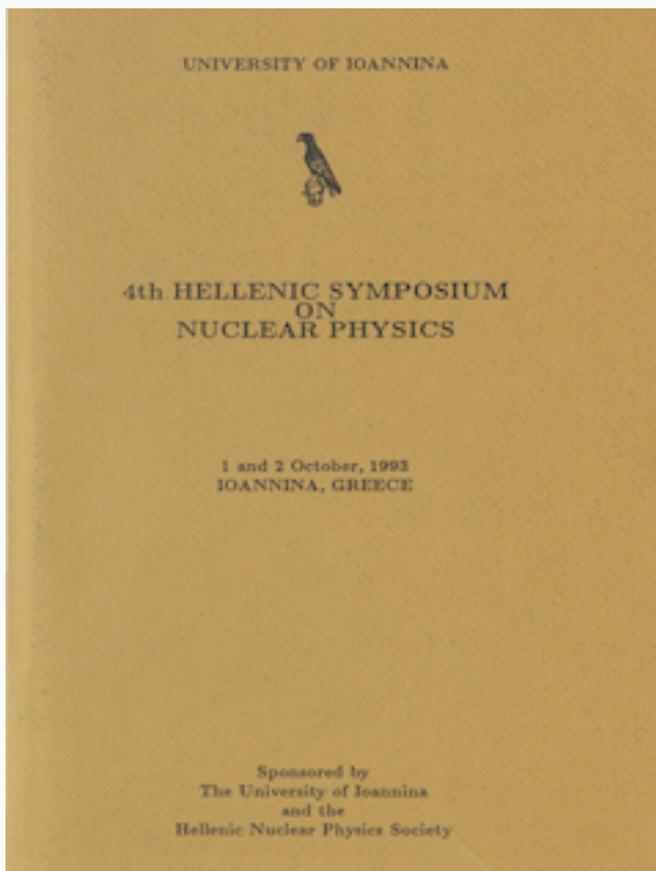


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The dependence of the nuclear charge form factor on short range correlations and surface fluctuation effects

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Abstract

An investigation is carried out in order to study effects, originating from fluctuations of the nuclear surface, to the elastic charge form factor of light nuclei in which the two-body part of the short range correlation factor is also included through a Jastrow-type correlation function. It is found that if these effects are taken into account in the uncorrelated (harmonic oscillator) part of such a form-factor for the ${}^4\text{He}$, ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$ nuclei, the quality of the fitting is improved. In addition, they lead to a drastic change in the asymptotic behaviour of the point-proton form factor which now drops off for large values of the momentum transfer q quite slowly that is as $\text{const.} \cdot q^{-4}$.

1 Introduction

In a series of papers [1, 2, 3] an expression of the elastic charge form factor, $F_{ch}(q)$, truncated at the two body term, was derived using the factor cluster expansion of Ristig et al [4, 5]. This expression, which is a sum of one-body and two-body terms, depends on the harmonic oscillator (HO) parameter b_1 and the correlation parameter λ through a Jastrow type correlation function which introduces the short range correlations (SRC). The fitting of $F_{ch}(q)$ to the experimental data was very good both for low and high values of momentum transfer except for the values around the last maximum for ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$. Better fitting can be obtained if the parameter λ is taken to be state dependent but in this case there is a big number of parameters, six for ${}^{16}\text{O}$ [1] and ten for ${}^{40}\text{Ca}$ [6].

Another possible way to make the agreement between theory and experiment better might be to introduce, in addition, other types of ground state

("long range") correlations which have been the subject of previous investigations by a number of authors (see, for example, [7, 8, 9, 10, 11, 12]). We focus our attention, as in ref.[10], on fluctuations of nuclear surface due to the zero point motion of collective surface vibrations [13, 14] which can affect the ground state charge density. The presence of surface fluctuation correlations (SFC), introduce another fitting parameter in addition to the HO and the SRC parameters. Thus, it appears to be of interest to develop the relevant formalism and to investigate what would be the effect of this additional parameter to the best fit values of the other parameters and to the quality of the fitting. The aim of this work is to report on results of certain investigations in this direction. This paper is a revised and extended version of the contribution to the Symposium.

In section 2, the above SFC are introduced to the HO densities and analytic expressions of the nuclear density, elastic form factor and of the n-th moment of the density distribution are given for the nuclei ${}^4\text{He}$, ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$. In section 3 the introduction of the SFC in addition to the SRC is studied. Numerical results are reported and discussed in section 4.

2 The effect of the collective surface vibrations on the harmonic oscillator density and form factor

Our starting point is the expression for the proton (or charge) density of a nucleus which has been deformed through the zero-point motions of the collective surface vibrations. This expression, according to ref. [10] (see also ref. [7, 8] for a rather similar expression) is the following:

$$\rho_{1\sigma}(r) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \rho_1(r - \xi) \exp\left[-\frac{(\xi - s_0)^2}{2\sigma^2}\right] d\xi \quad (1)$$

where $\rho_1(r)$ is the uncorrelated density, s_0 is a correction needed to conserve the number of particles in the correlated ground state and σ is a measure of the effect of the zero point fluctuations. The value of σ is related to β_λ , the deformation parameters for the states of multipolarity λ , with the relation $\sigma^2 \simeq \frac{R_0^2}{4\pi} \sum_\lambda \beta_\lambda^2(\tau = 0)$ while the β_λ parameters can be determined from the values of $B(E_\lambda)$ [10, 12].

