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*A. N. Antonov, P. E. Hodgson, G. A. Lalazissis, E. N. Nikolov, I. Zh. Petkov*

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# ALPHA PARTICLE MOMENTUM DISTRIBUTION IN NUCLEI WITHIN THE COHERENT DENSITY FLUCTUATION MODEL *

A. N. ANTONOV[1], P. E. HODGSON[2], G. A. LALAZISSIS[3],
E. N. NIKOLOV[1] and I. Zh. PETKOV[1]

1. Institute of Nuclear Research and Nuclear Energy, 1784 Sofia, Bulgaria
2. Nuclear Physics Laboratory, University of Oxford, Oxford OX1-3RH, U.K
3. Department of Theoretical Physics, University of Thessaloniki, 54006 Thessaloniki, Greece

**Abstract**

The alpha particle centre of mass momentum distribution in nuclei is determined on the basis of the four-body density matrix obtained within the coherent density fluctuation model. The calculations are carried out for a number of nuclei. The results are compared with those deduced from analyses of experimental data on alpha-particle knockout reactions induced by electrons,protons and alpha-particles which provide information on the alpha-particle momentum distribution.

## 1. Introduction

Many aspects of nuclear structure and reactions suggest that nucleons can combine to form transient sub-structures or clusters and among these the alpha cluster is the most likely for reasons of energy and symmetry[1]. It is important to determine the degree of alpha-clustering not only to facilitate an economical description of nuclear structure and reactions, but also to learn more about the nucleon-nucleon correlations in the nuclear interior.

The character of alpha-clustering depends on the nuclear size. Light nuclei can be considered to consist of linked clusters of alpha-particles, deuterons and nucleons. In heavier

* Presented by G.A. Lalazissis

nuclei alpha-clusters can be expected in the region of the nuclear surface since condensation into alpha-clusters is energetically favourable at densities around one-third of that in the nuclear interior[2]. Many nuclei decay spontaneously by emitting alpha-clusters and heavy fragments. Fusion reactions are affected by clustering in intermediate states and breakup reactions provide evidence of cluster structure in the projectile. At higher energies some nuclear reactions preferentially proceed by cluster transfer or by knockout and pickup processes. At very high energies nuclei can be fragmented into a wide range of clusters of nucleons.

Here we consider nuclear reactions that involve the transfer of more than one nucleon. For nuclei with a particular cluster structure, it is expected that reaction channels involving the removal or replacement of that cluster will have an enhanced probability. Such reactions are for example the alpha knockout (p,p$\alpha$) and ($\alpha$,2$\alpha$) reactions as well as (e,$e'\alpha$), (e,$\alpha$) reactions. The theoretical analyses of their cross-sections show the importance of the alpha-particle momentum distribution for the understanding of these reactions. We recall that the cross-sections of the (p,p$\alpha$) and ($\alpha$,2$\alpha$) reactions in the plane wave impulse approximation (PWIA) depend on the momentum distribution of the alpha-cluster in the target nucleus[3-6]. In addition it has been shown[7,8] that the cross-sections for the A(e,$e'\alpha$)B reactions in the Born approximation can be expressed in terms of the momentum distribution of the alpha-cluster in the target nucleus and the cross-section of the elastic electron scattering on the alpha-cluster.

Alpha particle momentum distribution provides a sensitive probe of short-range and tensor nucleon-nucleon correlations in nuclei [9]. Such correlations are responsible for the high-momentum components of the nucleon and cluster momentum distributions which are obtained in theories going beyond the Hartree-Fock approximation [9-14]. It should be noted, however, that most of the methods for calculating these distributions are restricted to light nuclei, with notable exception of the coherent density fluctuation model (CDFM) [9].

The basic relations in the CDFM are obtained in the framework of the generator co-ordinate method (GCM) [15] by generalization of its delta-function limit in the case of many-fermion systems. The high momentum components in the nucleon momentum distributions obtained in the CDFM are due to the special choice of the intermediate generating states which allows the inclusion of certain type of nucleon-nucleon correlations.

In the present work we report calculations of the alpha-particle centre of mass momentum distribution in the framework of the CDFM. The basic formalism is described in section 2 while the results of the calculations of the α-particle momentum distributions for a number of nuclei are reported and discussed in section 3.

