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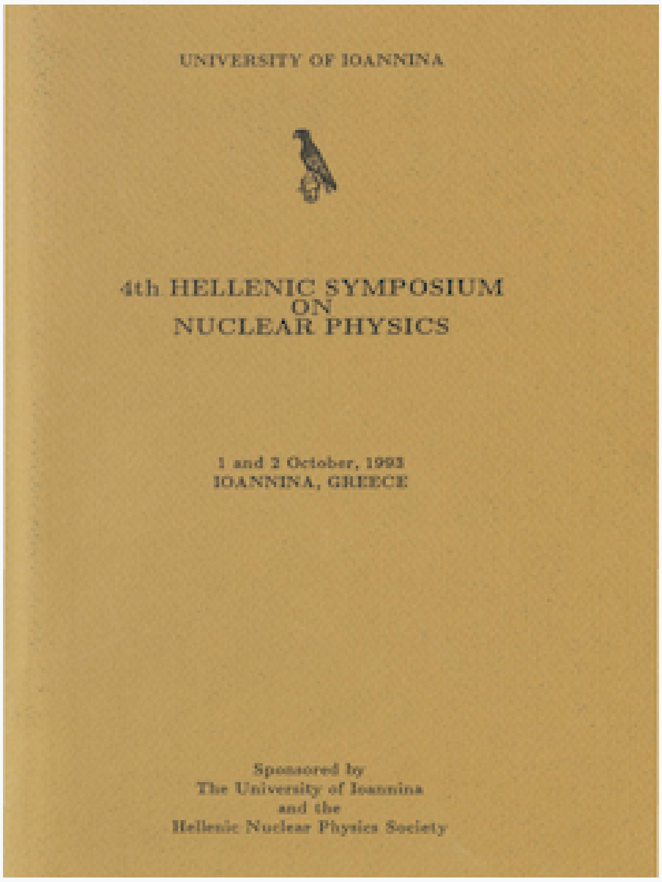

Local Theories and Bohmian Mechanics

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LOCAL THEORIES AND BOHMIAN MECHANICS

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ABSTRACT: *One of the last developments in the research for extending the scope of the quantum theory is the recently appearing work on the Bohmian Mechanics.The motivation for an extension is provided by the conclusions of the EPR paradoxon and the famous alternative concerning the physical reality.Discussed are some properties of Bohmian Mechanics concerning the self-consistency of the theory.*

1.INTRODUCTION

The publication of the EPR[1] paper in the year 1935 triggered a continuing discussion about the completeness or the incompleteness of Quantum Mechanics.The discussion concerned,conceivably, the very foundations of Quantum Theory and created new arguments about the existence of signals[2] moving with unlimited velocity.

This,of course,concerns directly the Theory of Relativity as well as the existence of waves transporting neither energy nor momentum[3].

A number of experiments have been carried out and a part of them definitely favor Quantum Mechanics[4,5].

The Aspect et al.experiments[6,7] have conclusively demonstrated that Quantum Mechanics cannot be complemented with hidden variables to make more complete its predictions.This should be clear also from the uncertainty principle.

This being impossible attempts have been made to give a looser interpretation of the configuration space of a quantum mechanical system in view of the random character of the coordinates distribution in Quantum Mechanics.

The purpose of this paper is to show on the basis of particular properties of the Bohmian Mechanics that such an effort presents some problems of principle which make the foundation of Bohmian Mechanics conceptually impossible.

In sect.2 the basics of Bohmian Mechanics are presented and in sect. 3 some consequences of the definition of the field velocity are derived which show that this definition leads to inconsistences.

2.THE BASICS OF BOHMIAN MECHANICS

I shall present here the basic ingredients of a Mechanics called Bohmian[8,9] to the honor of D.Bohm who first introduced the idea of hidden variables to Quantum Theory[10].

The aim of this new Mechanics is to give a deterministic character to Quantum Mechanics and make it more complete.

The idea of the authors working on Bohmian Mechanics is that quantum randommess is a local manifestation of a global quantum equilibrium state of the universe.

