave function:

#### 4d Space statistics

All space coordinates in the beam wave function after the reaction are set equal to the position vector  $\mathbf{r}$  of the observation "point" in the detector. It is clear that such a point is any point in the active volume of the detector, and that the differences  $\{|\mathbf{r}_i - \mathbf{r}_j|\}$  between the actual vectors  $(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \dots \mathbf{r}_N)$  are in general not a negligible part of  $r_i$  or  $r_j$ , and the reaction theory of nuclear reactions should take into account the quantities  $\{|\mathbf{r}_i - \mathbf{r}_j|, i, j = 1, 2, \dots\}$  in the expression  $\psi(\mathbf{r}_i; t_1, \dots, t_N) = [\psi(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N)]^{1/N}$ . 4.5

These differences in the space coordinates of the particles after the interaction contain one component stemming from the stochastic nature of the interacting fields.

This fact makes necessary the application of statistical methods to obtain a value for the representative position of the beam particles in  $\psi(\mathbf{r}, t)$ . We multiply

$[\psi(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N)]^{1/N}$

by the column vector  $\mathbf{r}_i$  and have

$$\mathbf{r}_i [\psi(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N)]^{1/N}. \quad 4.6$$

We subsequently form the row vector

$$\mathbf{r}_i^T [\psi^*(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N)]^{1/N} \quad 4.7$$

and multiply 4.6 by 4.7

$$\mathbf{r}_i^T \mathbf{r}_i [\psi^*(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N) \psi(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N)]^{1/N}$$

Next, we sum over all possible positions of the  $i$ -th particle in the active detector volume and obtain for the average value of the square  $r_i^2$  using the normalization condition for the wave function

$$\langle r_i^2 \rangle = \int \mathbf{r}_i^T \mathbf{r}_i |\psi^*(\mathbf{r}_1, t_1; \dots; \mathbf{r}_i, t_i; \dots; \mathbf{r}_N, t_N)|^{2/N} d\mathbf{r}_i^3$$

$$/ \int |\psi(\mathbf{r}_1; t_1, \dots, t_N)|^2 d\mathbf{r}_i^3. \quad 4.8$$

Summing 4.8 over all  $i$ -values and dividing by  $N$  we obtain the average square of the "position" vector of the beam in the detector

$$\overline{\langle r^2 \rangle} = \sum \langle r_i^2 \rangle / N. \quad 4.9$$

The position vector of the  $i$ -th beam particle is, therefore,  $\mathbf{r}_i = \hat{\mathbf{r}}_i (r_i + \langle r_i^2 \rangle^{1/2})$ . After taking the limit  $r \rightarrow \infty$ , as required by the scattering theory, stochasticity remains in the wave function through  $\langle r_i^2 \rangle$  which contain also the component coming from the stochasticity of the fields.

Another important fact to be pointed out is the reason for which the probability density,  $\rho(\mathbf{r}, t)$ , in Quantum Mechanics is given by  $|\psi(\mathbf{r}, t)|^2$  and not by  $|\psi(\mathbf{r}, t)|$ . This is due to the necessity for smoothness<sup>13</sup> of the gradient or the current density. While  $|\psi(\mathbf{r}, t)|$  has a discontinuous derivative for  $\mathbf{r}$  values for which  $\psi(\mathbf{r}, t) = 0$ , the derivative of  $|\psi(\mathbf{r}, t)|^m$ ,  $m > 1$ , is continuous (fig. 2). If  $z = \text{complex}$ , then  $z = |z|e^{i\theta}$ ,  $z^* = |z|e^{-i\theta}$ . If one takes  $m=2$ , then questions of uniqueness appear. For example, if  $m=v$ , ( $1 < v < 2$ ), then  $z = r \cdot \exp[i\theta \pm 2in\pi]$  with  $n$  any integer has the same value, while  $z^v = r^v \exp[iv\theta \pm 2niv]$  has not the same property for any  $n$ . This does not happen for  $m=2, 3, 4, \dots$ . Economy and Hilbert space considerations suggest taking  $m=2$ .

This completes the proof for the statistical nature of the wave function with respect to the space coordinate justified the Max Born choice.

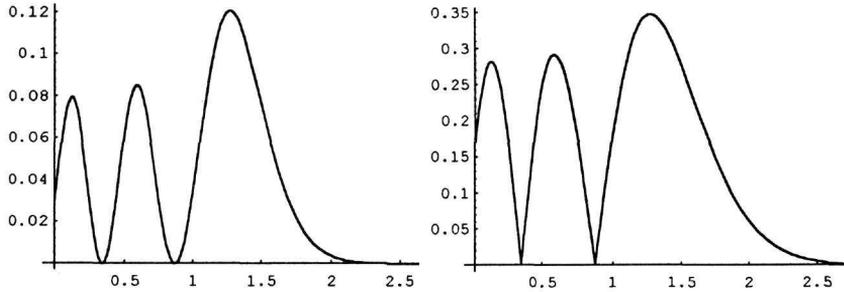


fig.2 Taking the square of the absolute value of a wave function makes the current density continuous at the zeros of the wave function. This is shown in above: (a)  $|\psi|^2$  has a continuous derivative and (b)  $|\psi|$  has a discontinuous derivative at the zeros.

#### 4e Time statistics.

To obtain the average result of the action of the evolution operator  $U(t_N, t'_N)U(t_{N-1}, t'_{N-1})\dots U(t_1, t'_1)$  on the initial beam wave function  $\psi_0(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2; \mathbf{r}_3, t_3; \dots \mathbf{r}_{N-1}, t_{N-1}; \mathbf{r}'_N, t'_N)$  we have to take the geometric mean of this evolution operator

$$\Theta(t, t') = [U(t_N, t'_N)U(t_{N-1}, t'_{N-1})\dots U(t_1, t'_1)]^{1/N}. \quad 4.10$$

From the form of the evolution operator and from the averaged wave function we obtain

$$\begin{aligned} \psi(\mathbf{r}; t) &= \Theta(t, t')\psi_0(\mathbf{r}'; t') \\ &= [U(t_N, t'_N)U(t_{N-1}, t'_{N-1})\dots U(t_1, t'_1)]^{1/N} \\ &\quad \times [\psi(\mathbf{r}_1, t_1; \dots \mathbf{r}_N, t_N)]^{1/N} \end{aligned} \quad 4.11$$

There have, thus, been quite naturally introduced two statistical aspects into the wave function: The space averaging and the time averaging.

