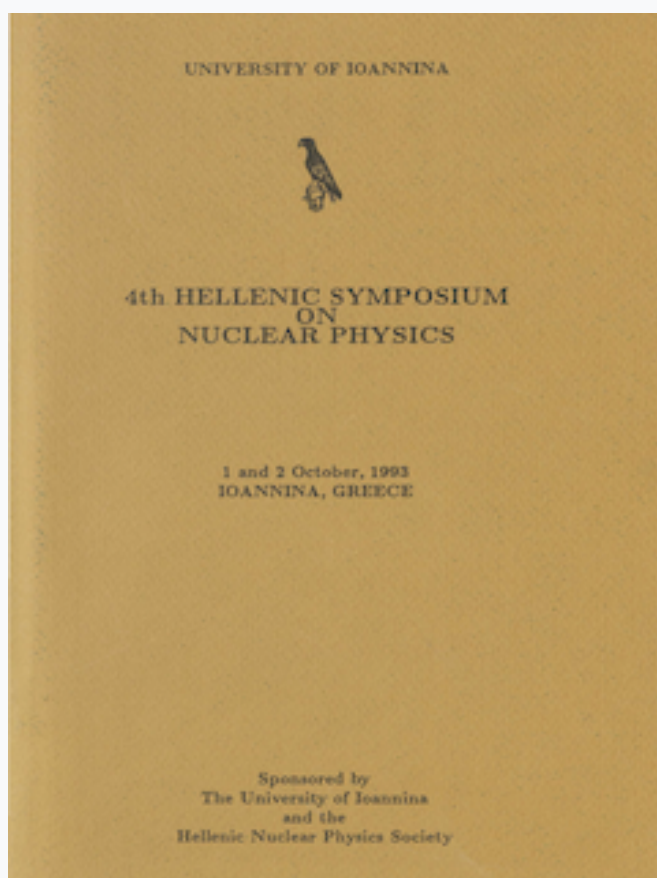


## HNPS Advances in Nuclear Physics

Vol 4 (1993)

HNPS1993



### Azimuthal Distribution in Heavy-Ion Collisions

G. Pandis, S. Das Gupta

doi: [10.12681/hnps.2887](https://doi.org/10.12681/hnps.2887)

#### To cite this article:

Pandis, G., & Das Gupta, S. (2020). Azimuthal Distribution in Heavy-Ion Collisions. *HNPS Advances in Nuclear Physics*, 4, 202–214. <https://doi.org/10.12681/hnps.2887>

## Azimuthal Distribution in Heavy-Ion Collisions

G. Pantis\* and S. Das Gupta

*Physics Dept., McGill University, Montreal, P.Q., H3A 2T8, Canada.*

### Abstract

We consider recent experimental data on azimuthal distributions of particles seen in heavy-ion collisions at  $35\text{MeV/nucleon}$  and at  $50\text{MeV/nucleon}$ . For  $^{12}\text{C}$  on  $^{12}\text{C}$  at  $50\text{MeV/nucleon}$  lab energy, the distribution shows features characteristic of flow; for  $^{12}\text{C}$  on  $^{197}\text{Au}$  experimental data show features characteristic of rotation. For  $^{40}\text{Ar}$  on  $^{51}\text{V}$  at  $35\text{MeV/nucleon}$ , effects of both flow and rotation are seen. In magnitudes the effects are small. We find that BUU calculations are able to reproduce these results.

---

PACS NO 25.70-z

\*Permanent Address; Dept. of Physics, University of Ioannina, P.O. Box 1186, GR-45110 Ioannina, Greece

The first microscopic calculations for azimuthal distributions were published about five years ago where the example considered was  $Nb$  on  $Nb$  at  $650 MeV$ / nucleon in the lab<sup>1</sup>. The magnitude of the peaking of the distribution near  $0^\circ$  was conjectured and shown to be linked to the nuclear equation of state. Since then many experiments<sup>2-7</sup> and more calculations have been done. At hundreds of  $MeV$  the only collectivity seen is due to the flow. This flow is repulsive; as the energy of collision goes down the flow turns attractive. The attractive nature of flow was demonstrated by Tsang et. al. with polarization measurements<sup>6</sup>. Many other details of collectivity are being measured and a comprehensive account may be found in Tsang<sup>5</sup> et. al. and references therein.