In (1) we consider for $\rho_1(r)$ the HO proton density in which the centre of

mass correction has been taken into account for nuclei ${}^4\text{He}$ to ${}^{40}\text{Ca}$ which is:

$$\rho_1(r) = \frac{1}{Z\pi^{3/2}} \frac{1}{\tilde{b}_1^3} \exp\left[-\frac{r^2}{\tilde{b}_1^2}\right] \sum_{k=0}^2 N_{2k} \left(\frac{r}{\tilde{b}_1}\right)^{2k} \quad (2)$$

where

$$\begin{aligned} N_0 &= 2\eta_{1s} + 6 \left(1 - \frac{b_1^2}{\tilde{b}_1^2}\right) \eta_{1p} + \left(10 - 20\frac{b_1^2}{\tilde{b}_1^2} + 10\frac{b_1^4}{\tilde{b}_1^4}\right) \eta_{1d} + \\ &\quad \left(2 - 4\frac{b_1^2}{\tilde{b}_1^2} + 5\frac{b_1^4}{\tilde{b}_1^4}\right) \eta_{2s} \\ N_2 &= 4\eta_{1p} \frac{b_1^2}{\tilde{b}_1^2} + \left(\frac{8}{3}\frac{b_1^2}{\tilde{b}_1^2} - \frac{20}{3}\frac{b_1^4}{\tilde{b}_1^4}\right) \eta_{2s} + \left(\frac{40}{3}\frac{b_1^2}{\tilde{b}_1^2} - \frac{40}{3}\frac{b_1^4}{\tilde{b}_1^4}\right) \eta_{1d} \\ N_4 &= \left(\frac{8}{3}\eta_{1d} + \frac{4}{3}\eta_{2s}\right) \frac{b_1^4}{\tilde{b}_1^4} \end{aligned} \quad (3)$$

and $\tilde{b}_1^2 = b_1^2 \left(1 - \frac{1}{A}\right)$, A is the mass number and $b_1 = \sqrt{\frac{\hbar}{m\omega}}$ the harmonic oscillator parameter. η_{nl} is the occupation probability (0 or 1 in the present treatment) of the nl state. It is easily checked that when $\tilde{b}_1 = b_1$, that is when the centre of mass correction is not taken into account, the coefficients in polynomial 2 are reduced to the well known expressions:

$$N_0 = 2\eta_{1s} + 3\eta_{2s}, \quad N_2 = 4\eta_{1p} - 4\eta_{2s}, \quad N_4 = \frac{8}{3}\eta_{1d} + \frac{4}{3}\eta_{2s}$$

General expressions of similar structure for the density and the form factor in the HO model have been given in ref [15].

From (1) and (2) an analytic expression of $\rho_{1\sigma}(r)$ can be derived. This is:

$$\rho_{1\sigma}(r) = \frac{1}{Z\pi^{3/2}} \frac{1}{\tilde{b}_1^2 \sqrt{\tilde{b}_1^2 + 2\sigma^2}} \exp\left[-\frac{(r - s_0)^2}{(\tilde{b}_1^2 + 2\sigma^2)}\right] \sum_{k=0}^4 C_k r^k \quad (4)$$

where the coefficients C_k depend on $N_0, N_2, N_4, \sigma, s_0$ and \tilde{b}_1 and are given by the following formulae:

$$\begin{aligned} C_0 &= N_0 + B^2 \left(\tilde{b}_1^2 \sigma^2 + 2\sigma^4 + \tilde{b}_1^2 s_0^2\right) N_2 + \\ &\quad \left(3B^2 \sigma^4 + 6B^3 \tilde{b}_1^2 \sigma^2 s_0^2 + B^4 \tilde{b}_1^4 s_0^4\right) N_4 \\ C_1 &= -2B^2 \tilde{b}_1^2 s_0 N_2 - 4B^4 \tilde{b}_1^2 s_0 \left(3\tilde{b}_1^2 \sigma^2 + 6\sigma^4 + \tilde{b}_1^2 s_0^2\right) N_4 \\ C_2 &= B^2 \tilde{b}_1^2 N_2 + 6B^4 \tilde{b}_1^2 \left(\tilde{b}_1^2 \sigma^2 + 2\sigma^4 + \tilde{b}_1^2 s_0^2\right) N_4 \\ C_3 &= -4B^4 \tilde{b}_1^4 s_0 N_4 \\ C_4 &= B^4 \tilde{b}_1^4 N_4 \end{aligned} \quad (5)$$

