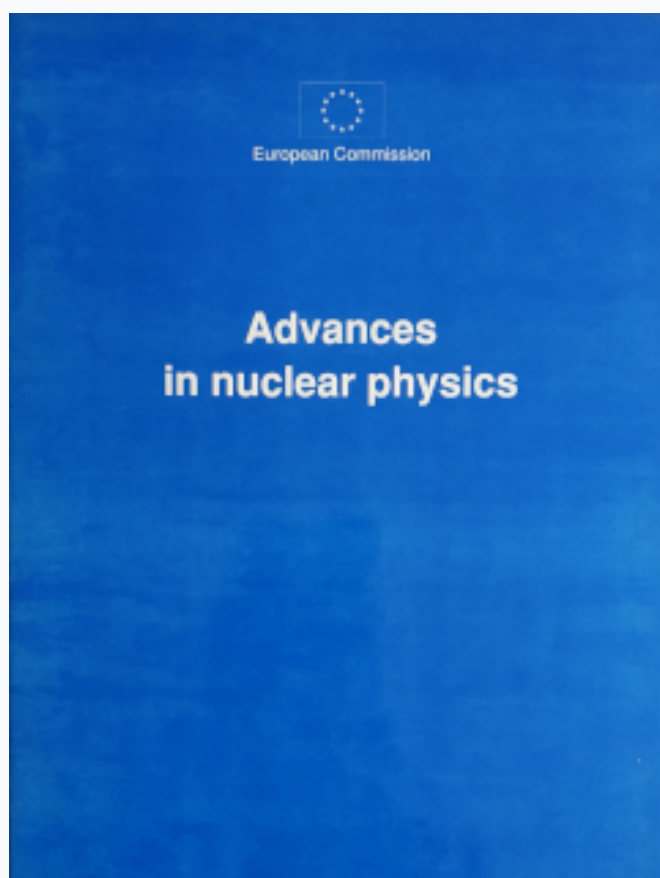


## Annual Symposium of the Hellenic Nuclear Physics Society

Τόμ. 5 (1994)

HNPS1994



### The form factor and the density distribution of the $4\text{He}$ nucleus using the Morse potential

K. Ypsilantis, S. Dimitrova, C. G. Koutroulos, M. E. Grypeos, A. Antonov

doi: [10.12681/hnps.2900](https://doi.org/10.12681/hnps.2900)

### Βιβλιογραφική αναφορά:

Ypsilantis, K., Dimitrova, S., Koutroulos, C. G., Grypeos, M. E., & Antonov, A. (2020). The form factor and the density distribution of the  $4\text{He}$  nucleus using the Morse potential. *Annual Symposium of the Hellenic Nuclear Physics Society*, 5, 171–179. <https://doi.org/10.12681/hnps.2900>

## The form factor and the density distribution of the ${}^4\text{He}$ nucleus using the Morse potential

K. Ypsilantis, S. Dimitrova<sup>+</sup>, C. Koutroulos, M. Grypeos

Department of Theoretical Physics  
Aristotle University of Thessaloniki, Greece

A. Antonov

Bulgarian Academy of Sciences  
Institute of Nuclear Research and Nuclear Energy, Sofia, Bulgaria.

<sup>+</sup> Also Bulgarian Academy of Sciences

Institute of Nuclear Research and Nuclear Energy, Sofia, Bulgaria.

### Abstract

The form factor and the density distribution of the  ${}^4\text{He}$  nucleus are calculated approximately using the Morse single-particle potential. The parameters are determined by fitting the theoretical charge form factor to the corresponding experimental data of the elastic electron scattering by  ${}^4\text{He}$  which are extended to large values of the momentum transfer. The corrections due to the center of mass motion (in the fixed center of mass approach) and of the finite proton size have been taken into account. The calculations can be performed partly analytically and the results show a considerable improvement with respect to those obtained with the oscillator shell model.

### Introduction

The study of the charge form factor of  ${}^4\text{He}$  [1-3] has received considerable attention by quite a number of authors who used various theoretical approaches. One of the main reasons is the simplicity of this nucleus. Moreover, it is one of the very few nuclei for which the charge form factor has been measured in a very wide range of momentum transfers.

