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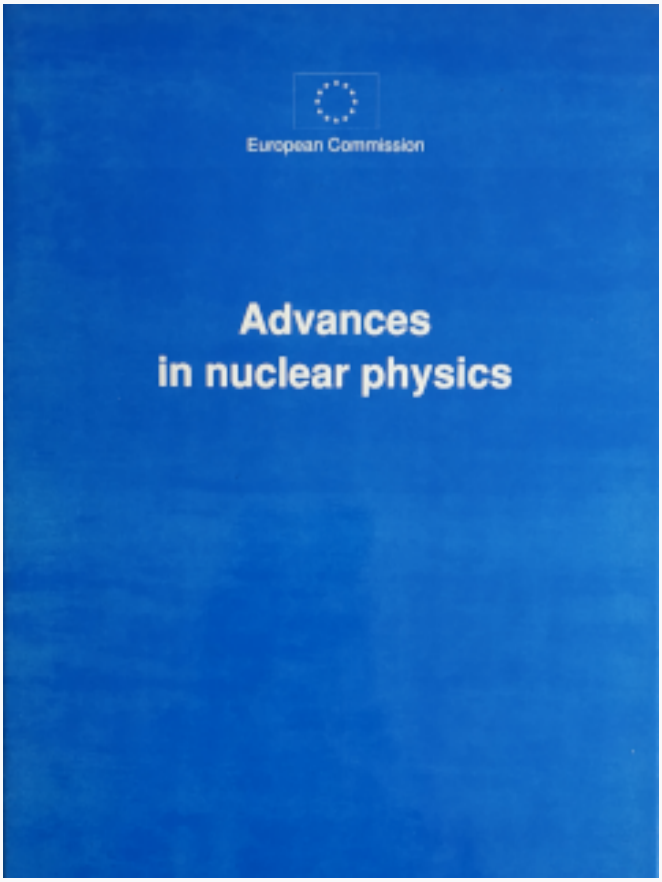

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QUANTUM ALGEBRAIC DESCRIPTION OF THE PAIRING CORRELATIONS IN A SINGLE-J NUCLEAR SHELL *

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Abstract

Pairing in a single-j shell is described in terms of two Q-oscillators, one describing the $J = 0$ fermion pairs and the other corresponding to the $J \neq 0$ pairs, the deformation parameter $T = \ln Q$ being related to the inverse of the size of the shell. Using these two oscillators an $SU_Q(2)$ algebra is constructed, while a pairing Hamiltonian giving the correct energy eigenvalues up to terms of first order in the small parameter can be written in terms of the Casimir operators of the algebras appearing in the $U_Q(2) \supset U_Q(1)$ chain, thus exhibiting a quantum algebraic dynamical symmetry. The additional terms introduced by the Q-oscillator are found to improve the agreement with the experimental data for the neutron pair separation energies of the Sn isotopes, with no extra parameter introduced.

* Presented by Dennis Bonatsos

1. Introduction

Quantum algebras (also called quantum groups) [1-4] are recently attracting much attention in physics, especially after the introduction of the q-deformed harmonic oscillator [5,6]. Applications in conformal field theory, quantum gravity, quantum optics, atomic physics, as well as in the description of spin chains have already been reported. In nuclear physics attention has been focused on the q-deformed rotator with $SU_q(2)$ symmetry and its use for the description of rotational spectra and B(E2) transition probabilities of deformed and superdeformed nuclei ([7-9] and references therein), as well as on the construction of exactly soluble nuclear models with quantum algebraic dynamical symmetries 10,11]. Similar efforts have been made for the description of rotational [12-14] and vibrational ([15-17] and references therein) spectra of diatomic molecules. In many cases the deformation parameter q turns out to be related to some well defined physical quantity. In the case of the q-deformed rotator, for example, the deformation parameter has been related to the softness parameter of the Variable Moment of Inertia (VMI) model [18].

