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ALPHA PARTICLE MOMENTUM DISTRIBUTION IN NUCLEI WITHIN THE COHERENT DENSITY FLUCTUATION MODEL *

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Abstract

The alpha particle centre of mass momentum distribution in nuclei is determined on the basis of the four-body density matrix obtained within the coherent density fluctuation model. The calculations are carried out for a number of nuclei. The results are compared with those deduced from analyses of experimental data on alpha-particle knockout reactions induced by electrons,protons and alpha-particles which provide information on the alpha-particle momentum distribution.

1. Introduction

Many aspects of nuclear structure and reactions suggest that nucleons can combine to form transient sub-structures or clusters and among these the alpha cluster is the most likely for reasons of energy and symmetry[1]. It is important to determine the degree of alpha-clustering not only to facilitate an economical description of nuclear structure and reactions, but also to learn more about the nucleon-nucleon correlations in the nuclear interior.

The character of alpha-clustering depends on the nuclear size. Light nuclei can be considered to consist of linked clusters of alpha-particles, deuterons and nucleons. In heavier

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nuclei alpha-clusters can be expected in the region of the nuclear surface since conden-
sation into alpha-clusters is energetically favourable at densities around one-third of that
in the nuclear interior[2]. Many nuclei decay spontaneously by emitting alpha-clusters
and heavy fragments. Fusion reactions are affected by clustering in intermediate states
and breakup reactions provide evidence of cluster structure in the projectile. At higher
energies some nuclear reactions preferentially proceed by cluster transfer or by knockout
and pickup processes. At very high energies nuclei can be fragmented into a wide range of
clusters of nucleons.

Here we consider nuclear reactions that involve the transfer of more than one nucleon. For
nuclei with a particular cluster structure, it is expected that reaction channels involving the
removal or replacement of that cluster will have an enhanced probability. Such reactions
are for example the alpha knockout $(p,p\alpha)$ and $(\alpha,2\alpha)$ reactions as well as $(e,e'\alpha)$, (e,α)
reactions. The theoretical analyses of their cross-sections show the importance of the alpha-
particle momentum distribution for the understanding of these reactions. We recall that the
cross-sections of the $(p,p\alpha)$ and $(\alpha,2\alpha)$ reactions in the plane wave impulse approximation
(PWIA) depend on the momentum distribution of the alpha-cluster in the target nucleus[3-
6]. In addition it has been shown[7,8] that the cross-sections for the $A(e,e'\alpha)B$ reactions
in the Born approximation can be expressed in terms of the momentum distribution of the
alpha-cluster in the target nucleus and the cross-section of the elastic electron scattering
on the alpha-cluster.

Alpha particle momentum distribution provides a sensitive probe of short-range and tensor
nucleon-nucleon correlations in nuclei [9]. Such correlations are responsible for the high-
momentum components of the nucleon and cluster momentum distributions which are
obtained in theories going beyond the Hartree-Fock approximation [9-14]. It should be
noted, however, that most of the methods for calculating these distributions are restricted
to light nuclei, with notable exception of the coherent density fluctuation model (CDFM)
[9].

The basic relations in the CDFM are obtained in the framework of the generator co-ordinate
method (GCM) [15] by generalization of its delta-function limit in the case of many-
fermion systems. The high momentum components in the nucleon momentum distributions
obtained in the CDFM are due to the special choice of the intermediate generating states
which allows the inclusion of certain type of nucleon-nucleon correlations.

In the present work we report calculations of the alpha-particle centre of mass momentum

distribution in the framework of the CDFM. The basic formalism is described in section 2 while the results of the calculations of the α-particle momentum distributions for a number of nuclei are reported and discussed in section 3.