and $B = 1/(\tilde{b}_1^2 + 2\sigma^2)$

By using expression (4) one can find an analytic expression for the n th moment of the density. This is the following:

$$\begin{aligned} \langle r^n \rangle_{1\sigma} &= \frac{2\tilde{b}_1^n}{Z\sqrt{\pi}} \exp[-s_0^2 B] \sum_{k=0}^4 C_k \tilde{b}_1^k \left(1 + \frac{2\sigma^2}{\tilde{b}_1^2}\right)^{(k+n+2)/2} \times \\ &\left[\Gamma\left(\frac{k+n+3}{2}\right) {}_1F_1\left(\frac{k+n+3}{2}; \frac{1}{2}; s_0^2 B\right) + \right. \\ &\left. 2s_0 \sqrt{B} \Gamma\left(\frac{k+n+4}{2}\right) {}_1F_1\left(\frac{k+n+4}{2}; \frac{3}{2}; s_0^2 B\right) \right] \end{aligned} \quad (6)$$

An approximate expression for $\langle r^n \rangle_{1\sigma}$ may be derived by truncation of the series at the second power for σ and the first power for s_0 :

$$\begin{aligned} \langle r^n \rangle_{1\sigma} &\simeq \frac{2\tilde{b}_1^n}{Z\sqrt{\pi}} \sum_{k=0}^4 C_k \tilde{b}_1^k \times \\ &\left[\left(1 + (k+n+2) \frac{\sigma^2}{\tilde{b}_1^2}\right) \Gamma\left(\frac{k+n+3}{2}\right) + \right. \\ &\left. \frac{2s_0}{\tilde{b}_1} \left(1 + (k+n+1) \frac{\sigma^2}{\tilde{b}_1^2}\right) \Gamma\left(\frac{k+n+4}{2}\right) \right] \end{aligned} \quad (7)$$

By taking into account that

$$\langle r^0 \rangle_{1\sigma} = \langle r^0 \rangle_1 = 1$$

the approximate expression for the parameter s_0 is:

$$s_0 \simeq -\frac{\sqrt{\pi}}{4} \frac{\sigma^2}{\tilde{b}_1} \frac{2N_0 + N_2 + \frac{3}{2}N_4}{N_0 + N_2 + 2N_4} \quad (8)$$

That expression was used as a first approximation in our calculations. More accurate values were obtained by varying s_0 until normalization of $\rho_{1\sigma}(r)$ was achieved to a good approximation. From expressions (7) and (8) and from the known expression of the moments of the HO density one can find the approximate expression of the contribution of the SFC, $\Delta \langle r^2 \rangle_{1\sigma}$, to the mean square radius for nuclei ${}^4\text{He}$ to ${}^{40}\text{Ca}$. This is given by the following expression:

$$\begin{aligned} \Delta \langle r^2 \rangle_{1\sigma} &\simeq \frac{2}{Z} \sigma^2 \left[3\left(N_0 + \frac{3}{2}N_2 + \frac{15}{4}N_4\right) - \right. \\ &\left. \frac{(2N_0 + N_2 + \frac{3}{2}N_4)(N_0 + 2N_2 + 6N_4)}{N_0 + N_2 + 2N_4} \right] \end{aligned} \quad (9)$$

Finally, for the elastic point proton form factor the well known expression in Born approximation

$$F_{1\sigma}(q) = 4\pi \int_0^\infty \rho_{1\sigma}(r) \frac{\sin(qr)}{qr} r^2 dr \quad (10)$$

is used. Substitution of $\rho_{1\sigma}(r)$ from (4) leads to the following analytic expression of $F_{1\sigma}(q)$ in terms of the confluent hypergeometric function

$$F_{1\sigma}(q) = \frac{1}{Z\sqrt{\pi}} \frac{\sqrt{B}}{\tilde{b}_1^2} \frac{1}{q} \sum_{k=0}^4 C_k I_k \quad (11)$$

where

$$I_k = \frac{1}{B^{(k+2)/2}} \exp[-s_0^2 B] \times \\ \text{Im} \left[2\Gamma\left(\frac{k+2}{2}\right) {}_1F_1\left(\frac{k+2}{2}; \frac{1}{2}; z^2\right) + 4\Gamma\left(\frac{k+3}{2}\right) z {}_1F_1\left(\frac{k+3}{2}; \frac{3}{2}; z^2\right) \right] \quad (12)$$

The complex quantity z is given by: $z = \sqrt{B}s_0 + iq/(2\sqrt{B})$.