For nuclear physics the description of correlated fermion pairs in terms of q-deformed bosons is of particular interest for several reasons. Correlated fermion pairs in a single-j nuclear shell or several non-degenerate j-shells ([19] and references therein) are known to satisfy commutation relations which resemble boson commutation relations, containing though corrections due to the presence of the Pauli principle. This fact has led to the development of boson mapping techniques for the description of many fermion systems (see [20] for an authoritative review). Boson mappings are of additional interest as a necessary tool in building a bridge between the phenomenologically successful algebraic models of nuclear collective motion, such as the Interacting Boson Model (IBM) ([21], see [22,23] for recent overviews) and the shell model. Since q-bosons also satisfy commutation relations different from the usual ones, it is reasonable to check to what extend correlated fermion pairs can be described in terms of q-deformed bosons. For fermion pairs of $J = 0$ this question has been answered in refs [24,25], where an approximate mapping of these fermion pair operators onto Q-bosons [26-28] has been constructed, which correctly reproduces the commutation relations and the pairing energies up to first order corrections in the small parameter $T = \ln Q$, which is related to the inverse of the size of the single-j shell. The

same problem has been solved exactly in refs [25,29] through the use of a generalized deformed oscillator [30]. The extension of this formalism to pairs of $J \neq 0$ is of obvious interest, since the $J \neq 0$ pairs are known to play an important role in the formation of nuclear properties.

In the present work a first step in this direction is taken. First, it is realized that for the description of pairing correlations in a single-j nuclear shell it suffices to represent the $J \neq 0$ pairs by a Q-oscillator similar to the one used for the $J = 0$ pairs. The two oscillators are then used for building a quantum algebraic dynamical symmetry. Finally, the higher order terms introduced by the Q-oscillator are found to lead to improved agreement with the experimental data, without the introduction of any new parameter.

2. Deformed oscillator description of $J = 0$ pairs

In the usual formulation of the theory of pairing in a single-j shell [31], fermion pairs of angular momentum $J = 0$ are created by the pair creation operators

$$S^+ = \frac{1}{\sqrt{\Omega}} \sum_{m>0} (-1)^{j+m} a^+_{jm} a^+_{j-m}, \tag{1}$$

where a^+_{jm} are fermion creation operators and $2\Omega = 2j + 1$ is the degeneracy of the shell. In addition, pairs of nonzero angular momentum are created by the $\Omega - 1$ operators

$$B^+_J = \sum_{m>0} (-1)^{j+m} (jmj - m|J0) a^+_{jm} a^+_{j-m}, \tag{2}$$

where $(jmj - m|J0)$ are the usual Clebsch Gordan coefficients. The fermion number operator is defined as

$$N_F = \sum_m a^+_{jm} a_{jm} = \sum_{m>0} (a^+_{jm} a_{jm} + a^+_{j-m} a_{j-m}). \tag{3}$$

The $J = 0$ pair creation and annihilation operators satisfy the commutation relation

$$[S, S^+] = 1 - \frac{N_F}{\Omega}, \tag{4}$$

while the pairing Hamiltonian is

$$H = -G\Omega S^+ S. \tag{5}$$

The seniority V_F is defined as the number of fermions not coupled to $J = 0$. If only pairs of $J = 0$ are present (i.e. $V_F = 0$), the eigenvalues of the Hamiltonian are

$$E(N_F, V_F = 0) = -G\Omega\left(\frac{N_F}{2} + \frac{N_F}{2\Omega} - \frac{N_F^2}{4\Omega}\right). \tag{6}$$

For non-zero seniority the eigenvalues of the Hamoltonian are

$$E(N_F, V_F) = -\frac{G}{4}(N_F - V_F)(2\Omega - N_F - V_F + 2). \tag{7}$$

We denote the operators N_F, V_F and their eigenvalues by the same symbol for simplicity.

In [24] it has been proved that the behaviour of the $J = 0$ pairs can be described, up to first order corrections, in terms of Q-bosons. Q-bosons [26-28] are defined by the commutation relations

$$[N, b^+] = b^+, \qquad [N, b] = -b, \qquad bb^+ - Qb^+b = 1, \tag{8}$$

where b^+ (b) are Q-boson creation (annihilation) operators and N is the relevant number operator. Q-numbers [26-28] are defined as

$$[x]_Q = \frac{Q^x - 1}{Q - 1}. \tag{9}$$

For $Q = e^T$ their Taylor expansion is

$$[x]_Q = x + \frac{T}{2}(x^2 - x) + \frac{T^2}{12}(2N^3 - 3N^2 + 1) + \frac{T^3}{24}(N^4 - 2N^3 + N^2) + \ldots \tag{10}$$

