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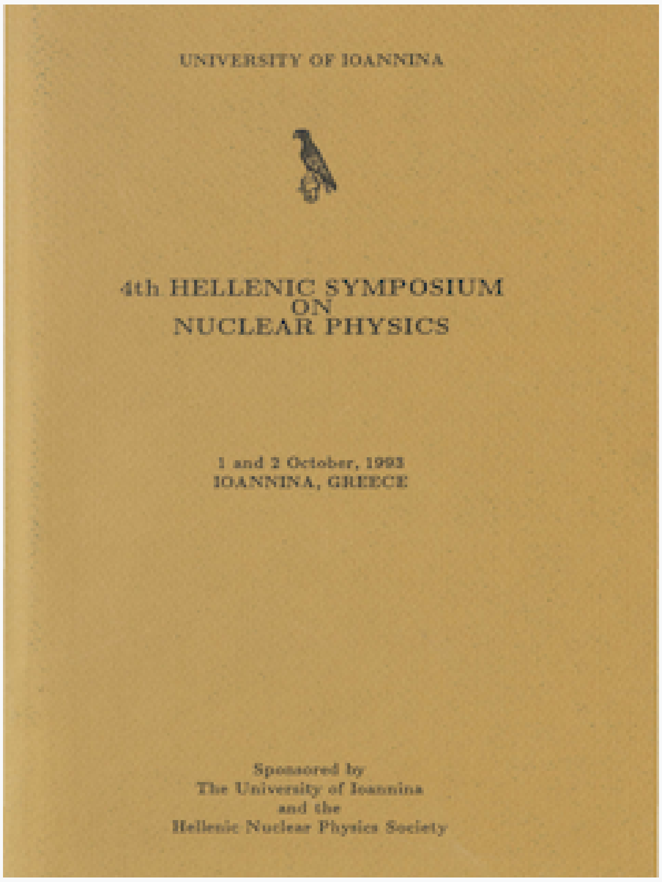

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Nuclear structure dependence of the coherent (μ^-,e^-) conversion matrix elements [1]

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Abstract

Coherent rates for the neutrinoless muon to electron conversion, (μ^-, e^-) in the presence of nuclei, are studied throughout the periodic table. The relevant ground state to ground state transition matrix elements are obtained in the context of the quasi-particle RPA. The results are discussed in view of the existing experimental data extracted at TRIUMF and PSI for ^{48}Ti and ^{208}Pb nuclei and compared with: (i) the single particle shell model results calculated with a determinantal ground state wave function and (ii) the results deduced in a local density approximation.

1. Introduction

The neutrinoless muon to electron conversion in the field of a nucleus, represented by the reaction

$$\mu^- + (A,Z) \rightarrow e^- + (A,Z)^* \tag{1}$$

is forbidden in the Standard Model by lepton flavor conservation and plays an important role in the study of the muon number violation [1]-[6]. Within the last decade, experiments at TRIUMF and PSI aiming to search for $\mu - e$ conversion electrons have mainly employed ^{48}Ti as target but up to now they have not measured any event. Instead, for the upper limit on the branching ratio

$$R_{eN} = \frac{\Gamma(\mu^-, e^-)}{\Gamma(\mu^-, \nu_\mu)} \tag{2}$$

the two independent experiments have obtained about the same value, i.e. at TRIUMF [7]

[1] Presented by T. S. Kosmas

$$R_{eN} < 4.6 \times 10^{-12}, \qquad (90\% \ Confidence \ Level) \tag{3}$$

and at PSI [8]

$$R_{eN} < 4.9 \times 10^{-12}, \qquad (90\% \ Confidence \ Level) \tag{4}$$

The experimental sensitivity is expected to be further improved by two to three orders of magnitude by on going experiments at PSI (to 10^{-14}) [8], at TRIUMF (to 10^{-14}) [7] and at INS (to $10^{-14} - 10^{-16}$) [9]. The most interesting result of these experiments would be not a new upper limit but some events of (μ^-, e^-) which will signal the break down of the muon number conservation and will reveal "new physics mechanisms" [3,5] beyond the Standard Model. For a demonstration of the motivation for the present work we would go through a brief historical review of the heroic experimental efforts to observe events of process (1) and of the development of the theoretical background for the (μ^-, e^-) conversion.