Furhtermore,in the framework of this kind of Mechanics whenever one is talking about particles one has to have in mind the picture of particles in the classical sense.So every particle has to have at any time a definite position and velocity.

The state of a system of N particles is given by $(\mathbf{Q},\Psi)$,where $\Psi(q_1,\ldots q_N,t)$ is the wave function and $\mathbf{Q} = (q_1,\ldots q_N)$ is the configuration space vector.

The evolution of the state as defined above is governed by the two equations

$$i\hbar\partial\psi(q,t)/\partial t = H\psi(q,t) \quad \text{(Schrödinger)}, \qquad (2.1)$$

and

$$d\mathbf{Q}/dt = u^{\Psi}(\mathbf{Q}) \qquad \text{(Bohmian)}. \qquad (2.2)$$

$u^{\Psi}(\mathbf{Q})$ in (2.2) is a velocity vector field defined on the configuration space by

$$u_n^{\Psi} := (\hbar/m_n)\ Im[\nabla_n\psi(\mathbf{q},t)/\psi(\mathbf{q},t)], \qquad (2.3)$$

where n = 1,2,3,...N is the particles number of the system.

In (2.1) -(2.3) a distinction is made between $\{q_1,q_2,\ldots q_N\}$, the generic and $\{Q_1,Q_2,\ldots Q_N\}$ the actual configuration space.

3. SOME ADVERSE CONSEQUENCES

The definition (2.3) gives rise to a number of remarks of which the following are presented here.

A) *The dynamics*

The first remark concerns the dynamics of the theory. If the wave function is purely real or purely imaginary, then the velocity field vanishes identically.

$$\text{(i)} \qquad \left.\begin{array}{ll} \text{If} \quad Re\,\psi(\mathbf{q},t) = 0 & \text{or} \\ Im\,\psi(\mathbf{q},t) = 0, & \text{then} \\ u^{\psi}(\mathbf{q},t) \equiv 0\ . & \end{array}\right\} \qquad (3.1)$$

This can trivially be seen as follows:

Let $\bar{\psi}(\mathbf{q},t) = Re\psi(\mathbf{q},t)$, *and* $\bar{\bar{\psi}}(\mathbf{q},t) = Im\psi(\mathbf{q},t)$ so that

$$\psi(\mathbf{q},t) = \bar{\psi}(\mathbf{q},t) + i.\bar{\bar{\psi}}(\mathbf{q},t)$$

and

$$\nabla\bar{\psi}(\mathbf{q},t) = \nabla\bar{\psi}(\mathbf{q},t) + i.\nabla\bar{\bar{\psi}}(\mathbf{q},t).$$

Then,

$$u^{\psi}(\mathbf{q},t) = (\hbar/m).Im\,\frac{[\nabla\bar{\psi}(\mathbf{q},t) + i.\nabla\bar{\bar{\psi}}(\mathbf{q},t)].[\bar{\psi}(\mathbf{q},t) - i.\bar{\bar{\psi}}(\mathbf{q},t)]}{[\bar{\psi}(\mathbf{q},t) + i.\bar{\bar{\psi}}(\mathbf{q},t)].[\bar{\psi}(\mathbf{q},t) - i.\bar{\bar{\psi}}(\mathbf{q},t)]} \qquad (3.2)$$

and

$$u^{\psi}(\mathbf{q},t) = (\hbar\ /)\quad Im\,\{[\bar{\psi}(\mathbf{q},t)\nabla\bar{\psi}(\mathbf{q},t)+\bar{\bar{\psi}}(\mathbf{q},t)\nabla\bar{\bar{\psi}}(\mathbf{q},t)]$$

$$+\ i.[\bar{\psi}(\mathbf{q},t)\nabla\bar{\bar{\psi}}(\mathbf{q},t)-\bar{\bar{\psi}}(\mathbf{q},t)\nabla\bar{\psi}(\mathbf{q},t)]\}/\ [\bar{\psi}(\mathbf{q},t)^2+\ \bar{\bar{\psi}}(\mathbf{q},t)^2]$$

$$= (\hbar/m)[\bar{\psi}(\mathbf{q},t)\nabla\bar{\bar{\psi}}(\mathbf{q},t)-\bar{\bar{\psi}}(\mathbf{q},t)\nabla\bar{\psi}(\mathbf{q},t)]/\ [\bar{\psi}(\mathbf{q},t)^2+\ \bar{\bar{\psi}}(\mathbf{q},t)^2]. \qquad (3.3)$$