Expression (12) may be reduced to a somehow more convenient form:

$$F_{1\sigma}(q) = \frac{1}{Z} \exp\left[-\frac{q^2}{4B}\right] \sum_{n=0}^2 \left[\tilde{C}_{2n} \cos(qs_0) + \tilde{\tilde{C}}_{2n} \frac{\sin(qs_0)}{q} \right] \left(\frac{q}{2\sqrt{B}}\right)^{2n} + \\ \frac{2}{\sqrt{\pi} Z \tilde{b}_1^2 \sqrt{B}} \exp[-s_0^2 B] \frac{1}{q} \text{Im}[I] \quad (13)$$

where

$$I = \sum_{n=0}^2 \frac{C_{2n}}{B^n} \Gamma(n+1) {}_1F_1\left(n+1; \frac{1}{2}; z^2\right) + 2z \sum_{n=0}^1 \frac{C_{2n+1}}{B^{n+\frac{1}{2}}} \Gamma(n+2) {}_1F_1\left(n+2; \frac{3}{2}; z^2\right) \quad (14)$$

The coefficients \tilde{C}_{2n} and $\tilde{\tilde{C}}_{2n}$ depend also on $N_0, N_2, N_4, \sigma, s_0$ and \tilde{b}_1 and are given by the following expressions:

$$\begin{aligned} \tilde{C}_0 &= N_0 + \frac{3}{2}N_2 + \frac{15}{4}N_4 + \frac{\sigma^2}{\tilde{b}_1^2} (2N_0 + N_2 + \frac{3}{2}N_4) \\ \tilde{C}_2 &= -N_2 - (5 - 4\sigma^2 B)N_4 \\ \tilde{C}_4 &= \tilde{b}_1^2 B N_4 \\ \tilde{\tilde{C}}_0 &= \frac{s_0}{\tilde{b}_1^2} (2N_0 + N_2 + \frac{3}{2}N_4) \\ \tilde{\tilde{C}}_2 &= -s_0 B (2N_2 + 6N_4) \\ \tilde{\tilde{C}}_4 &= 2s_0 \tilde{b}_1^2 B^2 N_4 \end{aligned} \quad (15)$$

It may be easily checked from expression (13) that when the SFC are switched off, that is when the limiting case $\sigma \rightarrow 0$ is considered, expression (13) for $F_{1\sigma}(q)$ goes over to the well known harmonic oscillator one, as should be the case, on the basis of expressions (10) and (1). Furthermore, by using the asymptotic expansion of the confluent hypergeometric function, we find that the behaviour for $F_{1\sigma}(q)$ at large values of the momentum transfer is the following:

$$F_{1\sigma}(q) \simeq \frac{1}{\sqrt{\pi}Z} \frac{\exp[-s_0^2 B]}{b_1^2 \sqrt{B}} \left(A_4 \left(\frac{q}{2\sqrt{B}}\right)^{-4} + A_6 \left(\frac{q}{2\sqrt{B}}\right)^{-6} + A_8 \left(\frac{q}{2\sqrt{B}}\right)^{-8} \right) \quad (16)$$

where

$$\begin{aligned} A_4 &= -s_0 C_0 - \frac{1}{2B} C_1 \\ A_6 &= (-3s_0 + 2Bs_0^3)C_0 + (3s_0^2 - \frac{3}{2B})C_1 + \frac{3s_0}{B}C_2 + \frac{3}{2B^2}C_3 \\ A_8 &= \left(-\frac{45s_0}{4} + 15Bs_0^3 - 597B^2s_0^5\right)C_0 + \left(-\frac{45}{8B} + \frac{45s_0^2}{2} + \frac{315}{2}Bs_0^4\right)C_1 + \\ &\quad \left(\frac{45s_0}{2B} - 15s_0^3\right)C_2 + \left(\frac{45}{4B^2} - \frac{45s_0^2}{2B}\right)C_3 - \frac{45s_0}{2B^2}C_4 \end{aligned} \quad (17)$$

Thus, it is seen that for sufficiently large values of q , the form factor tends to zero rather slowly, namely as the inverse fourth power of the momentum transfer. On the contrary, the HO form factor goes rapidly to zero for large q , namely as a Gaussian or as a Gaussian times an even power of q (depending on the nucleus).

The value of q at which the $F_{1\sigma}(q)$ approaches the value given by the asymptotic expression (16) does not seem to depend very strongly on the nucleus, at least when the values of the parameters b_1 and σ are determined in the way described in the following two sections. In Fig. 1 the $F_{1\sigma}(q)$ has been plotted for the ^{16}O nucleus, using the values $b_1 = 1.647\text{fm}$ and $\sigma = 0.224\text{fm}$ (see section 4), together with its asymptotic behaviour $\text{const.} \cdot q^{-4}$ and the improved asymptotic expression (16), respectively. It is seen that $F_{1\sigma}(q)$ becomes close to the asymptotic behaviour $\text{const.} \cdot q^{-4}$ at quite large values of the momentum transfer (larger than 10fm^{-1}) while the corresponding values of q pertaining expression (16) are quite smaller.