## 2. Theoretical method

We introduce the four-nucleon momentum distribution by using the definition of the four-body density matrix [16].

$$\rho^{(4)}(\xi_1,\xi_2,\xi_3,\xi_4;\xi'_1,\xi'_2,\xi'_3,\xi'_4) = \frac{A(A-1)(A-2)(A-3)}{4!}$$

$$\sum_{n_5,...n_A} \int d\mathbf{r}_5...d\mathbf{r}_A \Psi^{\dagger}(\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,...\xi_A)\,\Psi(\xi'_1,\xi'_2,\xi'_3,\xi'_4,\xi_5...\xi_A) \qquad (1)$$

In eq(1) $\Psi(\{\xi_i\})$, (i=1,2...A) is the total wave function of a system of A nucleons. Each co-ordinate $\xi_i$ is a compination of a space ($\mathbf{r}_i$), spin ($\sigma_i$) and isospin ($\tau_i$) co-ordinates: $\xi_i \equiv (\mathbf{r}_i;\sigma_i,\tau_i) \equiv (\mathbf{r}_i;n_i)$ with $n_i \equiv (\sigma_i,\tau_i)$

In the coherent density fluctuation model (CDFM) the many-body wave function has the GCM-form [9,15,17]

$$\Psi(\xi_1,\xi_2,...\xi_A) = \int_0^{\infty} dx f(x)\Phi(x;\xi_1,\xi_2,...\xi_A). \qquad (2)$$

The function $\Phi(x;\{\xi_i\})$ corresponds to a state of a system of A nucleons uniformly distributed in a sphere of radius x (the so called "flucton"). In the model $\Phi$ is a Slater determinant built up from plane waves in a volume V(x)=$\frac{4}{3}\pi x^3$. The weight function $f(x)$ can be determined from the nuclear density distribution $\rho(\mathrm{r})$. In the case of monotonically-decreasing density ($d\rho/dr < 0$) [17].

$$|f(x)|^2 = -\frac{1}{\rho_0(x)}\frac{d\rho(r)}{dr}|_{r=x} \qquad (3)$$

whith

$$\rho_0(x) = 3A/4\pi x^3 \qquad (4)$$

The main approximation in the CDFM is related to the delta-function limit of the overlap kernel in the GCM [15]

$$< \Phi(x'; \{\xi_i\}) | \Phi(x; \{\xi_i\}) >= \delta(x - x'), \tag{5}$$

which applies to the case of a large number of fermions.
Here we make a generalization of the delta-function limit in CDFM concerning the four-body density matrix of the many nucleon system by assuming that the following relation for the function $\Phi$ holds

$$\frac{A(A-1)(A-2)(A-3)}{4!} \sum_{n_5,...n_A} \int d\mathbf{r}_5...d\mathbf{r}_A \Phi^{\dagger}(x'; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, ...\xi_A)$$

$$\Phi(x; \xi_1', \xi_2', \xi_3', \xi_4', \xi_5...\xi_A) = \delta(x - x')\rho^{(4)}(\xi_1, \xi_2, \xi_3, \xi_4; \xi_1', \xi_2', \xi_3', \xi_4') \tag{6}$$

where $\rho_{(4)}(x; \xi_1, \xi_2, \xi_3, \xi_4; \xi_1', \xi_2', \xi_3', \xi_4')$ is the four-body density matrix in the plane wave case for a system with density $\rho_0(x)$ (4) described by the function $\Phi(x; \{\xi_i\})$.
By means of subsequent integrations over $\mathbf{r}_i$, summing over $n_i$ at $\xi_i = \xi_i'$ of eq(7) (i=4,3,2,1) and using the properties of the density matrices [16] one can obtain eq(6). This procedure makes clear the relation of the assumption (7) to the delta-function limit of the GCM (6).
Using eqs (5) and (7) one gets the following expression for the four-body density matrix in the CDFM

$$\rho^{(4)}(\xi_1, \xi_2, \xi_3, \xi_4; \xi_1', \xi_2', \xi_3', \xi_4') =$$

$$= \int_0^{\infty} dx |f(x)|^2 \rho^{(4)}(x; \xi_1, \xi_2, \xi_3, \xi_4; \xi_1', \xi_2', \xi_3', \xi_4') \tag{7}$$