In this paper we investigate in a semi-quantitative manner one feature of the collectivity. As we mentioned, at hundreds of  $MeV$  the only collectivity seen is due to flow. In a Key West, Florida Conference Roy Lacey presented data on azimuthal distribution (we state below how the azimuthal angle is defined) at much lower energy, where, in addition to collectivity expected from flow, signatures of collectivity due to rotation could be seen<sup>3</sup>. The effects are not large but are unmistakably present. As a continuation of work in this energy region, azimuthal correlation functions between different fragments have been measured<sup>4</sup>; these data also show collectivity due to rotation and flow and further in these correlation experiments there is no need to determine the reaction plane. In the theoretical calculation presented here we investigate if collectivity due to both rotation and flow can emerge in BUU calculations<sup>8</sup>. The reason we call the calculation semi-quantitative rather than quantitative is that as is typical in BUU calculations, clusters can not be produced without invoking additional mechanisms for fluctuations. Thus although the experimental results may be for protons (or for example, for alphas) our results will be an average value for all charges, free or bound. Because of this limitation in the present calculation we can not calculate for the data of ref. 4 but we will attempt semi-quantitative fits for data from references 3 and 7.

The reaction plane is defined to be the plane containing the impact parameter and

the beam direction. The azimuthal distribution is with respect to this plane. In each event that is analysed, experimentalists define as the positive x-direction that direction in which the projectile-like particles ( $y > y_{cm}$  where  $y$  is rapidity) have a net positive momentum. The net momentum in the y-direction is zero. Thus  $\langle p_x \rangle > 0$ ,  $\langle p_y \rangle = 0$  whether one has positive or negative angle scattering. For pure flow, particles with  $y < y_{cm}$  would peak at  $\phi = 180^\circ$  falling off on either side, particles with  $y > y_{cm}$  would peak at  $\phi = 0^\circ$  and  $360^\circ$ . For pure rotation, one would expect particles to be concentrated in the  $x - z$  plane for both  $y < y_{cm}$  and  $y > y_{cm}$ . This is depicted in Fig. 1. An intermediate situation where both rotations and flow are seen would be a superposition of Figs. 1(a) and 1(c) or of Figs. 1(b) and 1(c). (For cases where the flow is not well-developed distinguishing between  $\phi = 0^\circ$  and  $\phi = 180^\circ$  is difficult; see experimental refs. 5 and 7). We use the data from refs. 7 and 3; Fig. 3. in ref 7 for  $Ar$  on  $V$  at  $35MeV$  shows both flow and rotation; Fig. 5 in ref. 3 shows almost pure flow for  $C$  on  $C$  at  $50MeV$ ; for  $C$  on  $Au$  at  $50MeV$  the signatures are primarily those of rotation. The question we asked is; is the BUU model capable of showing these differences ? In the next section we give the necessary technical details and the last section gives our results.

### Details of Calculations

Since the BUU model has been described in full<sup>8</sup> before, we merely give some technical details. We use the Lenk-Pandharipande<sup>9</sup> prescription for solving the Vlasov part which is known to give very accurate energy and momentum conservation. We work in a configuration space of  $40fm^3$ . The Lenk-Pandharipande method requires dividing up this space into cubes. We use cubes of side 1 fm. One needs a smearing parameter  $n$  which is an integer ( see eq. 2.9 in ref. 9). We take  $n$  to be 1. A Skyrme interaction  $U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^{7/6}$  is used where  $A = -356MeV$ ,  $B = 303MeV$  and  $\rho_0 = .16fm^{-3}$ . For  $^{12}C$  on  $^{12}C$  we use 200 test particles per nucleon, for  $^{40}Ar$  on  $^{51}V$  we use 90 test particles per nucleon and for  $^{12}C$  on  $^{197}Au$  we use 40 test particles

per nucleon.