Hence, equations 4.4 contain  $N$  random time parameters. Consequently, if we represent the outgoing wave of the beam by a

single-particle wave function,  $\Psi(\mathbf{r}, t)$ , then this is related to the 4.4 through

$$\Psi(\mathbf{r}, t) \Rightarrow \Psi(\mathbf{r}, t_1; \dots \mathbf{r}, t_N) = \{U(t_N, t'_N)U(t_{N-1}, t'_{N-1}) \dots U(t_1, t'_1)\}^{1/N} \Psi_0(\mathbf{r}'_1, t'_1; \dots \mathbf{r}'_N, t'_N) \quad 4.12$$

In 4.11  $\Psi(\mathbf{r}_1, t_1; \dots \mathbf{r}_N, t_N)$  is replaced by  $\Psi(\mathbf{r}, t_1; \dots \mathbf{r}, t_N)$  and consequently by  $\Psi(\mathbf{r}, t)$ , because all beam particles are observed at the times  $t_1, \dots, t_N$ .

#### 4f The bound state case.

The wave function of the particles beam has been considered sofar in a *reaction experiment*. The sescteness of the particles has made it possible to apply enumerably many times the evolution operator on the initial state to obtain the final state, and justify the necessity for the statistical interpretation.

How should one proceed in the case of the *bound state problem* to obtain the statistical character of the wave function?

To answer this question it is necessary to realize that no interaction is a continuous process. The interaction responsible for the bouding of any particles is mediated by the exchange of field quanta between the bound particles.

The field quanta are the discrete entities par excellence in Nature. The non-continuity of the exchanged quanta makes the quanta exchange discrete. Hence, the totality of interactions in the bound states is countable.

Each interaction adds to the position vector as well as to the proper time of the bound particles stochastic components. This makes the argumentation for the beam particles in the reaction experiment applicable on the bound states too.

In the same way the space-time coordinates of every quantum becomes stochastic. The stochastic quantities are introduced into the wave function - which is as a deterministic solution of the Schrödinger equation - by means of the stochastic interactions.

This concludes the demonstration that the statistical character of the wave function derives from the stochastic nature of the fields.

## 5 DISSIPATION AND IRREVERSIBILITY AS IMPLICATIONS OF THE STOCHASTIC FIELDS - THE TIME ARROW IN NATURE

### 5a The stochastic and infinitely divisible fields

It has been shown in section 4 that the stochastic nature of the evolution of a beam of particles due to interactions necessarily leads to a wave function whose significance must be statistical.

Since the evolution of a quantum field is described by the same evolution operator as the one acting on a wave function representing the distribution density of particles, it is reasonable to assume that the quantum fields too must be stochastic in the sense of the theory of the *generalized stochastic fields*<sup>14</sup>.

Furthermore, it will be assumed that any function of the fields, e.g., the Lagrangian density,  $L(\phi(x), \pi(x))$ , must equally well represent a generalized stochastic field.

Following this philosophy it was possible to demonstrate<sup>15</sup> that:

*Statistical Mechanics is derivable from Quantum Field Theory and is representable (not in Euclidean space but) in the same Minkowski space-time as the quantum fields themselves (with the metric imposed by Relativity).*

Here the following statements will be shown:

i) *The arrow of time and the irreversibility in Nature is a consequence of the stochasticity and of the infinite divisibility of the quantum fields.*

ii) *There exist on the quantum level creative and dissipative processes described by corresponding evolution operators as in ordinary Quantum Field Theory which violate the T-symmetry.*

The demonstration of the above statements is based on the following *fundamental principles*:

A        *The Lagrangian density,  $L$ , of the quantum field is a generalized stochastic field.*

It is recalled that a field  $L \in \mathbb{R}$  is said to be stochastic, if for  $L < \xi \in \mathbb{R}$  a probability  $P(\xi)$  is given such that the conditions

be satisfied:

- a)  $P(\xi_1) \leq P(\xi_2)$ , if  $\xi_1 \leq \xi_2$ ,  
 b)  $\lim_{\xi \rightarrow -\infty} P(\xi) = 0$  and  $\lim_{\xi \rightarrow \infty} P(\xi) = 1$ ,  
 c)  $\lim_{\xi \rightarrow a-0} P(\xi) = P(a)$ .

5.1

**B** The Lagrangian density,  $L$ , of the quantum field is an infinitely divisible field.

A random field  $L$  is said to be infinitely divisible, if for every positive integer  $N$  the decomposition is possible

$$L = L_1 + L_2 + \dots + L_N, \quad 5.2$$

in which  $\{L_j | j=1, 2, \dots, N, \forall N\}$  are mutually independent (stochastic) and have identical probability distributions  $\{P(\xi_j)\}$ .

Although principles A and B have not been spelled out explicitly by Feynman, he used them in his heuristic finding of the path integral from which he derived the quantum theory<sup>16</sup>. It will be shown in the next section that the Feynman path integral is a particular case of the present theory.