Among the methods which have been used in calculations of the form factor of nuclei, those based on many-body techniques are usually the most

satisfactory ones. They are, however, more complicated than those based on single-particle models. As is well known, the disadvantage of the latter is that, one cannot fit with such a model both the form factor and the momentum distribution. Thus, if the parameters are determined by fitting the theoretical charge form factor to the corresponding experimental results, the values of the momentum distribution are not expected to agree well with the experimental values of it [3,4]. Nevertheless, the choice of the single-particle potential seems to play an important role in improving the results and diminishing the above mentioned difference between the calculated and experimental values of  $\eta(k)$ . This is clear if a strong short range repulsion is included in the potential. If for example a potential of the form [5]

$$V(r) = -V_0 + \frac{1}{2}kr^2 + \frac{B}{r^2}, \quad 0 \leq r < \infty \quad (1)$$

$k > 0$ ,  $B > 0$  is used, then a considerable improvement is mostly observed not only for the form factor but also for the momentum distribution.

A characteristic of the potential (1) is that it has an "infinite soft core" near the origin, which seems to be an extreme. It is more natural to expect that there is a repulsion in the single-particle potential near the origin, as for example relativistic Hartree calculations indicate, but not with an infinite behaviour near the origin. Thus, a potential of the Morse type [6-12] which, as is well known, has many applications in Physics, seems to be a rather good candidate for the single-particle potential of a light nucleus. In fact the first preliminary work in using the Morse potential in calculating the form factor of  ${}^4\text{He}$  was made in ref. [9]. The disadvantage is that the Schrödinger eigenvalue problem can be solved analytically only for the s-states for this potential. Therefore, it is not so convenient to be used for heavier nuclei if one wishes to take advantage of its analytic properties.

The object of this paper is to report on our first results in using the Morse potential in calculating the charge form factor of  ${}^4\text{He}$  by further extending the work of ref. [9]. In a subsequent investigation improvements will be made and also the interesting problem of the calculation of the momentum distribution will be discussed. In the next section the notation is specified and basic formulae regarding the Schrödinger eigenvalue problem with the Morse potential are given and discussed. In section 3, the expressions of the density distribution as well as of the form factor are given. The final section is devoted to numerical results and comments.

## 2 The approximate single-particle ground state wave function.

The well-known Morse potential is given by the following expression: [6]

$$V(r) = D[e^{-2a(r-r_0)} - 2e^{-a(r-r_0)}] = -D + D[1 - e^{-a(r-r_0)}]^2,$$

$$0 \leq r < \infty \quad (2)$$

It has its minimum value  $(-D)$  at  $r = r_0$ , tends asymptotically to zero as  $r \rightarrow \infty$  and is repulsive near the origin taking the value  $De^{2ar_0}(1 - 2e^{-ar_0})$  at  $r = 0$ . The corresponding radial Schrödinger equation for the s-state wave functions  $\varphi_{n_0}(r) = rR_{n_0}(r)$ :

$$\frac{d^2 \varphi_{n_0}(r)}{dr^2} - \frac{2\mu}{\hbar^2} [D(e^{-2a(r-r_0)} - 2e^{-a(r-r_0)}) - E] \varphi_{n_0}(r) = 0 \quad (3),$$

may be solved analytically. The approximate solution is

$$\varphi_{n_0}(r) = N_{n_0} e^{-de^{-a(r-r_0)}} e^{-\frac{a}{2}(2d-2n-1)(r-r_0)} \cdot L_{2d-2n-1}^{2d-2n-1}(2de^{-a(r-r_0)}) \quad (4),$$

where  $L$  are the generalized Laguerre polynomials and  $N_{n_0}$  is the normalization constant [6]. Also  $d$  is given by the expression:

$$d = \left( \frac{2\mu D}{\hbar^2 a^2} \right)^{\frac{1}{2}} \quad (5),$$

and  $n$  is an integer in the interval

$$0 \leq n \leq \frac{(2d-1)}{2}$$

The approximate energy eigenvalues are given by the expression

$$E_{n_0} = -a^2 \frac{\hbar^2}{8\mu} (2d-1-2n)^2 = -D + \hbar\omega_0 \left(n + \frac{1}{2}\right) - \left(\frac{\hbar^2 \omega_0^2}{4D}\right) \left(n + \frac{1}{2}\right)^2 \quad (6),$$