For two-body collisions we use the parametrisation of Appendix B in ref. 8. When two test particles collide their phase-space co-ordinates change from  $(\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2)$  to  $(\vec{r}_1, \vec{p}_1', \vec{r}_2, \vec{p}_2')$ . The scattering conserves total momentum, i. e.,  $\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$ . The angle  $\theta_s$  of scattering in the cm is chosen from Monte-Carlo sampling of differential scattering cross-section and it is usual to choose the azimuthal angle  $\phi$  arbitrarily. If we write total angular momentum as  $L = L_{cm} + L_{rel}$  then this prescription conserves the value of  $L_{cm}$  but will usually change both the direction and magnitude of  $L_{rel}$ . It requires no extra effort to maintain the direction of  $L_{rel}$  and we incorporate that in our scattering prescription. In the cm. of the colliding test particles, the momentum of one of the test particles before collision is  $\vec{p}$  (the other one has  $-\vec{p}$ ), which, after collision will change to  $\vec{p}' = \cos\theta_s \vec{p} + \sin\theta_s \vec{n}_\perp$ ;  $\vec{n}_\perp$  is a unit vector perpendicular to  $\vec{p}$  that we seek to find. After scattering  $\vec{p}'$  is perpendicular to  $\vec{L}'_{rel} = \vec{r} \times \vec{p}'$ . Here  $\vec{r} = \vec{r}_1 - \vec{r}_2$ . If the directions of  $\vec{L}'_{rel}$  and  $\vec{L}_{rel}$  are to remain the same,  $\vec{p}'$  is perpendicular to  $\vec{L}_{rel}$  as well. Since  $\vec{p}$  is already perpendicular to  $\vec{L}_{rel}$  this means  $\vec{n}_\perp$  is perpendicular to  $\vec{L}_{rel}$ . Hence  $\vec{n}_\perp = \frac{\vec{p} \times (\vec{r} \times \vec{p})}{\text{norm}}$ . Relativity makes the expression more complicated; the complete expression including relativity is given in eq.(13) of ref. 10. Pauli blockings for collisions are implemented as detailed in ref. 8.

## Results

We performed calculations for  $^{12}C$  on  $^{12}C$  at 50 MeV/nucleon lab energy,  $^{40}Ar$  on  $^{51}V$  at 35 MeV/nucleon and  $^{12}C$  on  $^{197}Au$  at 50 MeV/nucleon. For  $C$  on  $C$  and  $Ar$  on  $V$  we run the BUU code for 90 fm/c after the two nuclei touch. For  $C$  on  $Au$  the time is increased to 108 fm/c. At this time the azimuthal angle  $\phi$  for each test particle is determined from the ratio  $p_y/p_x$  and the signs of  $p_x$  and  $p_y$ . As the experimental effects we are looking for are small, steps had to be taken to diminish fluctuations that arise merely from the calculational limitations imposed by the use of finite number of test particles in Monte-Carlo simulation. One way to check this is

to repeat the calculation going through a different sequence of random numbers that the computer simulation will use and check if two runs give significant differences. We have found it necessary to take averages over  $\approx 10$  runs in each of the above cases. A better idea would be to increase the number of test particles ten times as this would also increase the accuracy of the mean field calculation but this was deemed to be too time consuming. In addition we use the symmetry that  $+y$  and  $-y$  are equivalent; this effectively increases the number of test particles by a factor of 2. Experimental results<sup>4,7,11</sup> suggest that results are not sensitive to impact parameters so long as the collisions are central or semi-central. In a limited search we found this to be true in our calculations also. For  $C$  on  $C$  we use impact parameter  $b = 2fm$ , in the other two cases an impact parameter value of  $b = 4fm$  is used. As we are looking for a qualitative effect rather than a specific number, the precise value of  $b$  is not important.