Very early the first experiments by Steinberger and Wolfe [10] using Cu as target found for the branching ratio R_{eN} the upper limit $R_{eN} < 10^{-4}$. Some years later two simultaneous experiments by Conversi *et al.* [11] using also Cu reduced the upper limit to $R_{eN} < 5 \times 10^{-5}$ and $R_{eN} < 5 \times 10^{-6}$, respectively. Using the same target one decade later Bryman *et al.* [12] improved the branching ratio to $R_{eN} < 1.6 \times 10^{-8}$. Experiments with targets different than Cu, have been performed on sulphar ^{32}S by Baderctsher *et al.* [13] ($R_{eN} < 7 \times 10^{-11}$) and recently at TRIUMF on ^{208}Pb [7] ($R_{eN} < 4.9 \times 10^{-10}$, value obtained from preliminary results).

On the theoretical side the basic background for the (μ^-, e^-) conversion has been set by Weinberg and Feinberg [1] who assumed that this process is mediated by virtual photons. Non-photonic contributions have been included later on (see ref. [3] and references therein) in the post gauge theory era. An interesting feature of the (μ^-, e^-) conversion process is the possibility of the ground state to ground state transitions. The strength of this channel appears enhanced because of the coherent contribution of all nucleons of the participating nucleus. Weinberg and Feinberg [1] estimated that, the coherent channel dominates the (μ^-, e^-) conversion process and that in the region of Cu the coherent rate is at least six times bigger than the incoherent one. This is the reason why the study of the coherent rate met a good priority by the authors investigating the (μ^-, e^-) conversion rates and why the majority of experiments were performed on targets around Cu.

Calculations of the coherent rate have been performed in terms of the nuclear form factors [2,3,14] in the framework of gauge theories. For the incoherent rate the first calculations were done only recently [15,4] in nuclei with closed shells or subshells throughout the periodic table by employing shell model sum-rules i.e by assuming closure approximation and using a single Slater determinant for the initial (ground) state. These shell model results showed that the coherent channel dominates the (μ^-, e^-) process for light and medium nuclei but in the region of ^{208}Pb, a great part of the rate goes to other inelastic channels. Also the dependence of the branching ratio R_{eN} on the nuclear mass A and charge Z showed a maximum around $A \sim 100$ in agreement with the estimates of ref. [1].

Recent studies of the coherent and incoherent $\mu - e$ conversion with two independent methods [16,17] provided us with new interesting information. In the first method [16] the local density approximation with a Lindhard function for the description of the elementary processes $\mu^- p \rightarrow e^- p$ and $\mu^- n \rightarrow e^- n$ was employed. The incoherent rate in this method was obtained by integrating over the excited states of a local Fermi sea. These results veryfied the estimates of Weinberg and Feinberg by showing that, the coherent contribution is dominant for all nuclei of the periodic table, but they have shown that the branching ratio R_{eN} becomes maximum in the region of ^{208}Pb and not in the region of Cu.

In the second study [17] the quasi-particle RPA (QRPA) was employed for explicit calculations of the final nuclear states entering the total (coherent and incoherent) rate. One of the advantages of this method is the possibility of calculating the mean excitation energy of the studied nucleus and thus checking the results of closure approximation which are sensitive to this property. An important result of the QRPA study [17] was that, in the (μ^-, e^-) process the mean excitation energy of the nucleus is very small, $\bar{E} \approx 2MeV$ for ^{48}Ti, and differs appreciably from that of (μ^-, ν_μ) reaction, $\bar{E} \approx 20MeV$, which had been used in shell model calculations [4]. This is mainly due to the fact that, the coherent channel is not possible in the latter process while in the (μ^-, e^-) this is the dominant one. The quasi-particle RPA results shown also that the coherent rate for ^{48}Ti is dominant.

The above discrepancies motivate a detailed study of all possible channels of the (μ^-, e^-) conversion for medium and heavy nuclei and in particular for nuclei around ^{208}Pb. In the present work we have done quasi-particle RPA calculations of the coherent (μ^-, e^-) conversion rate while detailed calculations for the incoherent channels with the same method are in progress and will appear elsewhere [18].