In the last expression it is clear that the nominator vanishes in any case if either $\bar{\psi}(\mathbf{q},t)$ or $\bar{\bar{\psi}}(\mathbf{q},t)$ is equal to zero.

It is concluded, therefore, that every quantum mechanical system for which ψ is real or imagimary is absolutely static.

B) *The self-consistency*

In Bohmian Mechanics the particle position in space appears in two distinct forms: One as a solution of equation (2.2) and the other as the expectation value of the position operator.

Hence, Bohmian Mechanics will be self-consistent, if and only if the quantum mechanical expectation value on the one hand and the value $<\mathbf{Q}(t)>$ of the coordinate of the particles on the other obtained as a solution of equation (2.2) and which both are functions of the time are equal.

Here, it is clear that

$$<\mathbf{Q}(t)>_{BM} = \mathbf{Q}(t)_{BM}, \tag{3.4}$$

and

$$\mathbf{Q}(t)_{BM} = \int_0^t dt' \upsilon^{\psi}[\mathbf{Q}(t')] \tag{3.5}$$

$\mathbf{Q}(t)_{BM}$ in (3.5) does not need any averaging integration, because **Q** represents the actual position vector in the configuration space.

On the other hand the quantum mechanical expectation value of the position operator is

$$<\mathbf{q}(t)>_{QM} = \int \psi^*(\mathbf{q},t)\mathbf{q}\psi(\mathbf{q},t)d\mathbf{q}. \tag{3.6}$$

The self-consistency of the theory requires that the solution of (2.2) must coincide with that given by (3.6).

Hence, (3.5) and (3.6) should give the self-consistensy condition

$$\hbar/m \; Im\{\int_0^t dt' [\nabla\psi(\mathbf{q},t')/\psi(\mathbf{q},t')]\} + C(\mathbf{q}) =? \int \psi^*(\mathbf{q},t)\mathbf{q}\psi(\mathbf{q},t)d\mathbf{q}, \tag{3.7}$$

where

$$C(\mathbf{q}) = \int \psi^*(\mathbf{q},0)\mathbf{q}\psi(\mathbf{q},0)d\mathbf{q}$$

is an integration "constant".

Relation (3.7) cannot be generally satisfied for the following obvious reasons:

i) If $\psi(\mathbf{q},t)$ has zeros in **q**, the r.h.s. vanishes, while the l.h.s. may become inifinite.

ii) If $\psi(\mathbf{q},t)$ 0 for t ∞, the r.h.s. vanishes, while the l.h.s. may be finite or infinite.

iii) If $\nabla\psi(\mathbf{q},t)$ has zeros in **q**, the l.h.s. vanishes, while the r.h.s. is different from zero.

iv) If $\psi(\mathbf{q},t)$ 0 for **q** ∞, then $\psi^*(\mathbf{q},t)\psi(\mathbf{q},t)$ vanishes, while $\nabla\psi(\mathbf{q},t)$

$/\psi(\mathbf{q},t)$ becomes infinite.

The above results A) and B) indicate that the theory might be mainetained,if the appropriate modifications in its foundations are possible.

4.CONCLUSIONS

The attempt to make more complete Quantum Mechanics in the framework of Bohmian Mechanics as defined in sect. 2 encounters at present substuntial difficulties on the conceptual level.

The main dificulties come from the definition of the velocity vector field and concern both the dynamics and the self-consistency of the theory.

It seems,therefore,that as the Aspect et al. experiments have shown Quantum Mechanics cannot be completed in the way of Bohmian Mechanics.On the other hand theoretical results agree sofar with the experiment to a very high accuracy.

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