### 5b The stochastic QFT-evolution operator

From the Schrödinger equation

$$i\hbar \frac{\partial \psi(r)}{\partial r} = H(r)\psi(r), \quad r \in (t_0, t) \quad 5.3$$

one formally finds the solution

$$\psi(t) = \psi(t_0) - e^{(-i\hbar)^{-1} \int_{t_0}^t H(r) dr} \psi(t_0)$$

This can be written also in the form

$$\psi(\Delta t + t_0) = \psi(t_0) - e^{(-i\hbar)^{-1} \int_{t_0}^{\Delta t + t_0} H(t') dt'} \psi(t_0), \quad \Delta t = (t - t_0) + t_0. \quad 5.4$$

$$\psi(\Delta t + t_0) = U(\Delta t + t_0, t_0) \psi(t_0), \quad 5.5$$

where  $U(t, t_0)$  is an operator bringing  $\psi$  from  $t_0$  to  $t = \Delta t + t_0$ . Hence,

$$U(\Delta t + t_0, t_0) = \Pi \left( e^{(-i\hbar)^{-1} \int_{t_0}^t H(t') dt'} \right). \quad 5.6a$$

Here we observe for later use, that omitting the imaginary unity "i" in the exponent of 5.6 implies the appearance of it as a factor to  $t$  in  $U(i\Delta t + it_0, it_0)$ ,

$$U(i\Delta t + it_0, it_0) = \Pi \left( e^{(-\hbar)^{-1} \int_{it_0}^{it_0 + i\Delta t} H(t') dt'} \right). \quad 5.6b$$

For simplicity we consider a scalar field and we recall that the Hamiltonian and the Lagrangian densities are related through

$$H(x) = \partial_0 \phi(x) \pi(x) - L(\phi(x), \pi(x)), \quad 5.7$$

where  $H(t) = \int d^3x H(x, t)$ .

If  $H(x)$  is replaced in 5.6b by its equivalent expression given by 5.7 and if the part of the exponential containing  $\pi(x) \partial_0 \phi(x)$  is expanded in a series of integrals one gets

$$U(t, t_0) = \left( 1 + \sum_n \frac{(i\hbar)^n}{n!} \left[ \prod_{j=1}^n \int d^3x_j \int d\phi(x_j) \pi(x_j) \right] \right) \times e^{(i\hbar)^{-1} \int_{t_0}^t [L(\phi(x), \pi(x))] d^4x}. \quad 5.8$$

We have used in 5.8 the notation:  $d\phi(x) = \partial_0 \phi(x) \cdot dt$ ,  $d^4x = d^3x \cdot dt$ .

Next the property 5.2 is used in 5.8, and the last exponential factor becomes for every value of  $n$

$$\prod_{j=0}^n \left( e^{(i\hbar)^{-1} \int_{t_0}^t [L_j(\phi(x_j), \pi(x_j))] d^4x_j} \right). \quad 5.9$$

From 5.8 and 5.9 it follows that

$$U(t, t_0) = e^{(i\hbar)^{-1} \int_{t_0}^t [L(\phi(x), \pi(x))] d^4x} + \left( \sum_n \frac{(i\hbar)^n}{n!} \left[ \prod_{j=0}^n \int d^3x_j \int d\phi(x_j) \pi(x_j) \right] \right) \times e^{(i\hbar)^{-1} \int_{t_0}^t [L_j(\phi(x_j), \pi(x_j))] d^4x_j} ]$$

It is observed in passing that the Feynman path integral

$$F = \frac{1}{2\pi} \int D\phi D\pi \exp[i\hbar^{-1} \int L[q(t), p(t)] dt]$$

follows formally from the above expansion by using the substitutions

$$\frac{1}{n!} \left[ \prod_{j=1}^n \int d^3x_j \int d\phi(x_j) n(x_j) \right] \rightarrow 1/(2n) \prod_{j=1}^n \int d\phi(x_j) dn(x_j)$$

$$p \rightarrow n(x) \quad \text{and} \quad q \rightarrow \phi(x)$$

for  $n \rightarrow \infty$ .

As a minor remark on the conceptual level: the product

$$d\phi(x_j) dn(x_j)$$

in the Feynman path integral would not be compatible with Heisenberg's uncertainty principle. This problem does not exist in our expansion, because  $n(x)$  does not appear in differential form.

Summing the series of integrals with application of the principles A and B we get

$$U(t, t_0) = \left( e^{-i/\hbar \int d^3x \int d\phi(x) n(x)} \exp[i/\hbar \int d^4x L(\phi(x), n(x))] \right). \quad 5.10$$

This is a new form of the evolution operator in field theory. Its exponent consists of two parts, a real and an imaginary.

The physical interpretation of its action on the state vector depends critically on time variable structure, i.e., whether the point of view of the *universal time* or of the *interaction proper time neighbourhood* is taken. This will be shown in what follows.

### 5c Non-unitarity and non-conservation in stochastic QFT

Next, it will be shown that the stochasticity and the infinite divisibility conditions modify the unitarity of the evolution operator in a very particular way. It imparts to the exponent three very important properties:

- i) A real part which violates the  $T$ -symmetry in PCT
- ii) A very special possibility for quantizing the field action and separating the unitary and the non-unitary parts of the evolution operator.
- iii) The unitary part obtained after the quantization of the field action contains as a special case the original evolution operator as before the application of the principles A and B.

Next, it will be demonstrated that the evolution operator 5.10 is not unitary

$$U^2(t_0, t)U(t, t_0) \neq \mathbb{1} \neq U(t_0, t)U^2(t_0, t).$$

If we write equation 5.10 in the more expressive form

$$U(t, t_0) = U_c(t, t_0)U_{n-c}(t, t_0),$$

where the sub-indices "c" = conservative and "n-c" = non-conservative, respectively and

$$U_c(t, t_0) = \left( e^{-i/\hbar \int d^3x \int d\phi(x) n(x) \{ \cos[1/\hbar \int d^4x L(\phi(x), \pi(x))] \}} \right) \quad 5.11a$$

$$U_{n-c}(t, t_0) = \left( e^{+i/\hbar \int d^3x \int d\phi(x) n(x) \{ \sin[1/\hbar \int d^4x L(\phi(x), \pi(x))] \}} \right) \quad 5.11b$$

we see immediately that the part with imaginary exponent obeys the condition

$$U_c(t, t_0)U_c^2(t, t_0) = \mathbb{1}$$

The operation "2" applied to  $U_c(t, t_0)$  changes once sign of the exponential and the above condition is satisfied.