In the set of isotopes we have chosen for study in the present work (see below *table 1*) we have included ^{48}Ti and ^{208}Pb for which recent experimental data exist for the upper limit on the branching ratio R_{eN} [7,8]. In the QRPA method nuclei with closed shells, like ^{60}Ni and ^{208}Pb, need a special treatment in order to determine the pairing parameters for protons (g^p_{pair}) and neutrons (g^n_{pair}). In this work we follow the manner used recently in the double beta decay [19].

2. Brief description of the formalism for the coherent (μ^-, e^-) process

The operator involved in the relevant nuclear matrix elements needed for the (μ^-, e^-) conversion rates has been described in detail in refs. [3,4,6]. Here we only give the non-relativistic expressions of the multipole expansion for the two components of this operator, i.e. the spin independent component (vector part)

$$T_M^{(l,0)J} = \tilde{g}_V \delta_{lJ} \sqrt{4\pi} \sum_{i=1}^{A} (3 + \beta\tau_{3i}) j_l(qr_i) Y_M^l(\hat{\mathbf{r}}_i) \tag{5}$$

and the spin-dependent component (axial vector part)

$$T_M^{(l,1)J} = \tilde{g}_A \sqrt{\frac{4\pi}{3}} \sum_{i=1}^{A} (\xi + \beta\tau_{3i}) j_l(qr_i) \left[Y^l(\hat{\mathbf{r}}_i) \otimes \sigma_i\right]_M^J \tag{6}$$

The summation in eqs. (5) and (6) runs over all nucleons of the considered nucleus (impulse approximation). The parameters $\tilde{g}_V$, $\tilde{g}_A$ and β depend on the assumed mechanism for lepton flavor violation [3,6] and take the values

$$\tilde{g}_V = \frac{1}{6}, \quad \tilde{g}_A = 0, \quad \beta = 3 \quad (photonic\ case) \tag{7}$$

$$\tilde{g}_V = \tilde{g}_A = \frac{1}{2}, \quad \xi = f_V/f_A, \quad f_V = 1, \quad f_A = 1.24 \quad (non-photonic\ case) \tag{8}$$

For the non-photonic case discussed in the present work, $\beta = 5/6$. In eqs. (5) and (6) $j_l(qr)$ are the spherical Bessel functions with q representing the magnitude of the momentum transferred to the nucleus. In a good approximation q is equal to the magnitude of the momentum of the outgoing electron i.e.

$$q \approx m_\mu - \epsilon_b - (E_f - E_{gs}) \tag{9}$$

where E_f, E_{gs} are the energies of the final and ground state of the nucleus, respectively, m_μ is the muon mass and ϵ_b the muon binding energy.

In the case of the coherent process ($E_f = E_i$), i.e. ground state to ground state ($0^+ \rightarrow 0^+$) transitions, only the vector component of the (μ^-, e^-) operator contributes and the corresponding rate is proportional to the muon-nuclear overlap

$$|< f \mid \Omega(q) \mid i, \mu >|^2 = \tilde{g}_V^2 \, (3 + f_V\beta)^2 \left[\tilde{F}_p(q^2) + \frac{3 - f_V\beta}{3 + f_V\beta} \tilde{F}_n(q^2) \right]^2 \tag{10}$$

where Ω is the responsible (μ^-, e^-) operator and

$$\tilde{F}_{p,n}(q^2) = \int d^3x \; \rho_{p,n}(\mathbf{x}) \; e^{-i\mathbf{q}\cdot\mathbf{x}} \; \Phi_\mu(\mathbf{x}) \tag{11}$$