2. Theoretical method

We introduce the four-nucleon momentum distribution by using the definition of the four-body density matrix [16].

$$\rho^{(4)}\left(\xi_1, \xi_2, \xi_3, \xi_4; \xi'_1, \xi'_2, \xi'_3, \xi'_4\right) = \frac{A\left(A-1\right)\left(A-2\right)\left(A-3\right)}{4!}$$

$$\sum_{n_5, \dots n_A} \int dr_5 \dots dr_A \Psi^\dagger\left(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \dots \xi_A\right) \Psi\left(\xi'_1, \xi'_2, \xi'_3, \xi'_4, \xi_5 \dots \xi_A\right) \qquad (1)$$

In eq(1) $\Psi(\{\xi_i\})$, (i=1,2...A) is the total wave function of a system of A nucleons. Each co-ordinate ξ_i is a compination of a space (r_i), spin (σ_i) and isospin (τ_i) co-ordinates: $\xi_i \equiv (r_i; \sigma_i, \tau_i) \equiv (r_i; n_i)$ with $n_i \equiv (\sigma_i, \tau_i)$

In the coherent density fluctuation model (CDFM) the many-body wave function has the GCM-form [9,15,17]

$$\Psi\left(\xi_1, \xi_2, \dots \xi_A\right) = \int_0^\infty dx f(x) \Phi\left(x; \xi_1, \xi_2, \dots \xi_A\right). \qquad (2)$$

The function $\Phi(x; \{\xi_i\})$ corresponds to a state of a system of A nucleons uniformly distributed in a sphere of radius x (the so called "flucton"). In the model Φ is a Slater determinant built up from plane waves in a volume $V(x) = \frac{4}{3}\pi x^3$. The weight function $f(x)$ can be determined from the nuclear density distribution $\rho(r)$. In the case of monotonically-decreasing density ($d\rho/dr < 0$) [17].

$$|f(x)|^2 = -\frac{1}{\rho_0(x)} \frac{d\rho(r)}{dr}|_{r=x} \qquad (3)$$

whith

$$\rho_0(x) = 3A/4\pi x^3 \qquad (4)$$

The main approximation in the CDFM is related to the delta-function limit of the overlap kernel in the GCM [15]

$$< \Phi(x'; \{\xi_i\}) | \Phi(x; \{\xi_i\}) > = \delta(x - x'),$$ (5)

which applies to the case of a large number of fermions.

Here we make a generalization of the delta-function limit in CDFM concerning the four-body density matrix of the many nucleon system by assuming that the following relation for the function Φ holds

$$\frac{A(A-1)(A-2)(A-3)}{4!} \sum_{n_5,...n_A} \int dr_5...dr_A \Phi^\dagger(x'; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, ...\xi_A)$$

$$\Phi(x; \xi'_1, \xi'_2, \xi'_3, \xi'_4, \xi_5...\xi_A) = \delta(x - x')\rho^{(4)}(\xi_1, \xi_2, \xi_3, \xi_4; \xi'_1, \xi'_2, \xi'_3, \xi'_4)$$ (6)

where $\rho_{(4)}(x; \xi_1, \xi_2, \xi_3, \xi_4; \xi'_1, \xi'_2, \xi'_3, \xi'_4)$ is the four-body density matrix in the plane wave case for a system with density $\rho_0(x)$ (4) described by the function $\Phi(x; \{\xi_i\})$.

By means of subsequent integrations over r_i, summing over n_i at $\xi_i = \xi'_i$ of eq(7) (i=4,3,2,1) and using the properties of the density matrices [16] one can obtain eq(6). This procedure makes clear the relation of the assumption (7) to the delta-function limit of the GCM (6). Using eqs (5) and (7) one gets the following expression for the four-body density matrix in the CDFM

$$\rho^{(4)}(\xi_1, \xi_2, \xi_3, \xi_4; \xi'_1, \xi'_2, \xi'_3, \xi'_4) =$$

$$= \int_0^\infty dx |f(x)|^2 \rho^{(4)}(x; \xi_1, \xi_2, \xi_3, \xi_4; \xi'_1, \xi'_2, \xi'_3, \xi'_4)$$ (7)

The four-body momentum distribution $n^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ is expressed by the diagonal elements of the four-body density matrix in momentum space

$$n^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = \rho^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4; \zeta'_1, \zeta'_2, \zeta'_3, \zeta'_4),$$ (8)

where $\zeta_i \equiv (k_i; \sigma_i, \tau_i) \equiv (k_i; n_i)$, k_i being the momentum of thr i-th nucleon.

Using eqs (9) and (10) we get the following form of the CDFM four-nucleon momentum distribution of the nucleus

$$n^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = \int_0^\infty dx |f(x)|^2 n_x^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$$ (9)

where

$$n_x^{(4)}\left(\zeta_1,\zeta_2,\zeta_3,\zeta_4\right) \equiv \rho^{(4)}\left(x;\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4\right) \tag{10}$$

is the four-nucleon momentum distribution of the "flucton".