On the other hand for the operation  $t \rightarrow t' = -t; n(x) \rightarrow -n(x)$ . The cosine-factor conserves its sign under  $t \rightarrow t' = -t$  and  $L$  is time reversal invariant and hermitian. Hence,

$$U_c(t, t_0) = U_c^{-1}(t_0, t).$$

This proves the unitarity of  $U_c(t, t_0)$ .

Concerning the second factor in the  $U_{n-c}(t, t_0)$  the exponent is real and the operation "2" does not change anything in it. On the other hand  $t \rightarrow t' = -t; (n(x) \rightarrow -n(x'), d^4x \rightarrow -d^4x')$ , so that  $\sin[\int d^4x L(\phi(x'), \pi(x'))]$  changes only once sign.

This proves that  $U_{n-c}(t, t_0)$  is not unitary.

Hence, the evolution operator 5.10 shows a complex behavior and it is interesting to study its two contributions to the evolution: The *conservative* and the *non-conservative*.

Let us try to see, how it is possible to separate the two parts which exhibit so contrary properties.

The *quantization of the action integral* was for Niels Bohr a possibility to obtain the quantum description of the atomic energy structure. We shall follow the Bohr way and *quantize the field action*. This will offer the the basis for what will be done in the sequel.

We first define a number  $\Lambda(n,\sigma)$  depending on two integers by

$$\Lambda(n,\sigma) = n \begin{cases} n+1/2 & \text{for } \sigma = 1 \\ n & \text{for } \sigma = 2 \end{cases}, n=1,2,3,\dots \quad 5.12$$

Then we require the action to take integral or half-odd multiples of the Planck constant

$$\int d^4x L[\phi(x),\pi(x)] = \pm \hbar \Lambda(n,\sigma), \text{action quantization.} \quad 5.13$$

Next, we use the relation 5.7 and 5.13 becomes

$$\int d^3x \int d\phi(x)\pi(x) = \int_{t_0}^t dt H(t) \pm \hbar \Lambda(n,\sigma).$$

We observe in passing, that the quantization relation represents at the same time a kind of action renormalization. It is, further observed that the quantization condition 5.13 has as a consequence that the operator  $U_c(t, t_0)$  becomes the unit operator,  $\mathbb{1}$ , when  $U_{n-c}(t, t_0)$  determines the non-conservative evolution of the system.

Vice-versa, when  $U_c(t, t_0)$  determines the conservative evolution of the system, then  $U_{n-c}(t, t_0)$  becomes equal to  $\mathbb{1}$ . In more detail it is seen that  $U_c(t, t_0)$  corresponds to

$$\sin[1/\hbar \int d^4x L(\phi(x), \pi(x))] = 0, \text{ and } \Lambda(n,\sigma) = n \cdot n \text{ for } \sigma=2,$$

is time reversal invariant, unitary and, therefore, conservative, both in the *universal time* framework as well as in the *interaction proper time interval*,

$$U_c(t, t_0) = \exp[\mp i \hbar^{-1} \int_{t_0}^t dt H(t) \pm \Lambda(n,2)], \text{ conservative, } t \geq t_0, \quad 5.14$$

where "-" goes with 0,2,4,6..., and "+" goes with 1,3,5,7... .

In the conservative case there is no substantial difference, whether the time is the universal  $t \in (-\infty, +\infty)$  or the interaction proper-time neighbourhood  $t \in (t', t'')$  because of the oscillatory character of the evolution operator.

The non-conservative part,  $U_{n-c}(t, t_0)$ , corresponds to

$$\cos[1/\hbar \int d^4x L(\phi(x), \pi(x))] = 0 \text{ for } (\Lambda(n,\sigma) = n \cdot (n + \frac{1}{2}), \sigma=1),$$

the exponent is real and it *no longer* is a matter of indifference to which direction the time runs.

This evolution operator is a realization of the irreversibili-

ty on both the *quantum* and on the *macroscopic* level.

If the time runs in the positive direction, the probability of corresponding state acted on by  $U_{n-c}(t, t_0)$  increases. On the contrary, if the time is reversed, then the probability of the state decreases.

$$U_{n-c}(t, t_0) = \exp[\pm i \hbar^{-1} \int_{t_0}^t dt H(t) \pm \Lambda(n, 1)], \text{ non-conservative, } 5.15$$

where "+" implies an increase of the state vector norm and goes with  $n = 1/2, 5/2, 9/2 \dots$ , while "-" leads to a shrinkage of the norm and goes with  $n = 3/2, 7/2, 11/2 \dots$ .

It should be pointed out that the conclusions which may be drawn from 5.16 depend on whether we consider evolution in the universal time  $t \in (-\infty, +\infty)$  or in the interaction proper time neighbourhood  $t \in (t', t'')$ . In the first case and for  $n = 1/2, 5/2, 9/2 \dots$  the norm of the state vector explodes. In the second case and for  $n = 3/2, 7/2, 11/2 \dots$  the norm remains finite.

#### 5d The creative evolution - universal time versus proper time

Equation 5.15 gives the the non-conservative evolution operator,  $U_{n-c}(t, t_0)$  which violates  $T$ -symmetry. For  $t \rightarrow +\infty$  makes the norm of the state vector *explode* and is called *creative*. It interesting to note that even for  $t \rightarrow +\infty$   $U_{n-c}(t, t_0)$  may give finite results, provided renormalization is applied by giving to  $-\Lambda(n, 1)$  the appropriate value of  $n$  which is a free quantum number.

For  $t \rightarrow -\infty$  and  $n = 3/2, 7/2, 11/2 \dots$   $U_{n-c}(t, t_0)$  brings the state vector norm to extinction. For this reason it is called *destructive*. Here, too, renormalisation is possible by giving to  $\Lambda(n, 1)$  the appropriate value of  $n$ .