One can then easily see that

$$b^+b = [N]_Q, \qquad bb^+ = [N + 1]_Q. \tag{11}$$

Making the mapping

$$S^+ \to b^+, \qquad S \to b, \qquad N_F \to 2N, \tag{12}$$

the Hamiltonian of eq. (5) becomes

$$H(N, V = 0) = -G\Omega b^+b = -G\Omega[N]_Q. \tag{13}$$

Using eq. (10) we see that it coincides with eq. (6) up to first order corrections in the small parameter, which is identified as $T = -2/\Omega$. Furthermore, the Q-bosons satisfy the commutation relation

$$[b, b^+] = [N+1]_Q - [N]_Q = Q^N = 1 + TN + \frac{T^2N^2}{2} + \frac{T^3N^3}{6} + \ldots, \qquad (14)$$

which coincides with eq. (4) up to first order corrections in the small parameter, which is, consistently with the above finding, identified as $T = -2/\Omega$. Therefore the fermion pairs of $J = 0$ can be approximately described as Q-bosons, which correctly reproduce both the pairing energies and the commutation relations up to first order corrections in the small parameter.

3. Deformed oscillator description of $J \neq 0$ pairs

For the case of nonzero seniority, one observes that eq. (7) can be written as

$$E(N_F, V_F) = G\Omega\left(\frac{V_F}{2} + \frac{V_F}{2\Omega} - \frac{V_F^2}{4\Omega}\right) - G\Omega\left(\frac{N_F}{2} + \frac{N_F}{2\Omega} - \frac{N_F^2}{4\Omega}\right), \qquad (15)$$

i.e. it can be separated into two parts, formally identical to each other. Since the second part (which corresponds to the $J = 0$ pairs) can be adequately described by the Q-bosons b, b^+, and their number operator N, as we have already seen, it is reasonable to assume that the first part can also be described in terms of some Q-bosons d, d^+, and their number operator V (with $V_F \to 2V$), satisfying commutation relations similar to eq. (8):

$$[V, d^+] = d^+, \qquad [V, d] = -d, \qquad dd^+ - Qd^+d = 1. \qquad (16)$$

From the physical point of view this description means that a set of Q-bosons is used for the $J = 0$ pairs and another set for the $J \neq 0$ pairs. The latter is reasonable, since in the context of this theory the angular momentum value of the $J \neq 0$ pairs is not used explicitly. The $J \neq 0$ pairs are just counted separately from the $J = 0$ pairs. A Hamiltonian giving the same spectrum as in eq. (15), up to first order corrections in the small parameter, can then be written as

$$H(N, V) = G\Omega([V]_Q - [N]_Q). \qquad (17)$$

Using eq. (10) it is easy to see that this expression agrees to eq. (15) up to first order corrections in the small parameter $T = -2/\Omega$.

Two comments concerning eq. (17) are in place:

i) In the classical theory states of maximum seniority (i.e. states with $N = V$) have zero energy. This is also holding for the Hamiltonian of eq. (17) to all orders in the deformation parameter.

ii) A landmark of the classical theory is that $E(N,V) - E(N,V=0)$ is independent of N. This also holds for eq. (17) to all orders in the deformation parameter.

4. A deformed SU(2) algebra

Knowing the Schwinger realization of the $SU_q(2)$ algebra in terms of q-bosons [5,6], one may wonder if the operators used here close an algebra. It is easy to see that the operators b^+d, d^+b and $N - V$ do not close an algebra. Considering, however, the operators [32]

$$J_+ = b^+ Q^{-V/2} d, \qquad J_- = d^+ Q^{-V/2} b, \qquad J_0 = \frac{1}{2}(N - V), \tag{18}$$

one can easily see that they satisfy the commutation relations [32,33]

$$[J_0, J_\pm] = \pm J_\pm, \qquad J_+ J_- - Q^{-1} J_- J_+ = [2J_0]_Q. \tag{19}$$

Using the transformation

$$J_0 = \tilde{J}_0, \qquad J_+ = Q^{(1/2)(J_0 - 1/2)} \tilde{J}_+, \qquad J_- = \tilde{J}_- Q^{(1/2)(J_0 - 1/2)}, \tag{20}$$

one goes to the usual $SU_q(2)$ commutation relations

$$[\tilde{J}_0, \tilde{J}_\pm] = \pm \tilde{J}_\pm, \qquad [\tilde{J}_+, \tilde{J}_-] = [2\tilde{J}_0]_q, \tag{21}$$

where q-numbers are defined as

$$[x]_q = \frac{q^x - q^{-x}}{q - q^{-1}}, \tag{22}$$

and $q^2 = Q$.