In the case of single Slater determinant wave function (e.g. the flucton state wave function $\Phi(x;\{\zeta_i\})$) the many-body density matrices are expressed by means of a determinant built up by one-body density matrices[18].In our case the Slater determinant is built up with plane-wave functions and the corresponding one-body density matrix is written

$$\rho^{(1)}\left(x;\zeta_i;\zeta_j\right) = (2\pi)^3 \delta_{n_i n_j} \delta\left(\mathbf{k_i}-\mathbf{k_j}\right)\Theta\left(k_F(x)-\frac{|\mathbf{k_i}+\mathbf{k_j}|}{2}\right) \tag{11}$$

where

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{a}{x} \qquad with \qquad a \equiv (9\pi A/8)^{1/3} \tag{12}$$

It follows from eq.(11) that the single-nucleon momentum distribution of a flucton with a radius x has the form

$$n_x^{(1)}(\zeta) = \rho^{(1)}\left(x;\zeta;\zeta\right) = V(x)\Theta\left(k_F(x)-|\mathbf{k}|\right)\delta_{nn}. \tag{13}$$

Having all these in mind and the fact that we are interested in the case when two protons and two neutrons with antiparallel spins form an alpha-cluster the expression of the four-nucleon momentum distribution for alpha-clusters in the CDFM is

$$n^{(\alpha)}\left(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3},\mathbf{k_4}\right) = \sum_{n_1,n_2,n_3,n_4} \rho^{(\alpha)}\left(\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4\right) =$$

$$= \int_0^\infty dx |f(x)|^2 V^4(x)\Theta\left(k_F(x)-|\mathbf{k_1}|\right)\Theta\left(k_F(x)-|\mathbf{k_2}|\right).$$

$$\Theta\left(k_F(x)-|\mathbf{k_3}|\right)\Theta\left(k_F(x)-|\mathbf{k_4}|\right). \tag{14}$$

Introducing Jacobi momenta $\mathbf{P},\mathbf{p_1},\mathbf{p_2},\mathbf{p_3}$ [19] (see also [20]) the centre of mass alpha-particle momentun distribution $n_{c.m}^\alpha(P)$ takes the form

$$n_{c.m}^{(\alpha)}(P) = \int \frac{d\Omega_p}{(2\pi)^3}\int \frac{d\mathbf{p_1}}{(2\pi)^3}\int \frac{d\mathbf{p_2}}{(2\pi)^3}\int \frac{d\mathbf{p_3}}{(2\pi)^3} n^{(\alpha)}\left(\mathbf{P},\mathbf{p_1},\mathbf{p_2},\mathbf{p_3}\right) \tag{15}$$

with the normalization condition

$$\int_0^\infty n_{c.m}^{(\alpha)}(P)P^2 dP = 1 \tag{16}$$

where the following notation is introduced

$$\Omega_P \equiv \{\Theta_P, \varphi_P\} \tag{17}$$

Accounting for the explicit form of $n^{(\alpha)}(\mathbf{P}, \mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3})$, eq.(15) can be written as

$$n_{c.m.}^{(\alpha)}(P) = \left(\frac{4}{A}\right)^4 \frac{1}{(2\pi)^{12}} \int d\Omega_P \int d\mathbf{p_1} \int d\mathbf{p_2} \int d\mathbf{p_3} \int_0^\alpha dx |f(x)|^2 \left(\frac{4}{3}\pi x^3\right)^4, \tag{18}$$

with $\alpha = a/max\{S_1, S_2, S_3, S_4\}$, where $max\{S_1, S_2, S_3, S_4\}$ is the largest of the quantities

$$S_1 \equiv |\frac{1}{4}\mathbf{P} + \mathbf{p_1} + \frac{1}{2}\mathbf{p_2} + \frac{1}{3}\mathbf{p_3}|,$$

$$S_2 \equiv |\frac{1}{4}\mathbf{P} - \mathbf{p_1} + \frac{1}{2}\mathbf{p_2} + \frac{1}{3}\mathbf{p_3}|,$$

$$S_3 \equiv |\frac{1}{4}\mathbf{P} + \mathbf{p_2} + \frac{1}{3}\mathbf{p_3}|, \tag{19}$$

$$S_4 \equiv |\frac{1}{4}\mathbf{P} - \mathbf{p_3}|,$$

and a is given by eq.(12).