Fig. 2. shows the results of our calculation for  $^{12}C$  on  $^{12}C$ . These should be compared with those given in Fig. 5 of ref.3. The peak to valley ratio is somewhat bigger in our graph ( $\approx 1.5$ ) than in experiment ( $\approx 1.35$ ) but this is acceptable as we have made no integration on impact parameter and besides the data are for protons whereas ours would be an average over all charges, whether they appear singly or in composites. Fig. 3 gives our results for  $Ar$  on  $V$  and should be compared with Fig. 3 in ref. 7. The figures are very similar, in shape and in magnitude. There appears to be shallow minima between  $0^\circ$  and  $90^\circ$  and between  $270^\circ$  and  $360^\circ$ . The maximum at  $180^\circ$  is higher than the one at  $0^\circ$  and  $360^\circ$ . For  $y < y_{cm}$  this would be expected for flow plus rotation. Wilson et. al.<sup>3</sup> define collective parameters  $F_{ip}$  and  $F_{fs}$ . Here

$$F_{ip} = \frac{\int_{-45^\circ}^{45^\circ} d\phi \frac{dn}{d\phi} + \int_{135^\circ}^{225^\circ} d\phi \frac{dn}{d\phi}}{\int_{0^\circ}^{360^\circ} d\phi \frac{dn}{d\phi}} \quad (1)$$

$$F_{fs} = \frac{\int_{-90^\circ}^{90^\circ} d\phi \frac{dn}{d\phi}}{\int_{0^\circ}^{360^\circ} d\phi \frac{dn}{d\phi}} \quad (2)$$

Wilson et. al.<sup>3</sup> call  $F_{ip}$  the rotation sensitive parameter and  $F_{fs}$  the flow sensitive parameter. For pure rotation  $F_{ip}$  would be greater than  $\frac{1}{2}$  and would be independent of rapidity  $y$ . For pure flow,  $F_{fs}$  would be a function of  $y$  changing from a value less than  $\frac{1}{2}$  for  $y < y_{cm}$  to greater than  $\frac{1}{2}$  for  $y > y_{cm}$ . The values of  $F_{ip}$  and  $F_{fs}$  are plotted in Fig. 3 in ref. 3 as a function of rapidity  $y$  for  $H$  and  $He$  for the case of  $Ar$  on  $V$ . We do not have enough Monte-Carlo data to plot this as a function of rapidity but for all  $y < y_{cm}$  we get "average" values ( $F_{ip} = .56$  and  $F_{fs} = .48$ ) which are in between experimental values of  $H$  and  $He$ . Experimentally  $He$  has more pronounced collectivity than  $H$ . As stated before, as our calculations are for all charges, free or bound, an intermediate value is quite reasonable. Lastly, as in experiments, the theoretical curves for the case of  $C$  on  $Au$ , as depicted in Fig. 4, show predominantly the collectivity of rotation; the values of the maxima at  $0^\circ$  and  $180^\circ$  are about the same. However, the agreement in the case of  $C$  on  $Au$  may not be of much significance. Since  $C$  is much smaller than  $Au$  the 50 MeV/nucleon beam leads to rather small excitation energy; we are not entirely convinced that our simple calculation is very appropriate in such cases.

Our calculation suggests that the BUU model is able to explain at least semi- quantitatively these data on azimuthal distribution; it would be interesting to see if angular correlation between different fragments as measured in ref. 4 are also reproducible or not. That is a more difficult calculation as that entails, in addition, assumptions about clusterisation in the theory. But present success lead us to believe further theoretical work in this area will be promising. Calculations in ref. 5. include the productions of deuterons but these authors have not yet extended the calculations to see if simultaneous signatures of rotation and flow can be reproduced in a microscopic calculation. In the future we intend to extend our calculations including clusters in phenomenological models<sup>12</sup>.

We are thankful to Roy Lacey for very useful discussions. This work is supported in

part by NSERC and in part by the Quebec Department of Education. G. P. wishes to acknowledge the hospitality of the Physics Dept. at McGill University while spending part of his sabbatical there.