Since the universal time results as the union of many interaction proper-time neighbourhoods, it is a macroscopic quantity and applies to macrocosmos. Indeed, it is tempting to *describe the big-bang of the created Universe with an exploding state under the action of the creative operator*.

Both, the dissipative and the creative operators implement *ir-*

reversible processes.

Collecting both forms we may write

$$U(t, t_0) = \begin{cases} \exp[-i\hbar^{-1} \int_{t_0}^t dt H(t) \pm \Lambda(n,1)], & \text{conservative (a)} \\ \exp[+ \hbar^{-1} \int_{t_0}^t dt H(t) \pm \Lambda(n,2)], & \left\{ \begin{array}{l} \text{dissipative} \\ \text{or creative.} \end{array} \right. \text{(b)} \end{cases} \quad 5.16$$

a) *The universal time case* ( $t = \bigcup_a \tau_a$ ).

(ia) *The stochasticity and the infinite divisibility of the fields implies the existence of an arrow of time in Nature.*

This follows from the fact that the two signs " $\pm t$ " in the exponent of 5.16b lead to states of completely different character. It is perhaps of importance that the exponent appears automatically with the "+" sign, might mean that all fields are in the phase of explosion. This explosion is not continuous in the universal time everywhere. It advances by one step in every interaction proper time neighbourhood.

(ib) *The majority of the physical systems evolve in an irreversible way.*

This follows from the different powers of the two number sets:

The set of the quantum numbers leading to irreversible processes and the set of quantum numbers leading to reversible processes. The action values leading to irreversible processes correspond to the quantum numbers of the union of the two number sets

$$\text{Power of } \{\Lambda(n,2)\} \cup \{\mathbb{R} \setminus \Lambda(n,1)\} > \text{Power of } \{\Lambda(n,1)\}.$$

All above cases concern macroscopic level irreversibility. In what follows we consider transitions in the interaction proper time neighbourhood.

ii) *The interaction proper time neighbourhood.*

(iia) *Irreversibility in atomic and sub-atomic transitions.*

It has been always considered as the correct point of view in Science, due to the time-reversal-invariance of the fundamental equations of Physics, that there existed no irreversibility on atomic or sub-atomic level.

The validity of the PCT theorem allowed according to the view accepted until 1964 at most simultaneous P- and C-symmetry violations. Indeed, the  $\beta$ -decay of the weak interaction was the first in 1957 to yield the proof that for the existence of simultaneous P- and C-symmetry violations, while conserving PCT-symmetry<sup>17</sup>.

As a consequence of this fact the experimental observation of a small T-asymmetry in the decay of the neutral K-system into two, instead of three,  $n^\pm$  mesons was an inexplicable puzzle in the frame of standard field theories.

The present theory allows according to 5.15 for T-symmetry violations also in the individual elementary particle interactions.

The integration in 5.16b of the Hamiltonian is effected in the interaction proper time neighbourhood leads to a very small *microscopically finite irreversibility*. The small magnitude of the T-asymmetry makes the observation difficult.

(iib) *In the case of 5.16a the norm of the state vector remains invariant after the interaction proper time interval of the process. This case encompasses all individual interactions of elementary particles which are T-symmetric.*

The above conclusions regard the behavior of the state vector under a single action of the evolution operator describing one particular particle interaction. In the case in which one is interested in statistically describing a system with many particles, then one has to apply on the state vector all evolution operators for the  $N$  individual interactions and take the  $N$ -th root of the product of the  $N$ -factor product of  $U$  operators.

### 5e The arrow of time in nature and its relation to the entropy

The overwhelming majority of the macroscopic physical systems evolve in time in an irreversible way. The entropy, on the other hand, increases in every closed system evolving irreversibly. Although this increase is not proportional to the time - which does not explicitly appear in thermodynamics - imposes this loose analogy between time and these phenomena the impression of a causal relationship.

The  $H$ -theorem in which the time appears explicitly indicates that a non-Maxwellian velocity distribution of gas molecules is converted into a Maxwellian one. The entropy of the gas increases thereby. It is observed that the time the equilibrium takes to establish itself is of the order of the mean free time between collisions.

Since the same entropy increase may appear, in many gases with different mean free times, it is concluded that there is not a unique proportionality between entropy and time<sup>18</sup>.

It has been indicated above that entropy is not proportional to the time. Here are two further a little more detailed arguments. i) The entropy is essentially defined as the ratio of the heat  $Q$  involved in a thermodynamic process and the corresponding absolute temperature.

The expression  $dQ/T$  is integrated between two thermodynamic states. Here, however, neither the heat nor the temperature are fundamental physical quantities nor is any of them proportional to any fundamental quantity. There is, therefore, no obvious reason for the entropy to be more fundamental than the heat or the temperature.

This is a formal argument. A physical argument runs as follows: ii) The proof of the fact that the entropy is a non-decreasing (= constant or increasing) quantity is based on a series of mathematical facts that are *traced back to the physical process in which heat flows from a hotter to a colder body.*

This last fact belongs to a whole class of similar and equally fundamental phenomena as is, for example, the falling water in the clephydra or the lowering of the pendulum mass, etc., processes generating the impression of the flowing of the macroscopic time.

The series of facts leading to the *almost* increasing of the entropy are the following:

ii) The relationship between the heats of two heat engines, one reversible ( $Q_1, Q_2$ ) and one possibly irreversible ( $Q'_1, Q'_2$ )

$$\frac{Q_2}{Q_1} \geq \frac{Q'_2}{Q'_1} \quad 5.19$$

iib) The property of being negative of the integral

$$\oint \frac{dQ}{T} \leq 0, \quad 5.20$$

which is based<sup>19</sup> on iia).

Consequently, the characteristic property of the entropy

$$S(B) \geq S(A) + \int_A^B \frac{dQ}{T},$$

where the integration is done in an irreversible or reversible way respectively ( $\geq$ ), for which it receives a fundamental like look is traced back to a change (heat transfer) generating, alike the clepsydra, the impression of the time flow.