where  $\omega_0$  is the angular frequency of classical small vibrations around  $r_0$ :

$$\omega_0 = a\left(\frac{2D}{\mu}\right)^{\frac{1}{2}} \quad (7),$$

The ground-state radial wave function in which we are interested here is given by the expression:

$$\varphi_{00}(r) = N_{00} e^{-de^{-a(r-r_0)}} e^{-\frac{a}{2}(2d-1)(r-r_0)} \quad (8),$$

where the normalization constant  $N_{00}$  is given in terms of the parameters  $a$  and  $d$ ,

$$N_{00} = \left[ \frac{a(2d)^{2d-1}}{\Gamma(2d-1)} \right]^{\frac{1}{2}} \quad (9).$$

As it was stated, the above expressions for the wave functions and energy eigenvalues are approximate. The reason is that they were derived under the assumption that the wave function  $\varphi$  is zero when  $r \rightarrow -\infty$  and not at  $r = 0$ , which is the appropriate boundary condition. The exact wave function may be also obtained [7,8], but in this case the energy has to be determined by solving numerically a transcendental equation. Work in this direction is in progress.

### 3. Expressions for the density distribution and the form factor of ${}^4He$ .

In this section we give the expressions of the density distribution and the form factor of  ${}^4He$ .

The normalized to unity ( $\int \rho(r) d^3r = 1$ ) density distribution for central single-particle potentials is given by the general expression

$$\rho(r) = \frac{1}{4\pi Z} \sum_{nl} 2(2l+1) R_{nl}^2(r) \quad (10).$$

In the case of the  ${}^4He$  we have simply

$$\rho(r) = \frac{1}{4\pi} |R_{00}(r)|^2 \quad (11)$$

The point-proton form factor in the Born approximation and for spherically symmetric  $\rho(r)$  is:

$$F(q) = 4\pi \int_0^\infty \rho(r) \left( \frac{\sin qr}{qr} \right) r^2 dr \quad (12)$$

In the case of  ${}^4\text{He}$  we have

$$F(q) = \int_0^\infty |R_{00}(r)|^2 \left( \frac{\sin qr}{qr} \right) r^2 dr \quad (13)$$

The proton charge form factor is introduced using the Chandra and Sauer parametrization [13]

$$f_p(q) = \sum_{i=1}^3 A_{p_i} e^{-\frac{a_{p_i}^2 q^2}{4}} \quad (14)$$

where

$$A_{p_1} = 0.506, A_{p_2} = 0.327, A_{p_3} = 0.165$$

and

$$a_{p_1} = 0.431 \text{ fm}, a_{p_2} = 0.139 \text{ fm}, a_{p_3} = 1.525 \text{ fm}$$

The center of mass correction in the Form Factor of  ${}^4\text{He}$  is taken into account, using the "fixed center of mass correction" of Radhakant-Khadkikar-Banerjee [14]. Thus, the expression of the form factor corrected for the center of mass motion is:

$$\tilde{F}(q) = \frac{\int d^3\omega F(|\vec{q} + \vec{\omega}|) F^3(\omega)}{\int d^3\omega F^4(\omega)} \quad (15)$$

Therefore, the theoretical expression for the charge form factor is:

$$F_{ch}(q) = f_p(q) \cdot \tilde{F}(q) \quad (16)$$

The above formulae read as follows for  ${}^4\text{He}$ , if the approximate wave-function of the Morse potential is used,

$$\rho(r) = \rho_B(r) = \frac{1}{4\pi} \frac{1}{r^2} \left[ \frac{a(2d)^{2d-1}}{\Gamma(2d-1)} \right] e^{-2de^{-(r-r_0)}} e^{-a(2d-1)(r-r_0)} \quad (17)$$

$$F(q) = \frac{1}{q} \left[ \frac{a(2d)^{2d-1}}{\Gamma(2d-1)} \right] \int_0^\infty \frac{1}{r} e^{-2de^{-a(r-r_0)}} \sin(qr) e^{-a(2d-1)(r-r_0)} dr \quad (18)$$

The integration in (18) is performed numerically.