## References

- 1) G. M. Welke, M. Prakash, T. T. S. Kuo, S. Das Gupta and C. Gale, Phys. Rev. **C38**,2101(1988)
- 2) R. Madey, M. Elaaser, W. M. Zhang, J. Schambach, D. Keane, B. D. Anderson, A. R. Baldwin, J. W. Watson, G. Krebs, H. Weiman, G. D. Westfall, C. Gale and K. Haglin, Nucl. Phys. **A553**,779c(1993).
- 3) W. Wilson, R. Lacey, D. Cebra, M. Cronqvist, S. Howden, J. Karn, D. Kroftcheck, T. Li, A. Nadasen, C. A. Ogilvie, T. Reposeur, A. Vander Molen, J. Yee and G. D. Westfall in Advances in Nuclear Dynamics, Proceedings of the 7th Winter Workshop on Nuclear Dynamics, Key West, Florida, 1991, edited by W. Bauer and J. Kapusta,p 139.
- 4) R. A. Lacey et. al. ,Phys. Rev. Lett **70**,1224(1993)
- 5) M. B. Tsang, D. R. Bowman, N. carlin, P. Danielewicz, C. K. Gelbke, W. G. Gong, Y. D. Kim, W. G. Lynch, L. Phair, R. T. de Souza, and F. Zhu, Phys. Rev. **C47**,2717(1993)
- 6) M. B. Tsang, W. G. Lynch, R. M. Ronningen, Z. Chen, C. K. gelbke, T. Nayak, J. Pochodzalla, F. Zhu, M. Tohyama, W. Trautmann, and W. Dunnweber, Phys. Rev. Lett. **60**,1479(1988).
- 7) W. K. Wilson, R. Lacey, C. A. Ogilvie and G. D. Westfall, Phys. Rev. **C45**,738(1992)
- 8) G. F. Bertsch and S. Das Gupta, Phys. Rep. **160**,189(1988)
- 9) R. Lenk and V. Pandharipande, Phys. Rev. **C39**,2242(1989)
- 10) C. Gale and S. Das Gupta, Phys. Rev. **C42**,1577(1990)
- 11) Roy Lacey, private communication.
- 12) G. Montaru et. al. Phys. Rev. **C47**,2764(1993)

### Figure Captions

Fig. 1.(1a) Pattern of angular distribution due to collective flow alone for  $y < y_{cm}$ ; (1b) pattern of angular distribution due to collective flow alone for  $y > y_{cm}$ ; (1c) pattern due to pure rotation; (1d) pattern due to both effects combined for  $y < y_{cm}$ .

Fig. 2. Calculated azimuthal distribution for  $^{12}C$  on  $^{12}C$  at  $50 MeV$ / nucleon lab energy. The top figure is for  $y < y_{cm}$  and the bottom one is for  $y > y_{cm}$ .

Fig. 3. Azimuthal distribution for  $^{40}Ar$  on  $^{51}V$  at  $35 MeV$ /nucleon for  $y < y_{cm}$ . Experimental points are shown as solid dots and have been read off from fig. 3 of ref. 7.

Fig. 4. Azimuthal distribution for  $^{12}C$  on  $^{197}Au$  at  $50 MeV$ /nucleon for  $y < y_{cm}$ .