The four-body momentum distribution $n^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ is expressed by the diagonal elements of the four-body density matrix in momentum space

$$n^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = \rho^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4; \zeta_1', \zeta_2', \zeta_3', \zeta_4'), \tag{8}$$

where $\zeta_i \equiv (\mathbf{k}_i; \sigma_i, \tau_i) \equiv (\mathbf{k}_i; n_i)$, $\mathbf{k}_i$ being the momentum of thr i-th nucleon.
Using eqs (9) and (10) we get the following form of the CDFM four-nucleon momentum distribution of the nucleus

$$n^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = \int_0^{\infty} dx |f(x)|^2 n_x^{(4)}(\zeta_1, \zeta_2, \zeta_3, \zeta_4) \tag{9}$$

where

$$n_x^{(4)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4) \equiv \rho^{(4)}(x;\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4) \tag{10}$$

is the four-nucleon momentum distribution of the "flucton".
In the case of single Slater determinant wave function (e.g. the flucton state wave function $\Phi(x;\{\zeta_i\})$) the many-body density matrices are expressed by means of a determinant built up by one-body density matrices[18].In our case the Slater determinant is built up with plane-wave functions and the corresponding one-body density matrix is written

$$\rho^{(1)}(x;\zeta_i;\zeta_j) = (2\pi)^3\delta_{n_i n_j}\delta(\mathbf{k_i}-\mathbf{k_j})\,\Theta\left(k_F(x)-\frac{|\mathbf{k_i}+\mathbf{k_j}|}{2}\right) \tag{11}$$

where

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{a}{x} \qquad with \qquad a \equiv (9\pi A/8)^{1/3} \tag{12}$$

It follows from eq.(11) that the single-nucleon momentum distribution of a flucton with a radius x has the form

$$n_x^{(1)}(\zeta) = \rho^{(1)}(x;\zeta;\zeta) = V(x)\Theta(k_F(x)-|\mathbf{k}|)\,\delta_{nn}. \tag{13}$$

Having all these in mind and the fact that we are interested in the case when two protons and two neutrons with antiparallel spins form an alpha-cluster the expression of the four-nucleon momentum distribution for alpha-clusters in the CDFM is

$$n^{(\alpha)}(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3},\mathbf{k_4}) = \sum_{n_1,n_2,n_3,n_4} \rho^{(\alpha)}(\zeta_1,\zeta_2,\zeta_3,\zeta_4;\zeta_1,\zeta_2,\zeta_3,\zeta_4) =$$

$$= \int_0^\infty dx|f(x)|^2V^4(x)\Theta(k_F(x)-|\mathbf{k_1}|)\,\Theta(k_F(x)-|\mathbf{k_2}|).$$

$$\Theta(k_F(x)-|\mathbf{k_3}|)\,\Theta(k_F(x)-|\mathbf{k_4}|). \tag{14}$$

Introducing Jacobi momenta $\mathbf{P},\mathbf{p_1},\mathbf{p_2},\mathbf{p_3}$ [19] (see also [20]) the centre of mass alpha-particle momentun distribution $n^{\alpha}_{c.m}(P)$ takes the form

$$n^{(\alpha)}_{c.m}(P) = \int\frac{d\Omega_p}{(2\pi)^3}\int\frac{d\mathbf{p_1}}{(2\pi)^3}\int\frac{d\mathbf{p_2}}{(2\pi)^3}\int\frac{d\mathbf{p_3}}{(2\pi)^3}n^{(\alpha)}(\mathbf{P},\mathbf{p_1},\mathbf{p_2},\mathbf{p_3}) \tag{15}$$

with the normalization condition

$$\int_0^\infty n^{(\alpha)}_{c.m}(P)P^2dP = 1 \tag{16}$$

where the following notation is introduced

$$\Omega_P \equiv \{\Theta_P, \varphi_P\} \tag{17}$$

Accounting for the explicit form of $n^{(\alpha)}(\mathbf{P}, \mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3})$, eq.(15) can be written as

$$n^{(\alpha)}_{c.m}(P) = \left(\frac{4}{A}\right)^4 \frac{1}{(2\pi)^{12}} \int d\Omega_P \int d\mathbf{p_1} \int d\mathbf{p_2} \int d\mathbf{p_3} \int_0^{\alpha} dx |f(x)|^2 \left(\frac{4}{3}\pi x^3\right)^4, \tag{18}$$

with $\alpha = a/max\{S_1, S_2, S_3, S_4\}$, where $\max\{S_1, S_2, S_3, S_4\}$ is the largest of the quantities