It is clear that $N + V$ is the first order Casimir operator of the $U_Q(2)$ algebra formed above (since it commutes with all the generators given in eq. (18)), while $N - V$ is the first order Casimir operator of its $U_Q(1)$ subalgebra, which is generated by J_0 alone. Therefore the Hamiltonian of eq. (17) can be expressed in terms of the Casimir operators of the algebras appearing in the chain $U_Q(2) \supset U_Q(1)$ as

$$E(N,V) = G\Omega \left(\left[\frac{C_1(U_Q(2)) - C_1(U_Q(1))}{2} \right]_Q - \left[\frac{C_1(U_Q(2)) + C_1(U_Q(1))}{2} \right]_Q \right), \tag{23}$$

i.e. the Hamiltonian has a $U_Q(2) \supset U_Q(1)$ dynamical symmetry.

In ref. [34] a q-deformed version of the pairing theory was *assumed*, with satisfactory results when compared to experimental data. The present construction offers some justification for this assumption, since in both cases the basic ingredient is the modification of eq. (4). It should be noticed, however, that the deformed version of eq. (4) considered in ref. [34] is different from the one obtained here (eq. (14)). A basic difference is that in ref. [34] the deformed theory reduces to the classical theory for $q \to 1$, so that q-deformation is introduced in order to describe additional correlations, while in the present formalism the Q-oscillators involved for $Q \to 1$ reduce to usual harmonic oscillators, so that Q-deformation is introduced in order to attach to the oscillators the anharmonicity needed by the energy expression (eq. (6)).

5. Comparison to experimental data

In the construction given above we have shown that Q-bosons can be used for the approximate description of correlated fermion pairs in a single-j shell. The results obtained in the Q-formalism agree to the classical (non-deformed) results up to first order corrections in the small parameter. However, the Q-formalism contains in addition higher order terms. The question is then born if these additional terms are useful or not. For answering this question, the simplest comparison with experimental data which can be made concerns the classic example of the neutron pair separation energies of the Sn isotopes, used by Talmi [35,36].

In Talmi's formulation of the pairing theory, the energy of the states with zero seniority is given by [35,36]

$$E(N)_{cl} = NV_0 + \frac{N(N-1)}{2}\Delta, \tag{24}$$

where N is the numbrer of fermion pairs and V_0, Δ are constants. We remark that this expression is the same as the one in eq. (6), with the identifications

$$\Delta/(2V_0) = -1/\Omega, \qquad \Delta = 2G, \qquad N_F = 2N. \tag{25}$$

The neutron pair separation energies are given by

$$\Delta E(N+1)_{cl} = E(N+1)_{cl} - E(N)_{cl} = V_0\left(1 + \frac{\Delta}{V_0}N\right). \tag{26}$$

Thus the neutron pair separation energies are expected to decrease linearly with increasing N. (Notice from eq. (25) that $\Delta/V_0 < 0$, since $\Omega > 0$.)

In our formalism the neutron pair separation energies are given by

$$\Delta E(N+1)_Q = -G\Omega([N+1]_Q - [N]_Q) = -G\Omega Q^N = -G\Omega e^{TN}. \tag{27}$$

Since, as we have seen, T is expected to be $-2/\Omega$, i.e. negative and small, the neutron pair separation energies are expected to fall exponentially with increasing N, but the small value of T can bring this exponential fall very close to a linear one.

In Table 1 the neutron pair separation energies of the even Sn isotopes from ^{104}Sn to ^{130}Sn (i.e. across the whole sdg neutron shell) are shown. We have performed a least square fitting of the energies using both theories. The quality of the fit was measured by

$$\sigma = \sum_{i=1}^{n} (\Delta E_i(exp) - \Delta E_i(th))^2, \tag{28}$$

i.e. by the sum of the squares of the differences between the experimental and theoretical values. Furthermore, we have performed a least square fit of the logarithms of the energies, since eq. (27) predicts a linear decrease of the logarithm of the energies with increasing N. In this case the quality of the fit is measured by

$$\sigma' = \sum_{i=1}^{n} (\ln \Delta E_i(exp) - \ln \Delta E_i(th))^2. \tag{29}$$

Both fits give almost identical results. Eq. (27) (in which the free parameters are $G\Omega$ and T), gives a better result than eq. (26) (in which the free parameters are V_0 and Δ/V_0) for every single isotope, without introducing any additional parameter, indicating that the higher order terms can be useful.

One should, however, remark that ^{116}Sn lies in the middle of the sdg neutron shell. If we fit the isotopes in the lower half of the shell (^{104}Sn to ^{116}Sn) and the isotopes in the upper half of the shell (^{118}Sn to ^{130}Sn) separately, we find that both theories give indistinguishably good results in both regions. Therefore Q-deformation can be understood as expressing higher order correlations which manifest themselves in the form of particle-hole asymmetry.