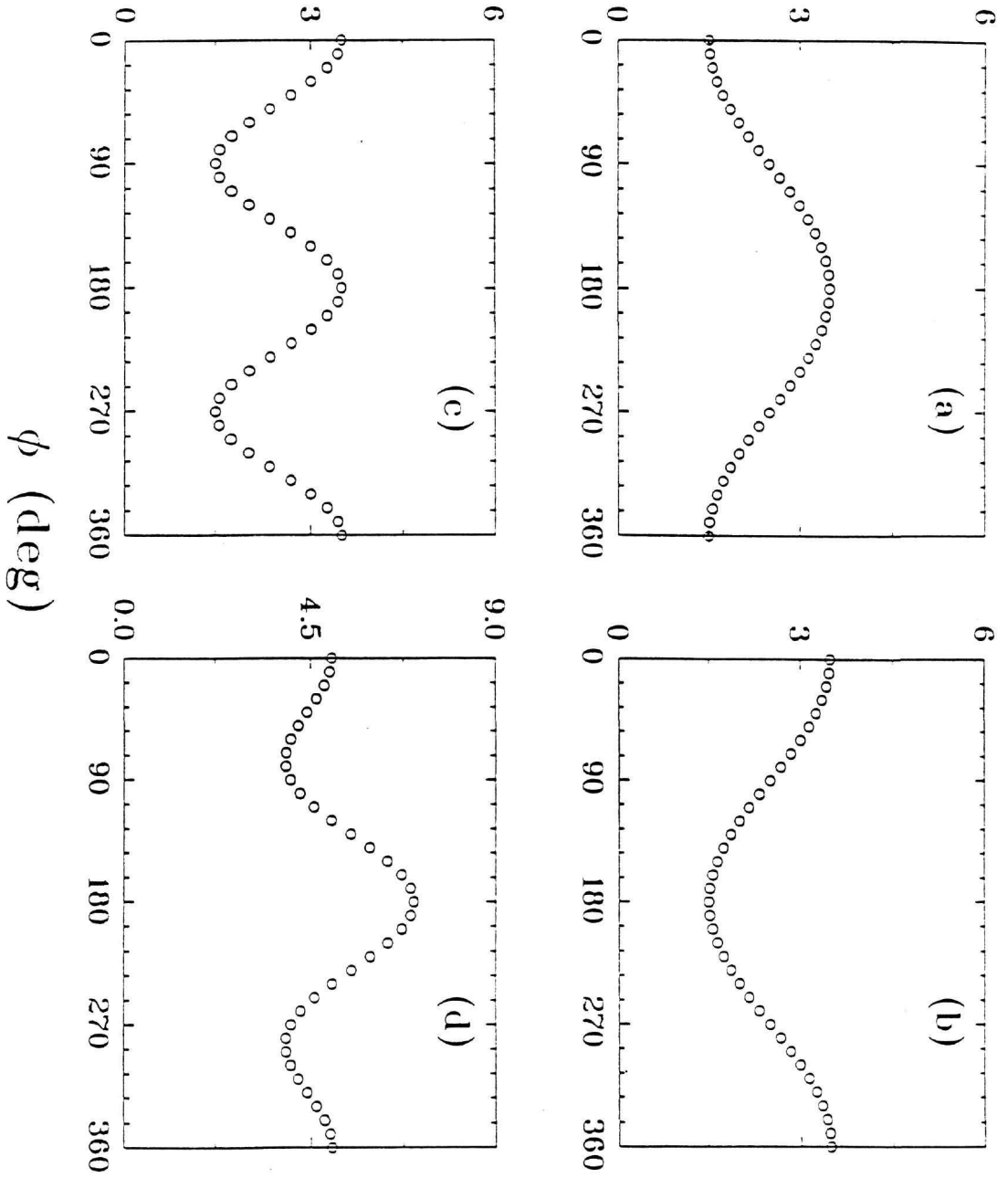
$$\frac{dn}{d\phi} \text{ (arb. units)}$$


Figure:

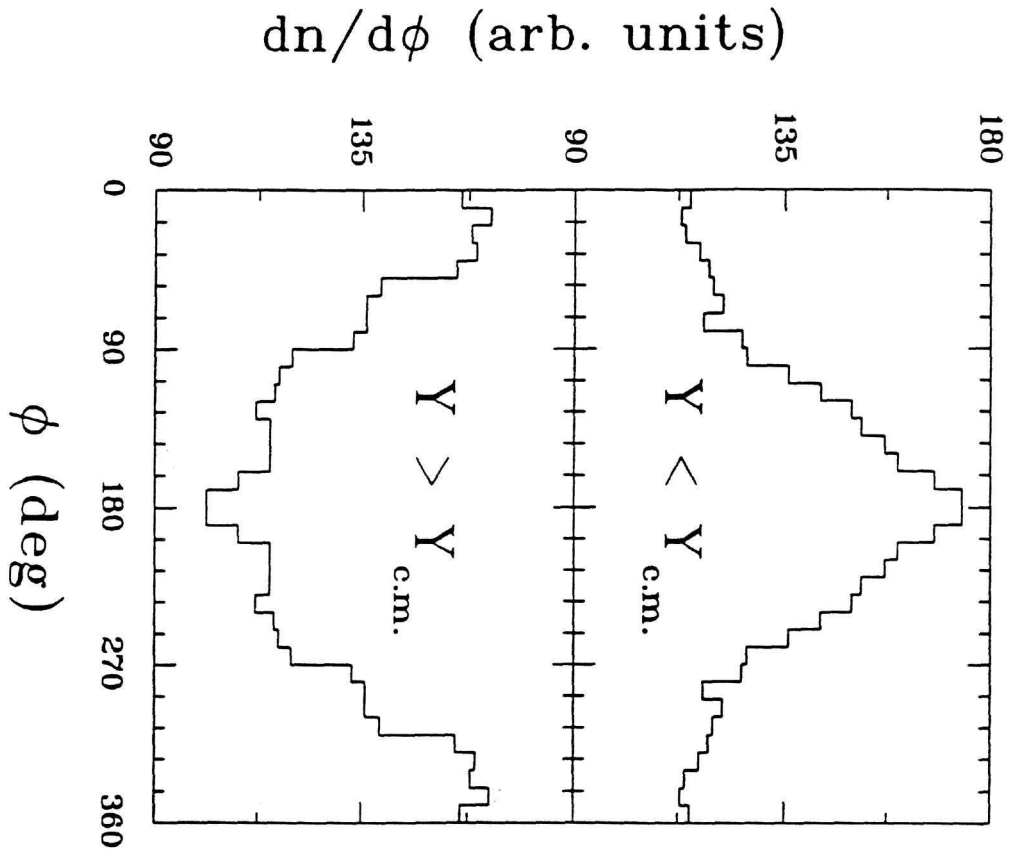


Figure: 2.