There are two parameters in expression (13), the HO parameter b_1 and the SFC parameter σ which can be determined from the deformation parameters β_λ associated with the low lying collective states of the nucleus or it can be treated for example as a free parameter. In the latter case, the fitting

of the form factor (13) (after correcting it for the finite proton size [1]) to the experimental data (refs. [16, 17] for ${}^4\text{He}$, [18] for ${}^{16}\text{O}$ and [19] for ${}^{40}\text{Ca}$) leads to zero value for the parameter σ except for ${}^4\text{He}$. For ${}^4\text{He}$, σ is different from zero ($\sigma = 0.706\text{fm}$ and $b_1 = 1.252\text{fm}$) and the value of χ^2 is smaller compared to the one obtained with the HO model. However the diffraction minimum is not reproduced. Because of these reasons the introduction of short range correlations is advisable. This is done in the next section.

3 The effect of the surface fluctuation and short range correlations on the charge form factor and density

A general expression for the charge form factor $F_{ch}(q)$ of light closed shell nuclei was derived [1, 3] using the factor cluster expansion of Ristig, Ter Low and Clark [4, 5]. This formula was subsequently simplified [1] by using normalized correlated wave functions of the relative motion which were parametrized through a Jastrow type wave function of the form:

$$\psi_{nIS}(r) = N_{nIS}[1 - \exp(-\lambda r^2/b^2)]\phi_{nl}(r) \quad (18)$$

where N_{nIS} are the normalisation factors, $\phi_{nl}(r)$ the harmonic oscillator wave functions and $b = \sqrt{2}b_1$ is the HO parameter for the relative motion. The expression for $F(q)$ is of the form

$$F(q) = F_1(q) + F_2(q) \quad (19)$$

$F_1(q)$ is the contribution of the one-body term to $F(q)$:

$$F_1(q) = \frac{1}{Z} \exp\left[-\frac{b_1^2 q^2}{4}\right] \sum_{k=0}^2 \tilde{N}_{2k} \left(\frac{b_1 q}{2}\right)^{2k} \quad (20)$$

where

$$\begin{aligned} \tilde{N}_0 &= 2(\eta_{1s} + \eta_{2s} + 3\eta_{1p} + 5\eta_{1d}) \quad , \quad \tilde{N}_2 = -\frac{4}{3}(2\eta_{2s} + 3\eta_{1p} + 10\eta_{1d}) \\ \tilde{N}_4 &= \frac{1}{3}(4\eta_{2s} + 8\eta_{1d}) \end{aligned} \quad (21)$$

while $F_2(q)$ is the contribution of the two-body term to $F(q)$ and is a function of q^2 through the matrix elements

$$A_{nIS}^{n''S'}(j_k) = \langle \psi_{nIS} | j_k(qr/2) | \psi_{n''S'} \rangle$$

It consists of simple polynomials and exponential functions of q^2 .

The point proton density can be obtained from (19) by Fourier transforming $F(q)$. The density is separated out again into two parts:

$$\rho(r) = \rho_1(r) + \rho_2(r) \quad (22)$$

$\rho_1(r)$ and $\rho_2(r)$ are the Fourier transforms of $F_1(q)$ and $F_2(q)$, respectively. $\rho_1(r)$ is given by expression (2) (with $\tilde{b}_1 = b_1$) while $\rho_2(r)$ is calculated numerically because of the complexity of $F_2(q)$ mainly for ^{40}Ca .

The correlation parameter λ and the HO parameter b_1 were determined by fitting $F_{ch}(q) = f_p(q)f_{CM}(q)F(q)$ to the experimental charge form factor. $f_p(q)$, $f_{CM}(q)$ are the corrections due to the finite proton size and the centre of mass motion [1], respectively.

As it was pointed out in the introduction, a possible way of improving the quality of the fitting in the approach outlined previously, should be to take into account the correlations originating from the fluctuation of the nuclear surface. For the sake of simplicity these SFC are introduced only in the one body density, $\rho_1(r)$, of equation (22) assuming that the effect of SFC to the two-body term of the density is small. This is also nessecary in order to avoid successive numerical integrations by Fourier transforming. These successive integrations apart from introducing inaccuracies for large values of q , because of the factor $\frac{\sin qr}{qr}$, need much computing time.

According to the above assumption, expression (22) becomes:

$$\rho_{tot}(r) = \rho_{1\sigma}(r) + \rho_2(r) \quad (23)$$

where $\rho_{1\sigma}$ is given analytically (expression 4) while $\rho_2(r)$ is the Fourier transform of the two-body term $F_2(q)$ corrected for the centre of mass motion: $\tilde{F}_2(q) = f_{CM} \cdot F_2(q)$, where (f_{CM} is the Tassie and Barker factor [20]) and can be found either analytically or numerically.

The form factor is of the form

$$F_{tot}(q) = F_{1\sigma}(q) + \tilde{F}_2(q) \quad (24)$$

where $F_{1\sigma}(q)$ is given by (13) and $F_2(q)$ is given in refs [1, 2, 3].

This expression of the form factor depends now on the three parameters, b_1 , λ and σ which can be determined by fitting $F_{ch}(q) = f_p(q)F_{tot}(q)$ to the experimental $F_{ch}(q)$.