$$S_1 \equiv |\frac{1}{4}\mathbf{P} + \mathbf{p_1} + \frac{1}{2}\mathbf{p_2} + \frac{1}{3}\mathbf{p_3}|,$$

$$S_2 \equiv |\frac{1}{4}\mathbf{P} - \mathbf{p_1} + \frac{1}{2}\mathbf{p_2} + \frac{1}{3}\mathbf{p_3}|,$$

$$S_3 \equiv |\frac{1}{4}\mathbf{P} + \mathbf{p_2} + \frac{1}{3}\mathbf{p_3}|, \tag{19}$$

$$S_4 \equiv |\frac{1}{4}\mathbf{P} - \mathbf{p_3}|,$$

and $a$ is given by eq.(12).
In the case of symmetrized Fermi density distribution [21]

$$\rho_{SF}(r) = \rho_0 \frac{sinh(R/b)}{cosh(r/b) + cosh(R/b)}, \qquad \rho_0 = \frac{3A}{4\pi R^3 \left[1 + \left(\frac{\pi b}{R}\right)^2\right]}, \tag{20}$$

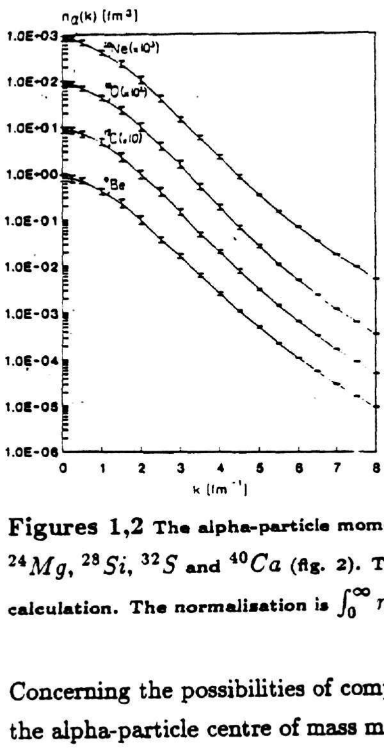
the weight function $|f(x)|^2$ (3) has the form

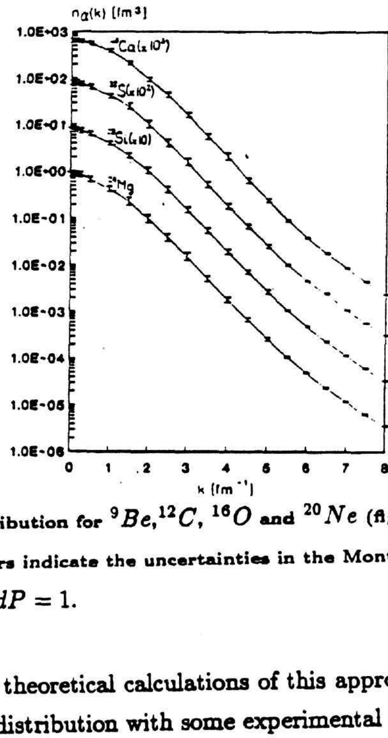
$$|f(x)|^2 = \frac{x^3}{bR^3 \left[1 + \left(\frac{\pi b}{R}\right)^2\right]} \{\frac{e^{(x-R)/b}}{\left(1 + e^{(x-R)/b}\right)^2} - \frac{e^{-(x+R)/b}}{\left(1 + e^{-(x+R)/b}\right)^2}\}. \tag{21}$$

### 3. Numerical Results and comments

The centre of mass alpha-particle momentum distribution $n^{(\alpha)}_{c.m}$ has been calculated for a number of nuclei in the region $9 \leq A \leq 40$ using eq.(18) for $n^{(\alpha)}_{c.m}$ and eq.(20) for the weight function $|f(x)|^2$. The twelve-dimensional integral in eq.(18) was reduced to a nine-dimensional one and evaluated by Monte Carlo method.The values of the parameters R (half-radius) and b (surface diffuseness) of the symmetrized Fermi density distribution (20) have been obtained from elastic electron scattering data .The corresponding parameters were taken from refs.21 and 22.

The results of the calculations of $n_{c.m}^{(\alpha)}$ in the CDFM are shown in figs. 1,2. The bars indicate the estimated accuracy of the computational method. The alpha-particle momentum distribution for $^9Be$,$^{24}Mg$ and $^{40}Ca$ are compared in fig. 3 and it is seen that their behaviour is very similar.In general (see also [20]), when the mass number increases $n_{c.m}^{(\alpha)}$ decreases at large momenta ($P > 5fm^{-1}$).