Because, if the universe consisted of two heat sources of equal temperatures, no macroscopic change whatsoever would be possible and no time flow would be observable. (This would correspond to a clepsydra having the two water compartments on the same level).

On the contrary, if the heat sources have different temperatures, a heat flow occurs and the time flow impression is generated.

Once it is established that the importance of the entropy for the arrow of time is traced back to a particular time - irreversible phenomenon, it is logical to consider directly the class of these phenomena separated from other accompanying secondary facts or logically following them.

In the case of the entropy secondary facts (following logically from the heat flow) are, e.g., the relationship 5.19 and the heat engines.

We are here interested in primary causes of the arrow of time.

The stochasticity and the infinite divisibility properties of the fields lead to a complex exponent of the operator - allowing in this way for the field action any value. This gives to the action the power of the *complex continuum* including the set of the half-odd numbers.

If the action has not been quantized, one might conclude that the evolution operator describes either quantum systems with continuous physical characteristics, or non-quantum (macroscopic) systems. This would, however, be an erroneous conclusion based exclusively on the assumption of a universal time.

Before we discuss in 5f the arrow of time in detail, it is appropriate to say that there are two levels of the arrow of time in Nature which are, of course, of common origine: the *quantum level and the macroscopic arrows of time*.

### 5f The quantum arrow of time

Let us demonstrate first the existence of an arrow of time on the quantum level. Since the proof for the existence of a quantum arrow is the existence of equation 5.15 itself, it is sufficient to discuss the conditions under which  $U_{n-c}(t, t_0)$  has been derived.

As a matter of fact all quantum scale systems are endowed with discrete values of their physical characteristics (except some properties of free particles: continuous energy and momentum). This suggested to quantize the action integral of the field.

The idea for so doing came not only from our intention to follow Bohr's example, but also from the observation that in this way the general evolution operator, 5.10, breaks down into two completely different evolution operators.

This last fact leads on the one hand to 5.11a which is unitary and contains as a particular case the evolution operator of the usual QFT differing only by the *subtracted zero point action*  $\frac{1}{2}\hbar$ .

On the other hand 5.11b is obtained with a real exponent. This highly desirable exponent in Statistical Mechanics was impossible to derive in the framework of the usual QFT in the Minkowski space. Just for this reason one resorted to the Euclidean field theories to get the real exponent (self-adjoint contraction semi-group leading to the Boltzmann factor<sup>20</sup>).

This evolution operator breaks the  $T$ -symmetry and imparts to the state vector (on which it acts) the arrow of time. Since the time integration is done on the interaction proper time neighbour-

hood it is the *quantum level arrow of time*.

The question arising thereby is, if and how this quantum-irreversibility leads to an *extinction* or to an *explosion* of the initial state and, therefore, to an easy observation of the T-symmetry violation.

The answer is evident, if we keep in mind that the time arrow we discuss arises from *elementary interactions*. It is clear that the time generated by one interaction, (the interaction proper time neighbourhood) can not go to infinity. So extinction or explosion of the state cannot occur on the quantum level. The *T*-asymmetric interaction only increases or diminishes the probability of corresponding state.

On the other hand the finiteness of the interaction proper time neighbourhood has as a consequence that *T*-asymmetry on the quantum level cannot be easily observed. This is due to the smallness of the factor  $\exp[\pm \hbar^{-1} \int H(t) dt]$  implying a small change of the state vector. One can easier observe simultaneous PC-symmetry violation instead of the *T*-symmetry violation.

It becomes sufficiently clear from the above that 5.11b demonstrates the existence of the arrow of time on the quantum level.

An example of a T-symmetry violation<sup>21</sup> in accordance with the *quantum arrow of time* is the branching of the neutral *kaon* decay.

In the presented example of the arrow of time the scalar field Hamiltonian has been used. However, is the formalism for the derivation of equation 5.15 model-independent and it applies to every covariant field theory.

### 5g The macroscopic level time arrow

The evolution problem consists in that the time-reversal-invariance of the fundamental (classical and quantum) equations of Physics and the irreversibility of the majority of the phenomena of the macrocosmos were logically independent in the framework of the standard theories.

Many, very diverse, attempts are known aiming at establishing a physical and logical connection between the microscopic and the macroscopic evolutions. This connection has not yet appeared in

the frame of the well-known field theories.

The main criterion of any successful macroscopic theory according to the present work is that the T-asymmetry must derive from the PCT-symmetrical theories by adding a new, missing, property to the fields.

This requirement is fulfilled by the present theory. The new property of the fields is that of the *generalized stochastic and infinitely divisible fields*.

Before proceeding it is important to state that in the case of stochastic fields *there is a difference between, on the one hand, a time-reversed dynamical quantity taking its value after an interaction and, on the other hand, the corresponding quantity following from the factually reversed interaction*.

The reason for this fundamental discrimination is that the interaction proper time neighbourhoods are (per definition stochastic in nature) and, hence, unpredictable. Therefore, the *direct* interaction proper time neighbourhood does not coincide with the *reverse* interaction proper time neighbourhood

Hence, the key for the explanation of the macroscopic irreversibility is the stochasticity of physical fields: This is demonstrated as follows:

a) Due to the stochasticity of the fields is the interaction of two particles stochastic: If  $H_I(x, t)$  is the interaction at time  $t$ , is it after the *direct* interaction proper time,  $\tau_d$ ,  $H_I(x, t + \tau_d)$ . Since  $\tau_d$  is -within limits- a stochastic number, is stochastic also  $H_I(x, t + \tau_d)$ .