#### 4. Numerical Results and Comments

Calculations of the charge form factor, and of the point-proton (or body) and charge density distributions were performed for  ${}^4\text{He}$  using the approximate solution of the Schrödinger equation. The potential parameters were determined by the least-squares method by fitting the theoretical expression of the charge form factor to the experimental data of the elastic electron scattering by  ${}^4\text{He}$ . We used the same experimental results (which are extended to large values of the momentum transfer), used in ref. [5] as well. The potential parameters obtained are:

$$\alpha = 1.6897 fm^{-1}, \quad d = 1.1134, \quad r_0 = 0.9095 fm \quad (19)$$

In figure 1 the variation of  $\log|F_{ch}(q)|$  with  $q^2$  for  ${}^4\text{He}$  is plotted (full line). The experimental points are indicated by crosses. Notice that the curve fits the experimental data very well. Also, in the same figure, we give the results obtained using a harmonic oscillator potential with potential parameter  $b = 1.432 fm$  (dashed curve). This seems to be in fairly good agreement with the experimental data at the small values of momentum transfer.

The body density distribution of  ${}^4\text{He}$  was calculated also, using the approximate wavefunction with the potential parameters given above. The corresponding curve is shown in figure 2.

It is seen that the approximate analytic expression for  $\rho_B(r)$  is not good for a small region near the origin ( $r \lesssim 0.2 fm$ ) as is expected. The pronounced dip of the point-proton density at small  $r$  is due to the short-range repulsion of the Morse potential. This is largely smeared out in the charge density (dashed line in fig.2) because of the proton charge density.

#### References

- [1] L. R. B. Elton, "Nuclear Sizes", Oxford University Press (1961); H.

- Überall "Electron Scattering From Complex Nuclei", Academic Press (N.Y., London) (1971).
- [2] R. C. Barret, D. F. Jackson, "Nuclear Sizes and Structure", Clarendon Press, Oxford (1974).
  - [3] A. N. Antonov, P. E. Hodgson and J. Zh. Petkov, Nucleon Momentum and Density Distributions in Nuclei, Clarendon Press Oxford (1988).
  - [4] C. Ciofi degli Atti, Nucl. Phys. A **97** (1989) 361C.
  - [5] a) M. E. Grypeos and K. N. Ypsilantis, J. Phys. G Nucl. and Part. Phys. **15** (1989) 1397  
 b) K. N. Ypsilantis, Ph. D. thesis, Aristotle University of Thessaloniki, Thessaloniki (1991)  
 c) K. N. Ypsilantis and M. E. Grypeos, Fourth Hellenic Symposium in Nuclear Physics (Ioannina 1993), submitted for publication.
  - [6] P. Morse, Phys. Rev. **34** (1929) 57.
  - [7] D. Ter Haar, Phys. Rev. **70** (1946) 222.
  - [8] S. Flügge, "Practical Quantum Mechanics I", Springer Verlag, Heidelberg, New York (1971) p. 182.
  - [9] A. Gersten and A. E. S. Green, Bull. Am. Phys. Soc. **13** (1968) 627.
  - [10] D. Bonatsos and C. Daskaloyannis, Phys. Let. **278** (1992) 1.
  - [11] F. Iachello and S. Oss, Phys. Rev. Let. **66** (1991) 2976.
  - [12] F. Iachello and S. Oss, Chem. Phys. Let. **187** (1991) 500.
  - [13] H. Chandra and G. Sauer, Phys. Rev. C **13** (1976) 245.
  - [14] S. Radakhant, S. Khadkikar and B. Banerjee, Nucl. Phys. A **142** (1970) 81.