In the case of symmetrized Fermi density distribution [21]

$$\rho_{SF}(r) = \rho_0 \frac{sinh(R/b)}{cosh(r/b) + cosh(R/b)}, \qquad \rho_0 = \frac{3A}{4\pi R^3 \left[1 + \left(\frac{\pi b}{R}\right)^2\right]}, \tag{20}$$
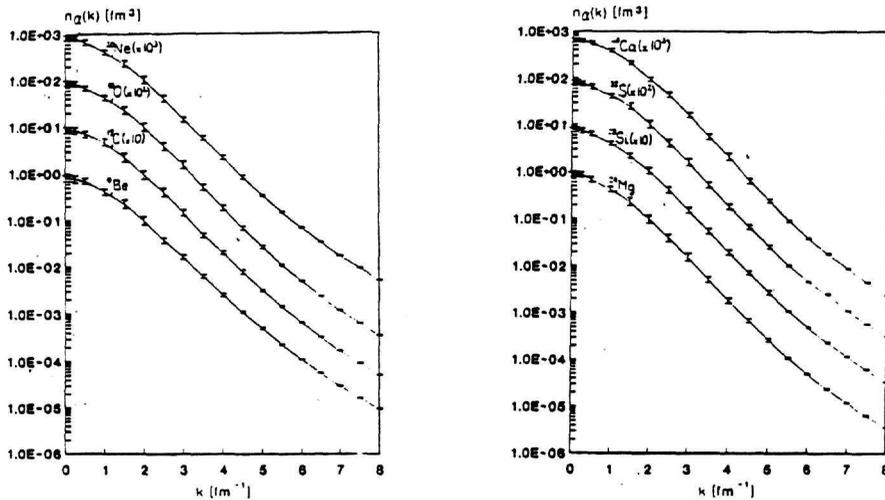
the weight function $|f(x)|^2$ (3) has the form

$$|f(x)|^2 = \frac{x^3}{bR^3 \left[1 + \left(\frac{\pi b}{R}\right)^2\right]} \{\frac{e^{(x-R)/b}}{\left(1 + e^{(x-R)/b}\right)^2} - \frac{e^{-(x+R)/b}}{\left(1 + e^{-(x+R)/b}\right)^2}\}. \tag{21}$$

3. Numerical Results and comments

The centre of mass alpha-particle momentum distribution $n_{c.m}^{(\alpha)}$ has been calculated for a number of nuclei in the region $9 \le A \le 40$ using eq.(18) for $n_{c.m}^{(\alpha)}$ and eq.(20) for the

weight function $|f(x)|^2$. The twelve-dimensional integral in eq.(18) was reduced to a nine-dimensional one and evaluated by Monte Carlo method. The values of the parameters R (half-radius) and b (surface diffuseness) of the symmetrized Fermi density distribution (20) have been obtained from elastic electron scattering data. The corresponding parameters were taken from refs.21 and 22.

The results of the calculations of $n_{c.m}^{(\alpha)}$ in the CDFM are shown in figs. 1,2. The bars indicate the estimated accuracy of the computational method. The alpha-particle momentum distribution for ^9Be, ^{24}Mg and ^{40}Ca are compared in fig. 3 and it is seen that their behaviour is very similar. In general (see also [20]), when the mass number increases $n_{c.m}^{(\alpha)}$ decreases at large momenta $(P > 5fm^{-1})$.

Figures 1,2 The alpha-particle momentum distribution for ^9Be, ^{12}C, ^{16}O and ^{20}Ne (fig. 1) ^{24}Mg, ^{28}Si, ^{32}S and ^{40}Ca (fig. 2). The error bars indicate the uncertainties in the Monte Carlo calculation. The normalisation is $\int_0^\infty n_{c.m}^{(\alpha)}(P)P^2dP = 1$.

Concerning the possibilities of comparing the theoretical calculations of this approach on the alpha-particle centre of mass momentum distribution with some experimental results, it should be noted that the available experimental data for $n_{c.m}^{(\alpha)}(P)$ are very scarce, mainly qualitative and for low momenta $(0 < P < 1fm^{-1})$. Though nucleon-nucleon correlation effects are reflected mainly in the behaviour of the momentum distribution at higher momenta $(P > 2fm^{-1})$ it is also interesting to compare our results with the existing data at low momenta.