On the other hand, the time-inversion transformation, leaves the equation of motion strictly invariant, because

$$t + \tau_d \rightarrow -t - \tau_d: H_I(x, t + \tau_d) = H_I(x, -t - \tau_d). \quad 5.21$$

However, the factual reversed motion is not invariant, unless the the fields were not stochastic, and the *reverse* interaction proper time,  $\tau_r$ , during the reversed motion equals the *direct* interaction proper time, i.e.,  $\tau_d = \tau_r$ , which is a rather unlikely event, and, hence,

$$H_I(x, t + \tau_d) \neq H_I(x, -t - \tau_r), \tau_d \neq \tau_r. \quad 5.22$$

b) Despite the stochasticity of the interaction of two particles their motion is in interaction free space between two successive interactions time-reversal-invariant,

$$H_0(x, t) = H_0(x, -t), \quad 5.23$$

where  $H_0(x, t)$  is the free field Hamiltonian.

c) Due to the stochasticity of the interaction is the state vector subjected to certain conditions:

The following tables A and B show how the state vector and its norm evolve under time inversion and under the action of the evolution operators.

*A Conservative evolution:*

A1  $t \rightarrow t' = -t: \psi(\mathbf{r}, t) \rightarrow \psi(\mathbf{r}, -t)$ , time inversion.

A2  $U_c(t, t_0) : \psi(\mathbf{r}, t) \rightarrow \hat{\psi}(\mathbf{r}, -t) = U_c(-t, t_0)\psi(\mathbf{r}, t_0)$ , conservative evolution from  $t_0$  to  $-t$ .

A3  $|\psi(\mathbf{r}, t)|^2 = |\psi(\mathbf{r}, -t)|^2 = |\hat{\psi}(\mathbf{r}, -t)|^2$ . Norm conservation

*B Non-conservative evolution:*

B1  $t \rightarrow t' = -t: \psi(\mathbf{r}, t) \rightarrow \psi(\mathbf{r}, -t)$

B2  $U_{n-c}(t, t_0) : \psi(\mathbf{r}, t) \rightarrow \hat{\psi}(\mathbf{r}, it) = U_{n-c}(-t, t_0)\psi(t_0)$

B3  $|\psi(\mathbf{r}, t)|^2 = |\psi(\mathbf{r}, -t)|^2 \neq |\hat{\psi}(\mathbf{r}, -it)|^2$ . Non conservation of the norm

In the stochastic evolution theory new aspects arise unknown to the unitary theories:

- The first one is that the norm is not conserved, and this gives the arrow to the time.

- To make clear the second aspect let us consider two interacting particles with state vector  $\psi(\mathbf{r}, -\tau_f/2)$  before and  $\psi(\mathbf{r}, +\tau_f/2)$  after the interaction. Let  $\tau_f$  denote the interaction proper time ( $f$  = forward in time) of the interaction from  $\psi(\mathbf{r}, -\tau_f/2)$  to  $\psi(\mathbf{r}, +\tau_f/2)$ . We put the zero of the time axis in the middle of the interaction proper time  $\tau_f$ .

If the interaction proceeds reversibly the state  $\psi(\mathbf{r}, +\tau_f/2)$  of the two particles can be brought back to  $\psi(\mathbf{r}, -\tau_f/2)$  in two ways:

*State vector and conservative evolution.*

First: By the time inversion  $t \rightarrow t' = -\tau_f/2: \psi(\mathbf{r}, +\tau_f/2) \rightarrow \psi(\mathbf{r}, -\tau_f/2)$ .

Second: By  $U_c(t, t_0): U_c(-\tau_f/2, +\tau_f/2)\psi(\mathbf{r}, +\tau_f/2) = \psi(\mathbf{r}, -\tau_f/2)$ .

If the interaction proceeds irreversibly the first way, which is mathematical operation, can be used and gives identical result to the one obtained in the first case of the conservative evolution.

In practice, however, if we wish to bring the system of two particles to the initial state, we must let them interact again in the opposite way. However - here is the key - the physical fields are stochastic and the interaction proper time duration now will be  $\tau_r$  ( $r$  = reverse interaction) instead of  $\tau_f$ .

In the direct interaction:

$$U_{n-c}(+\tau_f/2, -\tau_f/2)\psi(\mathbf{r}, -i\tau_f/2) = \psi(\mathbf{r}, +i\tau_f/2). \quad 5.24$$

In the reversed interaction aiming at bringing the system to the initial state the interaction proper time will be with high probability equal to  $\tau_r$ ,  $\tau_r - \tau_f \neq 0$ .

*State vector and non-conservative evolution.*

First:  $t \rightarrow t' = -\tau_f/2: \psi(\mathbf{r}, +i\tau_f/2) \rightarrow \psi(\mathbf{r}, -i\tau_f/2)$ .

Second:  $U_{n-c}(t, t_0): U_{n-c}(-\tau_r + \tau_f/2, +\tau_f/2)\psi(\mathbf{r}, +i\tau_f/2) = \psi(\mathbf{r}, i(-\tau_r + \tau_f/2))$ .

There is, of course, a certain probability that the state vector  $\psi(\mathbf{r}, -i\tau_r + i\tau_f/2)$  is identical to  $\psi(\mathbf{r}, -i\tau_f/2)$ . However, this probability is vanishing small (of measure zero) in comparison with the probability of all other possible states of the two particles.

It follows, therefore, that a stochastic interaction, which is formally time-reversal-invariant leads with high probability to irreversible and with zero probability to reversible phenomena.