Table 1

Neutron pair separation energies ΔE (in keV) for the Sn isotopes. N is the number of valence neutron pairs. Experimental data (denoted by exp) are taken from ref. [37]. The fits obtained with eq. (27) are denoted by Q, while the fits obtained with eq. (26) are denoted by cl. The parameter values obtained in each fit, as well as the quality measures of eqs (28), (29) are also given.

nucleus	N	ΔE	ΔE	ΔE	$\ln \Delta E$	$\ln \Delta E$	$\ln \Delta E$
		exp	Q	cl	exp	Q	cl
^{104}Sn	2	22820	22778	22277	10.035	10.029	10.000
^{106}Sn	3	21960	21767	21503	9.997	9.985	9.966
^{108}Sn	4	20950	20801	20728	9.950	9.940	9.931
^{110}Sn	5	19900	19878	19954	9.898	9.895	9.894
^{112}Sn	6	18967	18995	19179	9.850	9.851	9.856
^{114}Sn	7	18046	18152	18405	9.801	9.806	9.817
^{116}Sn	8	17109	17347	17630	9.747	9.761	9.776
^{118}Sn	9	16271	16577	16856	9.697	9.716	9.733
^{120}Sn	10	15592	15841	16081	9.655	9.672	9.688
^{122}Sn	11	14987	15138	15307	9.615	9.627	9.641
^{124}Sn	12	14443	14466	14532	9.578	9.582	9.592
^{126}Sn	13	13926	13824	13757	9.542	9.538	9.541
^{128}Sn	14	13560	13210	12983	9.515	9.493	9.486
^{130}Sn	15	12900	12624	12208	9.465	9.448	9.429
T			−0.0454			−0.0447	
$-G\Omega$ (keV)			23836			23718	
Δ/V_0				−0.0336			−0.0324
V_0 (keV)				23052			22766
$10^6\sigma$ (keV)^2			5.18	25.30			
$10^{-3}\sigma'$ (keV)^2						2.13	9.18

We have also attempted a fit of the neutron pair separation energies of the Pb isotopes from ^{186}Pb to ^{202}Pb. In this case both theories give indistinguishably good fits. This result is in agreement with the Sn findings, since all of these Pb isotopes lie in the upper half of the pfh neutron shell. Unfortunately, no neutron pair separation energy data exist for Pb isotopes in the lower part of the pfh neutron shell.

Concerning the values of T obtained in the case of the Sn isotopes ($T = -0.0454$, $= -0.0447$), we observe that they are slightly smaller than the value ($T = -0.0488$) which would have been obtained by considering the neutrons up to the end of the sdg shell as lying in a single-j shell. This is, of course, a very gross approximation which should not be taken too seriously. In the case of the Pb isotopes mentioned above, however, the best fit was obtained with $T = -0.0276$, which is again slightly smaller than the value of $T = -0.0317$ which corresponds to considering all the neutrons up to the end of the pfh shell as lying in a single-j shell.

6. Discussion

In conclusion, we have shown that pairing in a single-j shell can be described, up to first order corrections, by two Q-oscillators, one describing the $J = 0$ pairs and the other corresponding to the $J \neq 0$ pairs, the deformation parameter $T = \ln Q$ being related to the inverse of the size of the shell. These two oscillators can be used for forming an $SU_Q(2)$ algebra. A Hamiltonian giving the correct pairing energies up to first order corrections in the small parameter can be written in terms of the Casimir operators of the algebras appearing in the $U_Q(2) \supset U_Q(1)$ chain, thus exhibiting a quantum algebraic dynamical symmetry. The additional terms introduced by the Q-oscillators serve in improving the description of the neutron pair separation energies of the Sn isotopes, with no extra parameter introduced.

In ref. [29] a generalized deformed oscillator describing the correlated fermion pairs of $J = 0$ *exactly* has been introduced. This generalized deformed oscillator is the same as the one giving the same spectrum as the Morse potential [38,39], up to a shift in the energy spectrum. The use of two generalized deformed oscillators for the description of $J = 0$ pairs and $J \neq 0$ pairs in a way similar to the one of the present work is easy, while the construction out of them of a closed algebra analogous to the $SU_Q(2)$ obtained here is

an open problem. On the other hand, the construction of a generalized deformed SU(2) algebra, labelled as $SU_\Phi(2)$ (where Φ is a function characterizing the deformation), has been recently achieved [40,41]. The extension of the ideas presented here to the case of the BCS theory is under investigation.

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