In the latter definition, $\rho_p(\mathbf{x})$, $\rho_n(\mathbf{x})$ represent the proton, neutron densities normalized to Z and N, respectively and $\Phi_\mu(\mathbf{x})$ is the muon wave function. If we assume that the muon is at rest in the 1s atomic orbit and that its wave function varies a little inside nuclei (for light and medium nuclei this is a good approximation), we can factorize an average value $< \Phi_{1s} >$ of the muon wave function in eq. (11) and write

$$\tilde{F}_p(q^2) \approx < \Phi_{1s} > \; Z F_Z(q^2), \qquad \tilde{F}_n(q^2) \approx < \Phi_{1s} > \; N F_N(q^2) \tag{12}$$

with F_Z (F_N) the proton (neutron) nuclear form factors defined as

$$F_Z(q^2) = \frac{1}{Z} \int d^3x \rho_p(\mathbf{x}) e^{-i\mathbf{q}\cdot\mathbf{x}}, \qquad F_N(q^2) = \frac{1}{N} \int d^3x \rho_n(\mathbf{x}) e^{-i\mathbf{q}\cdot\mathbf{x}} \tag{13}$$

In the above approximation the nuclear part of the coherent rate, is analogous to the matrix element

$$M^2_{gs-gs}(q^2) = Z^2 F_Z^2(q^2) \left[1 + \frac{3 - f_V\beta}{3 + f_V\beta} \frac{N}{Z} \frac{F_N(q^2)}{F_Z(q^2)} \right]^2 \tag{14}$$

Thus, the nuclear structure dependence of the coherent (μ^-, e^-) conversion rate can be studied by calculating the matrix elements M^2_{gs-gs} of eq. (14) throughout the periodic table. In the photonic case only the protons of the considered nucleus contribute and the nuclear matrix element becomes $Z^2 F_Z^2(q^2)$.

In the present work the nuclear form factors $F_Z(q^2)$ and $F_N(q^2)$ are calculated by using quasi-particle RPA (see sect. 3 below) and compared with previous shell model results.

3. Coherent (μ^-, e^-) conversion matrix elements

The nuclear form factors involved in eq. (14) can either be obtained directly from experiment whenever possible [20] or be calculated by using various models as shell model [14], quasi-particle RPA [17] etc. For spherical nuclei in the Born approximation the point-proton (-neutron) nuclear form factors are given by

$$F_\tau(q^2) = \frac{1}{\tau}\sum_k \alpha_k^\tau (2j+1) < k \mid j_0(qr) \mid k >, \qquad \tau = Z,\ N \tag{15}$$

where α_j^τ are the occupation probabilities of the single particle states $\mid k >$ included in the used model space, $k \equiv (n, l, j)$. In the next subsections we describe in brief two methods of calculation of the nuclear form factors based on: (i) the shell model and (ii) the quasi-particle RPA.

A. Shell model form factors with fractional occupation probabilities

In the independent particle shell model, which is more appropriate for closed shell nuclei, the occupation probabilities α_k^τ in eq. (15) are *zero* for unoccupied states and *unity* for occupied states. For open-shell spherical nuclei or closed-shell spherical nuclei with diffused surfaces, the quantities α_k^τ are generally fractional numbers. If one uses harmonic oscillator wave functions, the point-nucleon form factors F_Z and F_N can be cast in compact analytical formulas as [21]

$$F_\tau(q^2) = \frac{1}{\tau} e^{-(qb)^2/4} \sum_{\lambda=0}^{N_{space}} \theta_\lambda^\tau (qb)^{2\lambda}, \qquad \tau = Z,\ N \tag{16}$$

where b is the harmonic oscillator parameter, N_{space} represents the maximum harmonic oscillator quanta included in the model space used and θ_λ^τ the coefficients

$$\theta_\lambda^\tau = \sum_{(n,l)j,\ \lambda \geq l} \alpha_{nlj}^\tau \frac{\pi^{\frac{1}{2}}(2j+1)n!C_{nl}^{\lambda-l}}{2\Gamma(n+l+\frac{3}{2})} \tag{17}$$

In eq. $\Gamma(x)$ is the known gamma function and

$$C_{nl}^m = \sum_{\kappa=0}^{m} \frac{(-)^m}{\kappa!(m-\kappa)!} \binom{n+l+\frac{1}{2}}{n-\kappa} \binom{n+l+\frac{1}{2}}{n+\kappa-m} \tag{18}$$

In the case of the independent particle shell model, the coefficients θ_λ^τ are the rational numbers of *table 2* ref. [21]. We should mention that a similar expression to that of eq. (17) is also obtained if one takes into account Gausian-type corrections in the point-nucleon form factors due to the nucleon finite size and center of mass motion of the nucleus (see ref. [21]).