**Figures 1,2** The alpha-particle momentum distribution for $^9Be$,$^{12}C$, $^{16}O$ and $^{20}Ne$ (fig. 1) $^{24}Mg$, $^{28}Si$, $^{32}S$ and $^{40}Ca$ (fig. 2). The error bars indicate the uncertainties in the Monte Carlo calculation. The normalisation is $\int_0^\infty n_{c.m}^{(\alpha)}(P)P^2dP = 1$.

Concerning the possibilities of comparing the theoretical calculations of this approach on the alpha-particle centre of mass momentum distribution with some experimental results, it should be noted that the available experimental data for $n_{c.m}^{(\alpha)}$(P) are very scarce, mainly qualitative and for low momenta ($0 < P < 1fm^{-1}$). Though nucleon-nucleon correlation effects are reflected mainly in the behaviour of the momentum distribution at higher momenta ($P > 2fm^{-1}$) it is also interesting to compare our results with the existing data at low momenta.

The PWIA analysis of the $^{12}C(p,p\alpha)$ reaction at $E_p$= 150 MeV [3] leads to values of $n^{(\alpha)}$(P = 0) = 1.9 to 5.3 $fm^3$ and $n^{(\alpha)}(P = 1fm^{-1})$ = 0.56 to 1.60 $fm^3$ depending on the errors in the determination of the effective number $N_{eff}(\alpha)$ of the alpha-clusters in $^{12}C$ (in [3] this number is estimated to be $N_{eff}(\alpha) = 0.30^{+0.23}_{-0.11}$). Our results for the alpha momentum distribution are $n^{(\alpha)}_{c.m}$(P = 0) = $0.9fm^3$ and $n^{(\alpha)}_{c.m}$ (P = $1fm^{-1}$) = $0.45fm^3$, which are of similar magnitude.

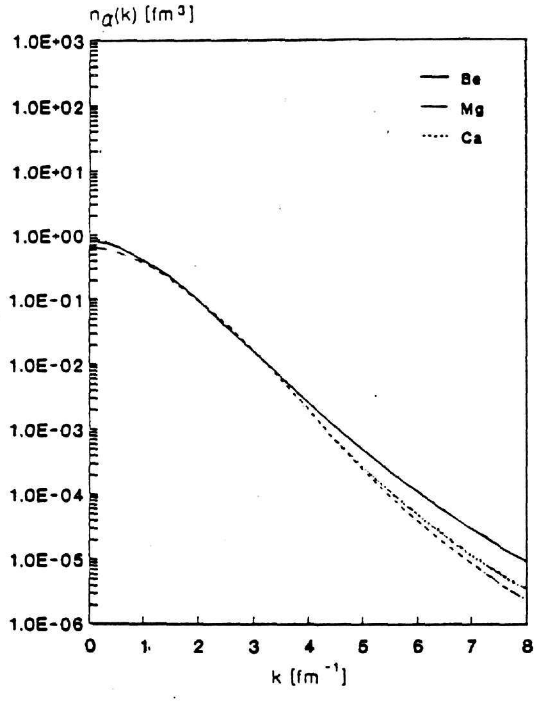



**Figure 3** The alpha-particle momentum distributions for $^{9}Be$ (solid line), $^{24}Mg$ (dashed line) and $^{40}Ca$ (dash-dotted line).

The values of the alpha momentum distributions at zero momenta extracted from PWIA analysis of $(\alpha, 2\alpha)$ ($E_\alpha$ = 700 MeV) reactions [6] for $^{12}C$ *and* $^{16}O$ are $n^{(\alpha)}_{^{12}C}(P = 0)$ = $1.36fm^3$ and $n^{(\alpha)}_{^{16}O}(P = 0) = 1.21fm^3$. Our result for both nuclei is $n^{(\alpha)}_{c.m}(P = 0) \simeq 0.9fm^3$. Thus the theoretical calculations are not in contradiction with the experimental data considered. The theoretical results for $n^{(\alpha)}_{c.m}(P = 0)$ are almost constant for A = 12 to 28 and decrease monotonically for larger A in agreement with the result in ref. 6.