The PWIA analysis of the $^{12}C(p, p\alpha)$ reaction at $E_p = 150$ MeV [3] leads to values of $n^{(\alpha)}(\text{P} = 0) = 1.9$ to $5.3\ fm^3$ and $n^{(\alpha)}(P = 1 fm^{-1}) = 0.56$ to $1.60\ fm^3$ depending on the errors in the determination of the effective number $N_{eff}(\alpha)$ of the alpha-clusters in ^{12}C (in [3] this number is estimated to be $N_{eff}(\alpha) = 0.30^{+0.23}_{-0.11}$). Our results for the alpha momentum distribution are $n^{(\alpha)}_{c.m}(\text{P} = 0) = 0.9 fm^3$ and $n^{(\alpha)}_{c.m}(\text{P} = 1 fm^{-1}) = 0.45 fm^3$, which are of similar magnitude.
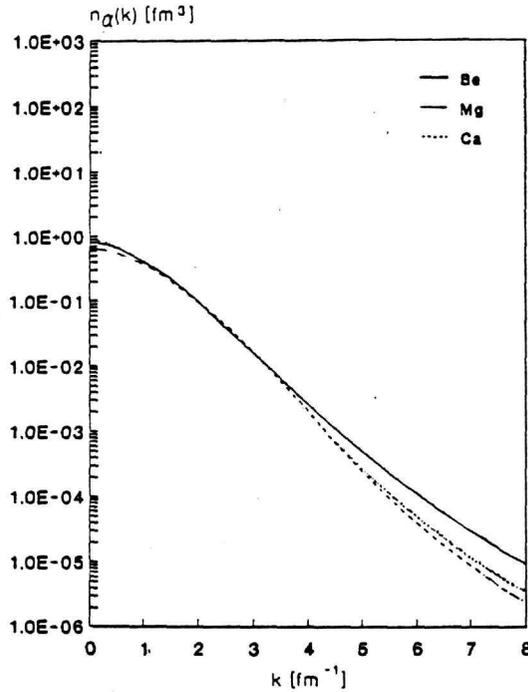

Figure 3 The alpha-particle momentum distributions for ^9Be (solid line), ^{24}Mg (dashed line) and ^{40}Ca (dash-dotted line).

The values of the alpha momentum distributions at zero momenta extracted from PWIA analysis of $(\alpha, 2\alpha)$ $(E_\alpha = 700$ MeV) reactions [6] for ^{12}C and ^{16}O are $n^{(\alpha)}_{^{12}C}(P = 0) = 1.36 fm^3$ and $n^{(\alpha)}_{^{16}O}(P = 0) = 1.21 fm^3$. Our result for both nuclei is $n^{(\alpha)}_{c.m}(P = 0) \simeq 0.9 fm^3$. Thus the theoretical calculations are not in contradiction with the experimental data considered. The theoretical results for $n^{(\alpha)}_{c.m}(P = 0)$ are almost constant for A = 12 to 28 and decrease monotonically for larger A in agreement with the result in ref. 6.

In the region $P \simeq 4$ to $7 \ fm^{-1}$ our alpha-particle momentum distribution $n_{c.m.}^{(\alpha)}(P)$ can be well reproduced by the function $exp(-P/P_0)$ with P_0 indepent of A within 10%. This agrees with the results from (p,α) inclusive reactions at intermediate energies ($E_p = 210$, 300 and 480 MeV) [23] and proton ($E_p = 90$ Mev) and alpha ($E_\alpha = 140$ MeV) induced inclusive reactions [24]. In the case of ^9Be $P_0 \simeq 140 MeV/c$ compared with $P_0 \simeq 75 MeV/c$ from ref.23. For ^{12}C P_0 is 122 MeV/c.

The alpha centre of mass momentum distributions obtained in the framework of the CDFM may be used in calculations of the cross-sections of alpha knockout reactions. It is expected that a comparison with the appropriate data would provide a sensitive test of their validity. Examples of suitable processes are the reactions with electrons (e,e'α) on light nuclei [25,26], as well as high energy (p,α) inclusive reactions [20] and ($\alpha, 2\alpha$) reactions [5,6]. These calculations are in progress.

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