B. Quasi-particle RPA Calculation

In the context of quasi-particle RPA, the form factors F_Z and F_N can be obtained by using as nuclear ground state either an uncorrelated vacuum or a correlated vacuum. The uncorrelated vacuum can be a BCS type vacuum or a Hartree - Fock - Bogolyubov (HFB) vacuum. In the majority of QRPA studies the considered BCS ground state contained only proton - proton and neutron - neutron pairing correlations. The proton - neutron pairing correlations could be included in the framework of the Hartree - Fock - Bogolyubov (HFB) theory. Recently, the QRPA theory has been extended so as the proton - neutron pairing to be taken into account by using a HFB ground state. Such a QRPA teatment of the nuclear double beta decay process. for example, indicates that the effect of proton - neutron pairing is significant [22,23].

In the present work, however, we consider the proton-neutron pairing to be negligible for the (μ^-, e^-) conversion and we shall use a BCS ground state. In this case, F_Z and F_N are calculated from eq. (15) by replacing the occupation probabilities α_k^τ with the quantities $\left(V_j^Z\right)^2$ (for protons) and $\left(V_j^N\right)^2$ (for neutrons), where V_j^Z, V_j^N are the amplitudes for the proton, neutron single particle states to be occupied which are determined by solving the BCS equations.

Since, as is well known, the QRPA ground state takes into account the short range nucleon-nucleon correlations, the effect of which has recently been proved to be very important [17], in the present work we will study the effect of these correlations on the coherent (μ^-, e^-) matrix elements. The short range nucleon - nucleon correlations can be included in the ground state by defining the correlated QRPA vacuum $| \tilde{0} >$ in terms of the uncorrelated vacuum $| 0 >$ as [24]-[27]

$$| \tilde{0} > \; = \; N_0 \, e^{\hat{S}^+} | \, 0 > \tag{19}$$

where N_0 is a normalization constant and $\hat{S}^+$ the operator

$$\hat{S}^+ \; = \; \frac{1}{2} \sum_{ij,JM} \; (-1)^{J-M} \; C_{ij}^J \, A_i^+(JM) \, A_j^+(J-M) \tag{20}$$

The operators $A_i^+(JM)$ denote the two quasi-particle creation operators in the angular momentum coupled representation. The indices i and j run over those two - quasiparticle configurations of the chosen model space which are coupled to a given J. The correlation matrix C^J (symmetric matrix) is constructed for each angular momentum J from the X^J and Y^J matrices, i.e. from the QRPA amplitudes for forward and backward excitation. A first order approximation for C^J is the following [25]

$$C_{ij}^J \; = \; \left(Y^J \left[X^J \right]^{-1} \right)_{ij} \tag{21}$$

Then, by keeping first order terms for the correlation matrix C in eq. (19), the normalization constant N_0 is given by

$$N_0^2 \; = \; \left[1 + \frac{1}{2} \sum_{ij,J} \; | \, C_{ij}^J \, |^2 \, \right]^{-1} \tag{22}$$

By using the correlated QRPA vacuum of eq. (19), the coherent rate matrix elements could be approximated in the form

$$< \tilde{0} \, | \, T \, | \, \tilde{0} > = \; N_0^2 \; < 0 \, | \, T \, | \, 0 > \tag{23}$$

This means that the correlated matrix elements are a rescaling of the uncorrelated ones.