4 Numerical results and discussion

The best fit values of the three parameters in the form factor, as well as the values of χ^2 , for the nuclei ${}^4\text{He}$, ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$ are displayed in table 1 where three cases are considered. In case 1 there are no correlations of any kind while in case 2 the SRC are included. These two cases have been studied in previous works [1, 2, 3]. Finally, in case 3, both the SFC and SRC are included.

Table 1: The values of the HO parameter b_1 , the SRC parameters λ , and $(b_1^2/\lambda)^{1/2}$, the parameter σ , the χ^2 and the RMS radius and the effects to it from the HO density and the various correlations for nuclei ${}^4\text{He}$, ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$ (distances in fm). For the various cases see text.

a: [21], b: [18], c: [19]

Case	Nucleus	b_1	λ	$\sqrt{\frac{b_1^2}{\lambda}}$	σ	χ^2	$\langle r_{ch}^2 \rangle^{1/2}$				
							total	HO	SRC	SFC	Exper.
1	${}^4\text{He}$	1.465				878	1.726	1.726			
2	${}^4\text{He}$	1.216	5.982	0.497		155	1.579	1.492	0.516		
3	${}^4\text{He}$	1.089	8.093	0.383	0.558	100	1.622	1.377	0.383	0.766	1.630 ^a
1	${}^{16}\text{O}$	1.786				9013	2.728	2.728			
2	${}^{16}\text{O}$	1.679	12.768	0.470		6226	2.659	2.577	0.654		
3	${}^{16}\text{O}$	1.647	11.440	0.487	0.224	6005	2.652	2.532	0.695	0.372	2.728 ^b
1	${}^{40}\text{Ca}$	1.950				26847	3.439	3.439			
2	${}^{40}\text{Ca}$	1.860	13.915	0.499		19930	3.420	3.289	0.936		
3	${}^{40}\text{Ca}$	1.814	11.786	0.529	0.364	19634	3.422	3.212	1.036	0.565	3.482 ^c

The value $\sigma = 0.364 fm$ for ${}^{40}\text{Ca}$ may be compared with the value $\sigma = 0.638 fm$ which is given in [11]. It is seen that the above analysis underestimates the value of σ . We observe also that for ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$ (see table 1), the introduction of the SFC and the SRC decreases the values of parameters b_1 and λ while the value of $(b_1^2/\lambda)^{1/2}$ (which is the real correlation parameter, since small values of $(b_1^2/\lambda)^{1/2}$ imply values of the correlation factors closer to unity) is increased.

For the three nuclei we have considered the introduction of the SFC has the effect of improving the fitting of $F_{ch}(q)$ to the experimental data. This can be seen also in figures 2, 3 and 4 where the $F_{ch}(q)$ and the corresponding densities for ${}^4\text{He}$, ${}^{16}\text{O}$ and ${}^{40}\text{Ca}$ have been plotted with the best fit values of the parameters and compared with the experimental $F_{ch}(q)$ and $\rho_{ch}(r)$.

Finally in figures 2, 3 and 4 the contribution in the charge density coming from SRC and SFC are shown. From these figures it can be seen that the contribution to the density of $\Delta\rho_{1\sigma}(r) = \rho_{1\sigma}(r) - \rho_1(r)$ is quite small and is characterized by oscillations. Furthermore, it is seen that the charge densities with SFC (case 3) are closer to the "experimental" ones.

In summary, the present analysis suggests that the inclusion of the correlations originating from the fluctuations of the nuclear surface in the uncorrelated (harmonic oscillator) part of the usual cluster expansion (truncated at the second term) of the charge form factor of light nuclei leads to improvement in the quality of the fitting to the experimental data. Furthermore, the inclusion of these correlations has a drastic effect on the asymptotic behaviour of the point proton form factor, which now drops off for large q quite slowly, that is as $const. \cdot q^{-4}$.

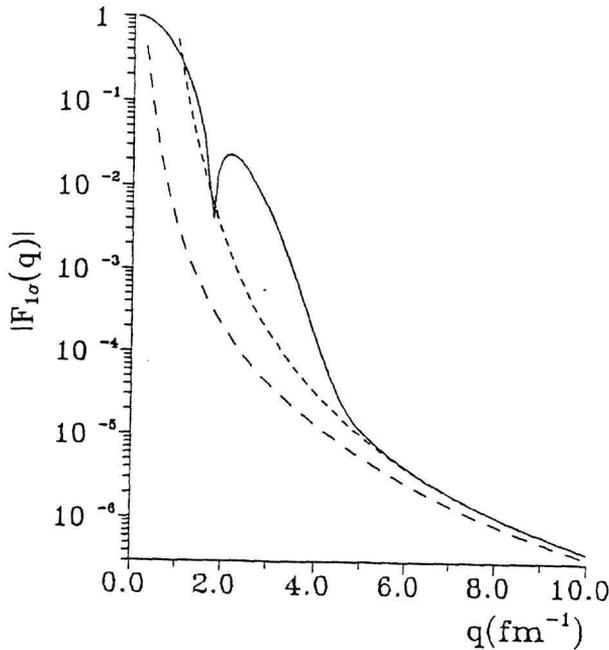


Fig. 1. The elastic point form factor in the HO model with SFC: $F_{1\sigma}(q)$ for ^{16}O with $b_1 = 1.647\text{fm}$ and $\sigma = 0.224\text{fm}$ (solid line) and its asymptotic behaviour $const. \cdot q^{-4}$ (dashed line) together with the values of the asymptotic expression (16) (dashed dot line).