In the region P $\simeq$ 4 to 7 $fm^{-1}$ our alpha-particle momentum distribution $n^{(\alpha)}_{c.m}(P)$ can be well reproduced by the function $exp(-P/P_0)$ with $P_0$ indepent of A within 10%. This agrees with the results from (p,$\alpha$) inclusive reactions at intermediate energies ($E_p$ = 210, 300 and 480 MeV) [23] and proton ($E_p$ = 90 Mev) and alpha ($E_\alpha$ = 140 MeV) induced inclusive reactions [24]. In the case of $^9Be$ $P_0 \simeq 140 MeV/c$ compared with $P_0 \simeq 75 MeV/c$ from ref.23. For $^{12}C$ $P_0$ is 122 MeV/c.

The alpha centre of mass momentum distributions obtained in the framework of the CDFM may be used in calculations of the cross-sections of alpha knockout reactions. It is expected that a comparison with the appropriate data would provide a sensitive test of their validity. Examples of suitablé processes are the reactions with electrons (e,e'$\alpha$) on light nuclei [25,26], as well as high energy (p,$\alpha$) inclusive reactions [20] and ($\alpha, 2\alpha$) reactions [5,6]. These calculations are in progress.

## References

[1] P.E. Hodgson In Proc. Fifth Int. Conf. Clustering Aspects in Nuclear and Subnuclear Systems, Kyoto, 1988, J. Phys. Soc. Jpn. 58, suppl. p. 755 (1989); Contemporary Physics 31, 99 (1990).

[2] D. Brink and J.J Castro Nucl. Phys. A216, 109 (1973)

[3] A.N. James and H.G. Pugh Nucl. Phys. 42, 441 (1963)

[4] J.W. Watson, H.G Pugh, P.G Roos, D.A. Goldberg, R.A. Riddle and D.I Bonbright Nucl. Phys. A172, 513 (1971); M. Jain, P.G. Roos, H.G Pugh and H.D Holmgren Nucl. Phys A153, 49 (1970).

[5] N. Chirapatpimol et al., Nucl. Phys. A264, 379 (1976).

[6] W.E. Dollhoph et al., Nucl. Phys. A316, 350 (1979).

[7] T.A. Griffi, R.J. Oakes and H.M. Schwartz Nucl. Phys. 86, 313 (1966).

[8] I.G. Evseev et al., Ukr. Fiz. Zh. 35, 839 (1990).

[9] A.N. Antonov, P.E. Hodgson and I.Zh. Petkov Nucleon Momentum and Density Distributions in Nuclei (Clarendon Press, Oxford, 1988).

[10] J.G. Zabolitzky and W. Ey Phys. Lett. 76B, 527 (1978).

[11] O. Bohigas and S. Stringari Phys. Lett. 95B, 9 (1980).

[12] C. Ciofi degli Atti, E. Pace and G. Salme Phys. Lett. 141B, 14 (1984).

[13] J.W. Van Orden, W. Truex and M.K. Banerjee Phys. Rev. C21, 2628 (1980).

[14] A.N. Antonov et al., Nuovo Cimento A91, 119 (1986); Nuovo Cimento A100, 779 (1988).

[15] J.J Griffin and J.A. Wheeler Phys. Rev. 108, 311 (1957).

[16] P.-O. Lõwdin Phys. Rev. 97, 1474 (1955).

[17] A.N. Antonov et al., Bulg. J. Phys. 6, 151 (1979); Z. Phys. A297, 257 (1980).

[18] P.-O. Lõwdin Phys. Rev. 97, 1490 (1955).

[19] K. Wildermuth and Y.C. Tang An Unified Theory of the Nucleus, p 369 (Academic Press ,N.Y., 1977).

[20] A.N. Antovov P.E. Hodgson, G.A. Lalazissis, E.N. Nikolov and I.Zh. Petkov 1991, Submitted for publication.

[21] V.V. Burov et al., Preprint JINR, E4-8029, Dubna (1974).

[22] J.M. Eisenberg and W. Greiner Nuclear Theory Vol. 2 (North-Holland, Amsterdam-London, 1970).

[23] D.H. Boal and R.M. Woloshyn Phys. Rev. C20, 1978 (1979).

[24] N.S. Wall et al., Phys. Rev. C20, 1079 (1979).

[25] J.P Genin et al., Phys. Lett. 52B, 46 (1974).

[26] V.F. Dimitriev et al., Nucl. Phys. A464, 237 (1987).