4. Results and Discussion

In the present work we have calculated the matrix elements $M_{gs \to gs}^2$ of eq. (14) for the coherent (μ^-, e^-) conversion rate in the context of quasi-particle RPA. We have used harmonic oscillator wave functions to compute the elastic nuclear form factors (F_Z and F_N) entering eq. (14) for the nuclei ^{48}Ti, ^{60}Ni, ^{72}Ge, ^{112}Cd, ^{162}Yb and ^{208}Pb. In the BCS description of the uncorrelated ground state, the single particle energies have been calculated from a Coulomb - corrected Wood - Saxon potential with spin - orbit coupling. The G - matrix elements of the realistic Bonn one - boson exchange potential have been considered. The values of pairing parameters g_{pair}^p and g_{pair}^n renormalizing the proton and neutron pairing channels in the G - matrix have been deduced by comparing the quasiparticle energies with experimental pairing gaps as described in ref. [28]. Since, ${}^{60}_{28}Ni$ is a closed-shell (for protons) nucleus and ${}^{208}_{82}Pb$ is double closed-shell nucleus, their pairing parameters have been deduced from the neighbouring nuclei ${}^{60}_{26}Fe$ and ${}^{208}_{84}Po$, respectively, in analogy to that done in nuclear double beta decay of ${}^{48}Ca$ [19]. The model space, the harmonic oscillator size parameters and the pairing parameters g^p_{pair}, g^n_{pair} used for each studied nucleus are shown in *table 1.*

The proton, $F_Z(q^2)$, and neutron, $F_N(q^2)$ nuclear form factors obtained in the way described in sect. 2, are listed in *table 2*. We distinquished the following two cases of the momentum transferred to the nucleus: (i) by neglecting the muon binding energy ϵ_b the elastic value of the momentum transfer is the same for all nuclei i.e. $q \approx m_\mu \approx .535 fm^{-1}$ (in table 2 these results are labeled as QRPA(i)). (ii) by taking into account ϵ_b the elastic momentum transfer is equal to $q \approx m_\mu - \epsilon_b$ and varies from $q \approx .529 fm^{-1}$ (for ${}^{48}Ti$, where $\epsilon_b \approx 1.3 MeV$) to $q \approx .482 fm^{-1}$ (for ${}^{208}Pb$, where $\epsilon_b \approx 10.5 MeV$). In table 2 these results are labeled as QRPA(i). In this way, we can test the approximation of neglecting the muon binding in the calculation of the ground state to ground state transition matrix elements. We recall that the shell model results of ref. [4] were obtained with $q_{el} = .535 fm^{-1}$ throughout the periodic table, i.e. they correspond to QRPA(i) case.

By comparing the QRPA(i) form factors with the shell model ones we see that the two methods give about the same results. However, the form factors of QRPA(ii) for heavy nuclei differ appreciably from those of QRPA(i) and shell model ones. For ${}^{208}Pb$, for example, the QRPA(ii) form factors are about 30% larger than the corresponding QRPA(i) and shell model results. This is because ϵ_b makes the momentum transfer to the nucleus smaller and consequently the form factors bigger. The larger ϵ_b (lead region) the bigger form factor.

Table 1. Renormalization constants for proton (g^p_{pair}) and neutron (g^n_{pair}) pairing interactions determined from the experimental proton (Δ^{exp}_p) and neutron (Δ^{exp}_p) pairing gaps.

Nucleus	Configuration Space	$b_{ho}(fm^{-1})$	$\Delta^{exp}_p(MeV)$	$\Delta^{exp}_n(MeV)$	g^p_{pair}	g^n_{pair}
${}^{48}_{22}Ti_{26}$	16 levels (no core)	1.92	1.896	1.564	1.082	1.002
${}^{60}_{28}Ni_{32}$	16 levels (no core)	2.02	1.718[a]	1.395[a]	1.033	0.901
${}^{72}_{32}Ge_{40}$	16 levels (no core)	2.07	1.611	1.835	0.924	0.995
${}^{112}_{48}Cd_{64}$	16 levels (core ${}^{40}_{20}Ca_{20}$)	2.21	1.506	1.331	1.099	0.950
${}^{162}_{70}Yb_{92}$	23 levels (core ${}^{40}_{20}Ca_{20}$)	2.32	1.170	1.104	0.894	0.951
${}^{208}_{82}Pb_{126}$	18 levels (core ${}^{100}_{50}Sn_{50}$)	2.40	0.807[a]	0.611[a]	0.861	1.042

[a] *For the closed shell nuclei the parameters g^p_{pair} and g^n_{pair} have been borrowed from the $(N \pm 2, Z \mp 2)$ nuclei i.e. the experimental gaps (columns 4 and 5) for ${}^{60}_{28}Ni_{32}$ and ${}^{208}_{82}Pb_{126}$, are those of ${}^{60}_{26}Fe_{34}$ and ${}^{208}_{84}Po_{124}$, respectively.*

Table 2. Nuclear form factors for protons (F_Z) and neutrons (F_N) calculated in the context of the shell model and quasi-particle RPA. The two cases of the quasi-particle RPA results refer to momentum transfer: (i) $q = m_\mu$ for all nuclei (columns labeled QRPA(i)) and (ii) $q = m_\mu - \epsilon_b$ which depend on the considered nucleus (columns labeled QRPA(i)).