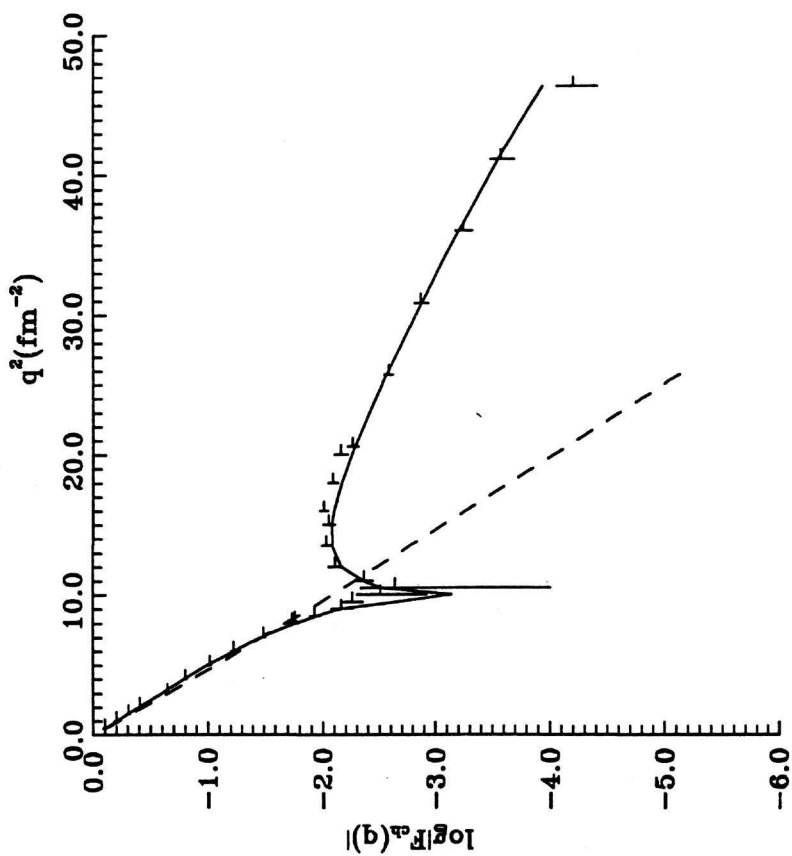


Fig. 1. The charge form factor of  ${}^4\text{He}$  with the Morse potential (full line) and with the harmonic oscillator (dashed line).

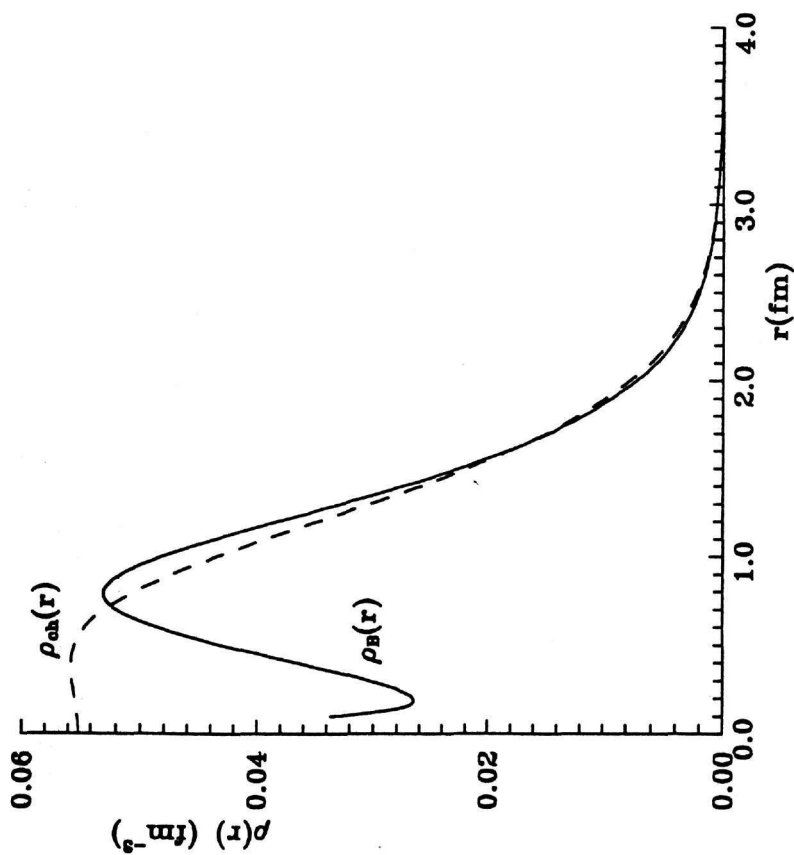


Fig. 2. The charge  $\rho_{ch}$  and point-proton (body)  $\rho_B$  density distributions of  ${}^4\text{He}$  using the potential parameters obtained by fitting the form factor.