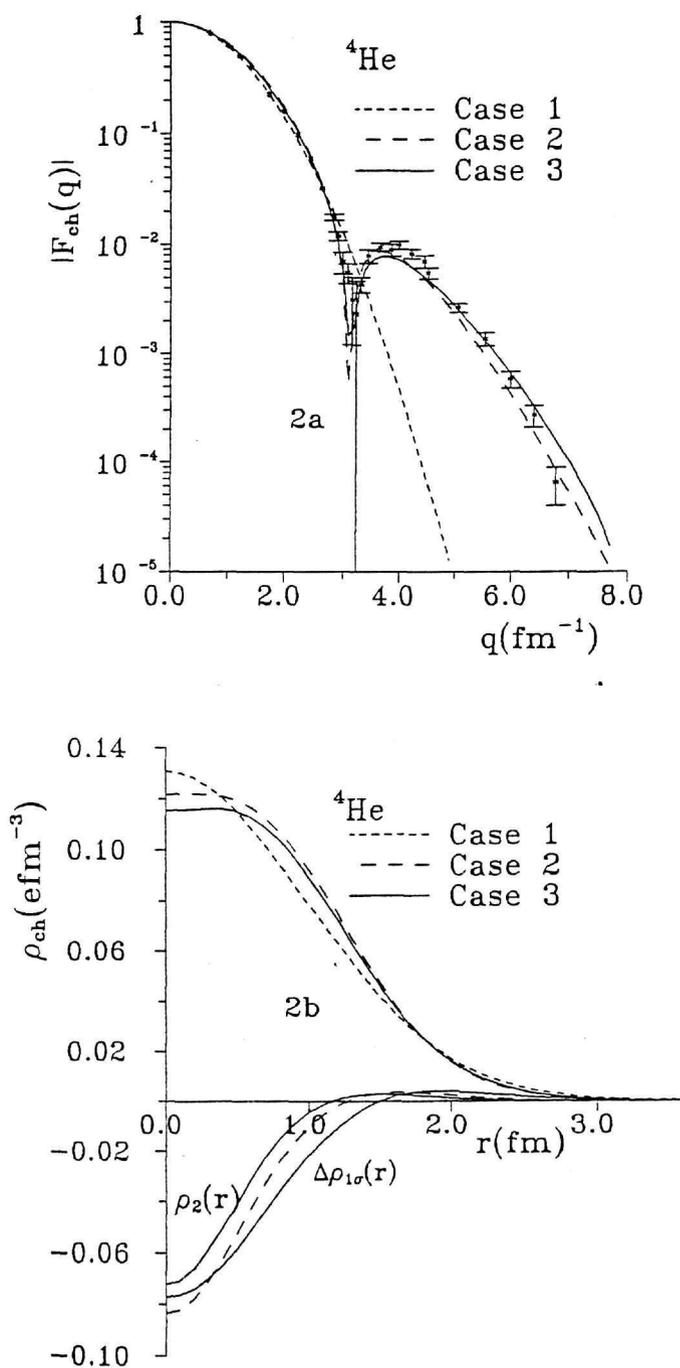


Fig. 2. The charge form factor (2a) and density distribution (2b) of ${}^4\text{He}$ (for various cases see text). The experimental points of the form factor are from refs. [16, 17].