Nucleus	Shell Model			QRPA (i)		QRPA (ii)		
(A, Z)	b_{ho} (fm^{-1})	F_Z	F_N	F_Z	F_N	ϵ_b (MeV)	F_Z	F_N
$^{48}_{22}Ti_{26}$	1.906	.543	.528	.528	.506	1.250	.537	.514
$^{60}_{28}Ni_{32}$	1.979	.489	.478	.489	.476	1.950	.503	.490
$^{72}_{32}Ge_{40}$	2.040	.470	.448	.456	.435	2.150	.472	.451
$^{112}_{48}Cd_{64}$	2.202	.356	.318	.349	.312	4.890	.388	.352
$^{162}_{70}Yb_{92}$	2.335	.261	.208	.252	.218	7.500	.314	.280
$^{208}_{82}Pb_{126}$	2.434	.194	.139	.207	.151	10.475	.294	.236

Table 3. Coherent (μ^-, e^-) conversion matrix elements calculated in the context of shell model and quasi-particle RPA. See caption of table 2.

Nucleus	Photonic Mechanism ($\beta = 3$)			Non-Photonic Mechanism ($\beta = 5/6$)		
(A, Z)	Shell Model	QRPA (i)	QRPA (ii)	Shell Model	QRPA (i)	QRPA (ii)
$^{48}_{22}Ti_{26}$	142.7	135.2	139.6	374.3	363.2	375.2
$^{60}_{28}Ni_{32}$	187.5	187.8	198.7	499.6	498.2	527.4
$^{72}_{32}Ge_{40}$	212.9	212.7	227.8	595.8	596.2	639.5
$^{112}_{48}Cd_{64}$	274.2	280.0	346.7	769.4	785.8	983.3
$^{162}_{70}Yb_{92}$	313.6	311.0	484.3	796.0	840.3	1412.1
$^{208}_{82}Pb_{126}$	240.2	287.5	582.9	631.4	767.5	1674.9

The nuclear matrix elements given from eq. (14) are listed in *table 3*. We see that the coherent matrix elements show the following characteristics: (i) The results obtained by neglecting the muon binding energy (cases QRPA(i) and shell model), increase up to $A \approx 160$ (^{162}Yb) where they start to decrease. (ii) By taking into account the muon binding energy, case QRPA (ii), the obtained matrix elements become even a factor of 2 bigger in the lead region. The latter conclusion is in agreement with ref. [16], where a local density approximation was used and ϵ_b was calculated by solving the Schrödinger equation.

Another important conclusion is the fact that, the QRPA(ii) matrix elements ($M^2_{gs \to gs}$) increase continuously up to lead. In ref. [16] it was found that the coherent matrix elements start to decrease around ^{238}U. This means that the coherent rate is bigger for heavy nuclei (lead region) and that, from an experimental point of view, one has to employ as heavy as possible nuclear targets provided that they also satisfy other additional criteria e.g. the minimization of the reaction background etc. In addition the (μ^-, e^-) conversion electrons are expected to show a pronounced peak around $E_e = m_\mu - \epsilon_b$, which in lead region is about $95 MeV$. The dependence of the branching ratio R_{eN} of the coherent process on the mass number A is shown in fig. 1.

We should mention that, in the present approach as wel as the one used in ref. [4], we use a mean value for the overlap between the muon and nuclear wave function (see eq. (12)). This is described by the effective charge Z_{eff} which feels the muon in the 1s atomic orbit [16,29]. In ref. [16] an exact muon wave function was used for the description of the muon - nucleus overlap and found that this approximation is not very reliable in the ^{208}Pb region and beyond.