If the transitions due to interactions are effected by means of the conservative evolution operator, equation 5.14, the observable probability density distribution given by

$$P_{rev} = |\psi(\mathbf{r}, +\tau_f/2)|^2. \quad 5.25$$

If, however, the evolution is effected by means of the non-con-

servative evolution operator, equation 5.15, then, of course, the observable probability density distribution is given by

$$P_{irr.} = |\Psi(\mathbf{r}, +i\tau_f/2)|, \quad 5.26$$

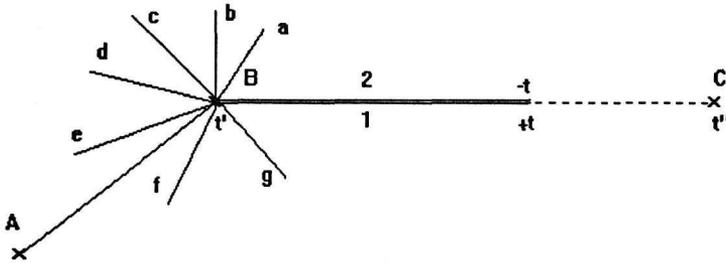
a quantity differing substantially from 5.25. The motion is irreversible, although the fundamental equations of motion are time-reversal-invariant. This is not a paradoxon, because the stochasticity implies fluctuations in the interaction proper time. This has as a consequence the inequalities

$$H_I(\mathbf{r}, \tau_r) \neq H_I(\mathbf{r}, \tau_f) \neq H_I(\mathbf{r}, -\tau_r) \quad 5.27$$

dispite the relation

$$H_I(\mathbf{r}, \tau_f) = H_I(\mathbf{r}, -\tau_f), \quad 5.28$$

The relations 5.27 and 5.28 are perfectly compatible.



**fig.3** A particle after an interaction at A with interaction proper time  $\alpha$  and action integral  $h[\alpha]$  interacts at B with interaction proper time  $\beta$  and action integral  $h[\beta]$  and flies free of interaction towards interaction point C. Halfway time is inverted and the particle flies in opposite direction interacting at B with interaction proper time  $\gamma$  and action integral  $h[\gamma] \neq h[\beta]$ . Due to the stochasticity of the interaction and of  $\gamma$  the alternatives  $\{A, a, b, c, \dots, g, \dots\}$  are open with certain probabilities to the particle. The probability measure of  $\{a, b, c, \dots, g\}$  is finite, whilst that of  $\{A\}$  is almost zero. This makes the interaction at B almost irreversible, although the interaction Hamiltonian is time-reversal-invariant. The same happens in systems with many particles. This reconciles the time-reversal-invariance of the fundamental equations of Physics and the irreversibility in Nature.

It has, thus, been demonstrated that two interacting particles do not by time reversal return from the final state to the initial state. What happens with two particles, happens *a fortiori* with many particles. This fully explains the origin and the becoming of the macroscopic arrow of time (fig.3).

## 6 DISCUSSION AND CONCLUSIONS

The idea that an interaction proper time neighbourhood is bound to the particular event which it describes, is a merit and, perhaps, the major achievement of Einstein's Relativity.

However, although this has been fully recognized, it has been not possible to find the way to practically apply it in reconciling the microscopic time-reversal invariance of the fundamental equations of Physics with the irreversibility of the most macroscopic physical phenomena.

Attempts to advance the concept of the entropy to the level of the fundamental quantity for the explanation of the evolution did not help to construct the missing bridge between the description provided by the fundamental equations of Physics - classical and quantal - and the macroscopic phenomena.

One possible reason for this long standing impasse in Physics might have been the fact that the *universal time* - a quantity whose existence never has been demonstrated - passed silently from Classical to Quantum Physics.

The generalized stochastic fields offer the possibility to recognize the importance of the interaction proper time neighbourhood in relating the microscopic and the macroscopic behaviors of the matter and to explain the long puzzling arrow of time.

It is interesting to note that the Feynman path integral follows as a particular case of the integrals series 5.9a for  $n=\infty$ , with a substantial difference: The product  $\Delta q \times \Delta p$  cannot on the quantum level take arbitrarily small values due to the Uncertainty Principle. This product represents the integration "measure" in the Feynman integral and is the "headache" of any measuretheoretical mathematician.

In equation 5.9a the Feynman integral automatically appears with the factor  $[n!]^{-1}$  for  $n=\infty$ . This factor has an important consequence: It makes the integral equal to zero and the problem of a "measure" which is no measure disappears altogether.

Also, the momentum in the present case does not appear as a differential. This makes the integral compatible with the Uncertainty Principle, if the field,  $\phi(x, t)$ , and the canonically conjugate

te momentum,  $n(x, t)$ , must be considered as operators.

The discontinuity property of the time parameter in systems with few particles gives the possibility to explain why the wave function can only be statistically interpreted.

This is intimately connected with the loss of the continuous group property of the evolution operator deriving from the non-additivity of disjoint time neighbourhoods. Also due to the same reason is the individual unpredictability of the interaction proper time neighbourhoods. This is analogous to the stochastic character of another quantity, the *impact parameter* of the reaction theory.

The most important result of this work is the derivation of a creative or destructive evolution operator which is not unitary and possesses the required properties for producing the *arrow of time*.

In analysing the effects of the stochastic fields, distinction must be made between the *time-inversed state vector* and the *state vector of the reverse reaction state*. They are different and at the root of the arrow of time, both the quantal and the macroscopic.

The obtained results have been possible by quantizing the field action integral, thus getting at the same time a generalized non-unitary operator whose parts are 1) one unitary evolution operator and 2) another non-unitary evolution operator describing irreversible processes.

The first one contains, as a special case, the evolution operator of Quantum Field Theory with a spontaneous action renormalization. The *zero point action renormalization* is equal to  $-\frac{1}{2}\hbar$ .

However, the double process of quantization - for integers and half-odd - numbers, is a fact reminding of the Bose-Einstein and of the Fermi-Dirac statistics. This fact guards - may be - the secret of the physical *raison d'être* of the stochastic fields in Nature and remains for us at the moment a kind of an enigma.

One very difficultly escapes the suspicion that the Heisenberg uncertainty principle is of the same stochastic origin as the stochastic fields.

Also, one might speculate that by correctly taking into account

the time structure (which is in relation to the space structure) the divergences in the integrals of the QFT- perturbation theory may possibly be eliminated. Also the spontaneous renormalization of the action integral through  $\pm\hbar\Lambda(n,\sigma)$  is corroborating the present view and may eventually make unnecessary the by now famous renormalization theory which did not earn the respect of Dirac.

Another problem remains unsolved for the moment: There is full freedom in which value of  $\pm\hbar\Lambda(n,\sigma)$  must be taken.

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