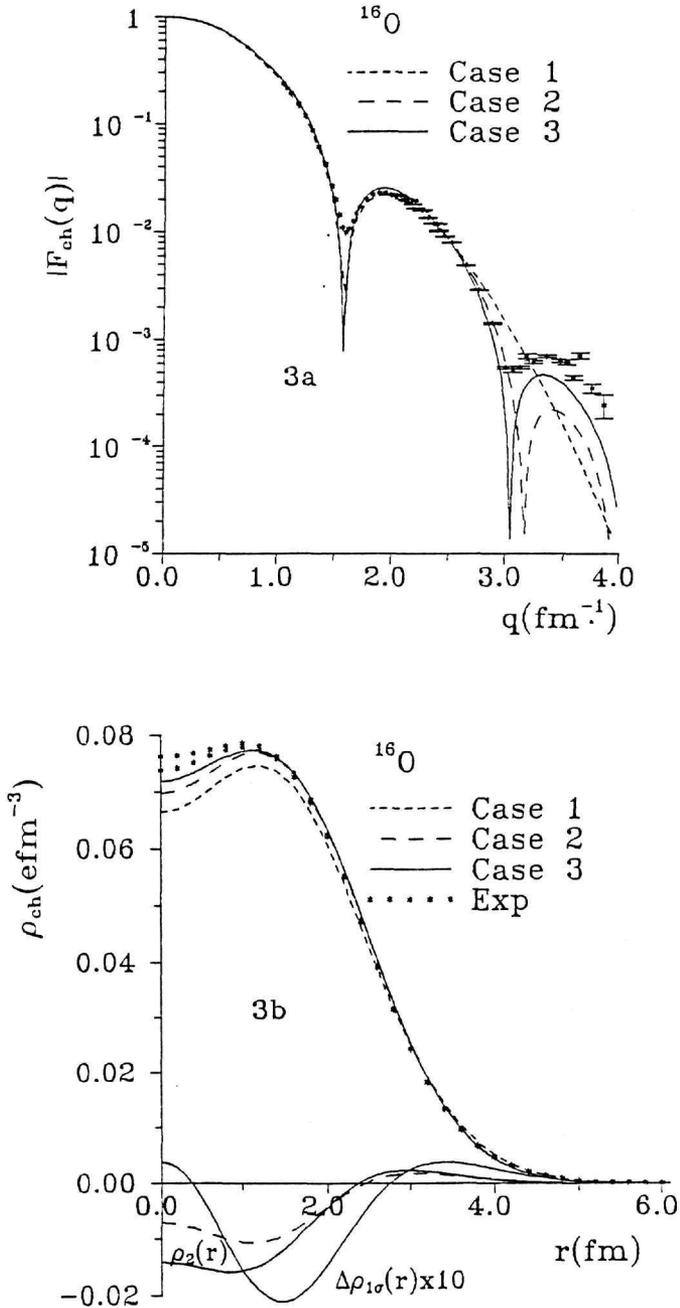


Fig. 3. The charge form factor (3a) and density distribution (3b) of ^{16}O (for various cases see text). The experimental points of the form factor are from ref. [18], while for the density from refs. [22, 23].

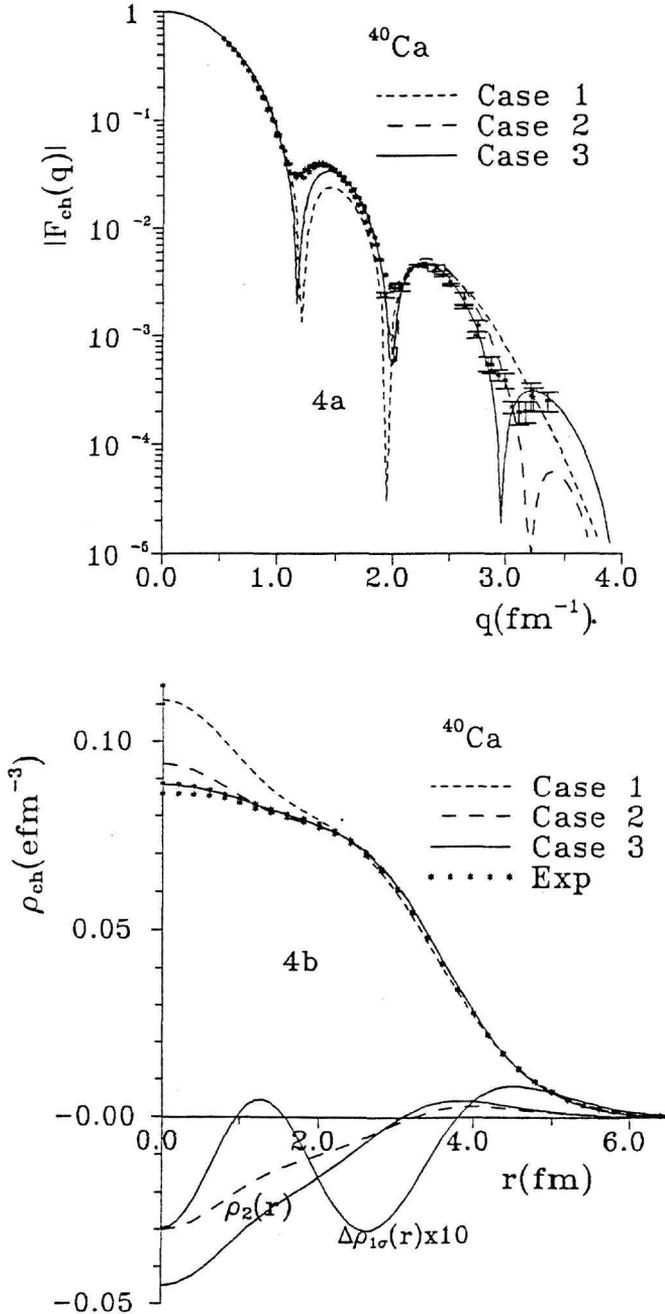


Fig. 4. The charge form factor (4a) and density distribution (4b) of ^{40}Ca (for various cases see text). The experimental points of the form factor are from ref. [19], while for the density from refs. [22, 23].

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