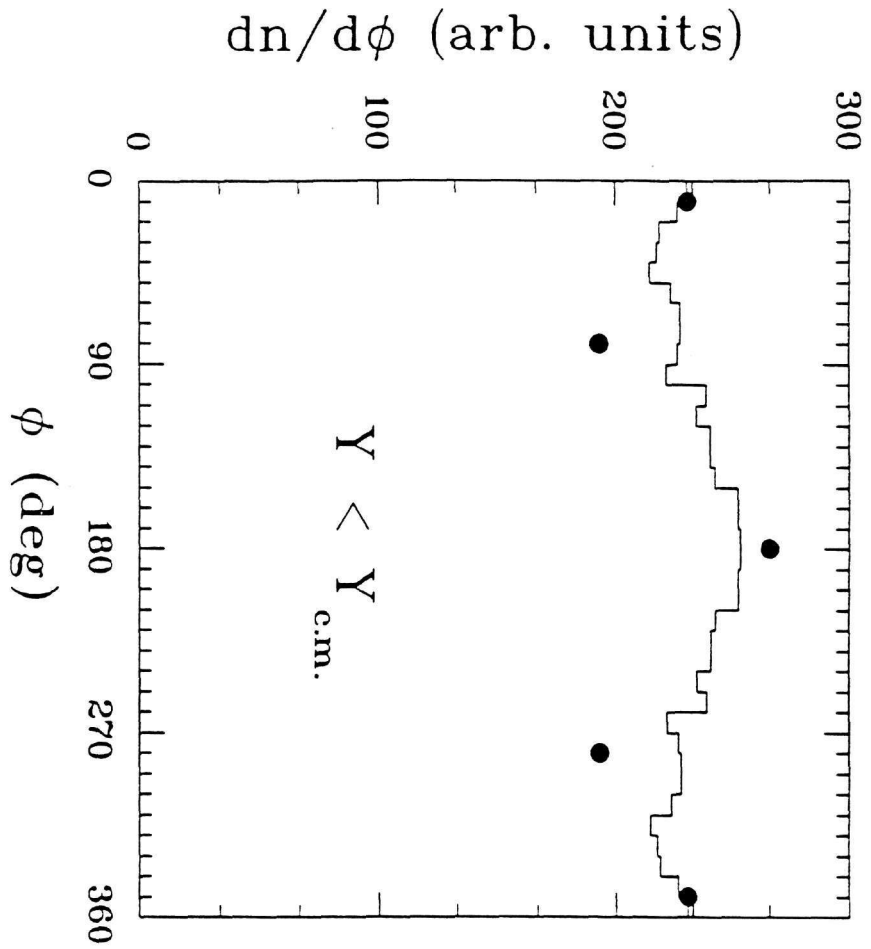


Figure: 3.

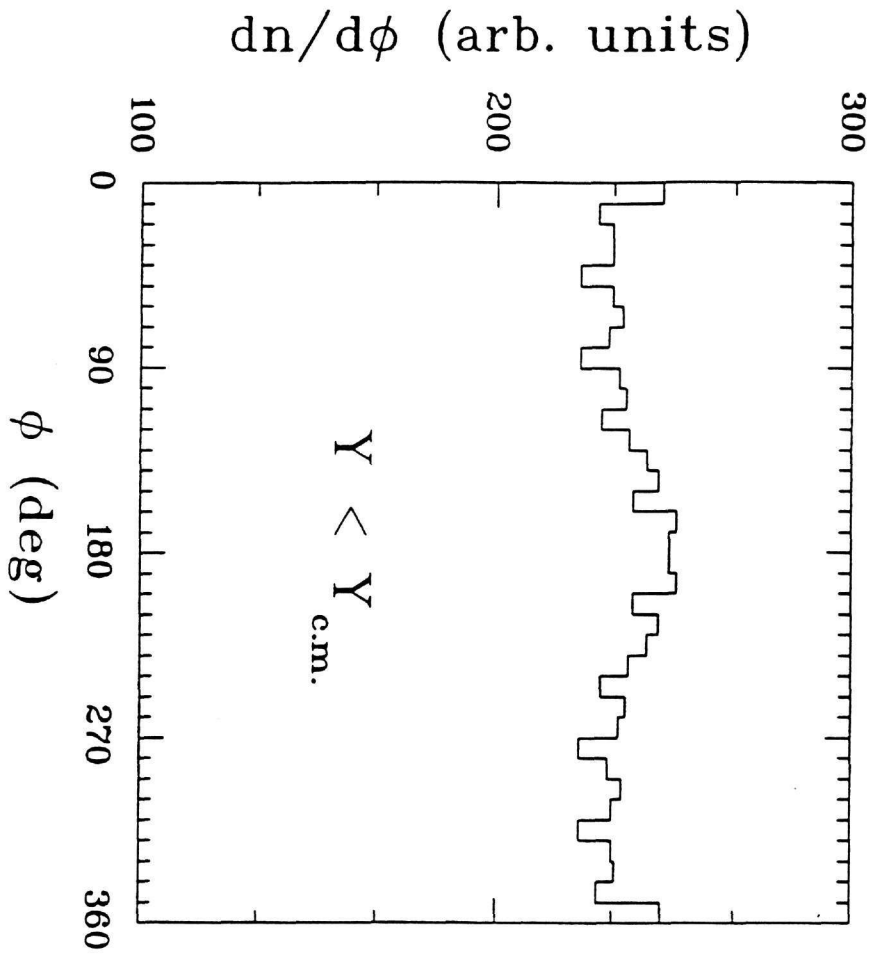


Figure: 4.