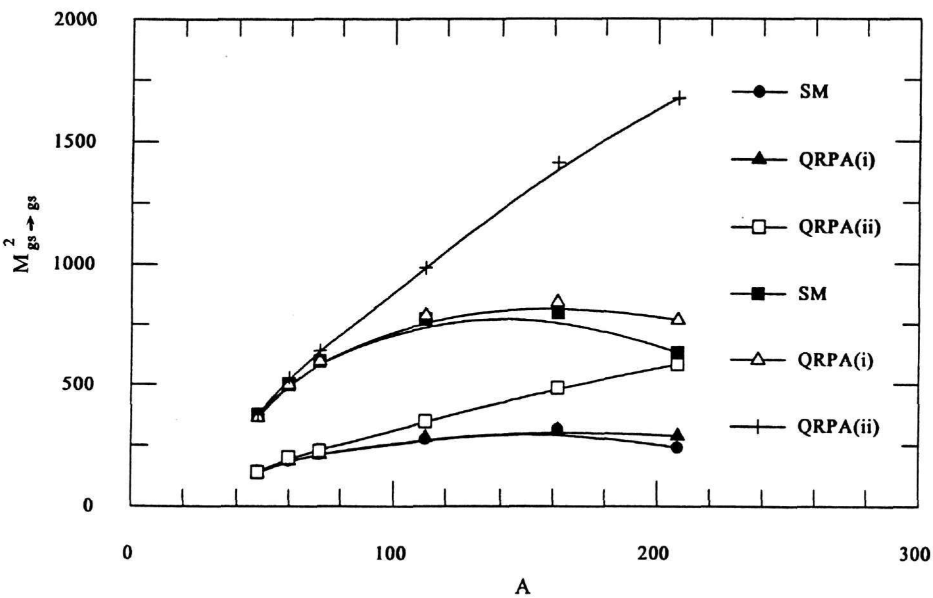

Figure 1. Variation of the coherent (μ^-, e^-) conversion matrix elements ($M^2_{gs \to gs}$) with respect to the mass number A for the photonic mechanism (three lower curves) and the non-photonic mechanism (three upper curves). Consideration of the muon binding energy ϵ_b (QRPA(ii) results) strongly affects the matrix elements for heavy nuclei. For comparison the results of ref. [7] (shell model results) are also shown.

We must also recall that, contrary to the present calculations, the shell model results of ref. [4] take into account the finite nucleon size by folding the nuclear point-density with a Gaussian proton (neutron) density distribution. This correction reduces the form factors by about 5%. However, these results do not include any corrections due to the smearing of the Fermi surface but the studied nuclei have been assumed of closed shells, i.e. with occupation probabilities zero and one. In the case of quasi-particle RPA, the occupation probabilities are fractional numbers for all states included in the model space, namely, they are about equal to unity for the inner most levels and progressively decrease as we go to the uppermost levels where they tend to zero. A similar picture in the context of shell model has been recently developped [21] by determining the fractional occupation probabilities from the elastic scattering form factor data.

In the present work we have also estimated the effect of the ground state correlations on the coherent matrix elements by using a correlated quasi-particle RPA vacuum instead of the uncorrelated one, as we have stated in sect. 3B. We found that the coherent matrix elements obtained by using eq. (22) are about $30-35\%$ smaller than those of eq. (14) which means that the ground state correlations strongly reduce the coherent matrix elements.

5. Conclusions

In the present work we have studied the dependence of the coherent (μ^-, e^-) conversion matrix elements on the nuclear parameters A and Z. We have employed the quasi-particle RPA method to determine the proton and neutron nuclear form factors for a set of six nuclei from ^{48}Ti to ^{208}Pb. We found that the coherent rate increase continuously up to the lead region.

We have also investigated the effect of consideration of the muon binding energy in the kinematic of the (μ^-, e^-) process and the nucleon-nucleon correlations in the QRPA ground state on the $\mu - e$ matrix elements. We found that the present result are in good agreement with those extracted in the framework of the Local Density Approximation. However, the quasi-particle RPA results differ appreciably for heavy nuclei from those obtained in the context of the shell model.

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