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PHENOMENOLOGICAL RELATIVISTIC STUDY OF THE ENERGY OF A Λ IN ITS GROUND AND EXCITED STATES IN HYPERNUCLEI *

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Abstract

The binding energy B_Λ of a Λ-particle in hypernuclei is studied by means of the Dirac equation containing attractive and repulsive potentials of orthogonal shapes. The energy eigenvalue equation in this case is obtained analytically for every bound state. An attempt is also made to investigate the possibility of deriving in particular cases approximate analytic expressions for B_Λ.

1. Introduction

In a previous publication [1] the Dirac equation containing attractive and repulsive potentials was used for the study of the binding energies of a Λ particle in its ground state in hypernuclei using orthogonal shapes for the potentials. The purpose of this paper is to extend the above study to the calculation of the binding energies of the Λ in the excited states and make relevant remarks. The outlay is as follows: In Section 2 we derive the eigenvalue equation which holds both, for the ground and for the excited states. In Section 3 we investigate the possibility of finding approximate formulae for the determination of the binding energy of the Λ in the various states, while in Section 4 we give our numerical results and we comment on them.

2. The eigenvalue equation for the ground and the excited states

We assume as in ref.[1] that the average Λ-nucleus potential is made up of an attractive $U_s(r)$ and a repulsive $U_v(r)$ component, both of orthogonal shape and that the differential equation discribing its motion is the Dirac equation [2-5]:

$$[c\vec{\alpha}\vec{p} + \beta\mu c^2 + \beta U_s(r) + U_v(r)]\Psi = E\Psi \tag{1}$$

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Instead of the potentials $U_s(r)$ and $U_v(r)$, the potentials

$$U_{\pm}(r) = U_s(r) \pm U_v(r) \tag{2}$$

are used, ,which are assumed to be of the form

$$U_{\pm}(r) = -D_{\pm}[1 - \Theta(r - R)]$$

where Θ is the unit step function and $R = r_0 A^{\frac{1}{3}}$ $(A = A_{core})$. By a method analogous to the one given in the refs.[6,1] we have derived the following eigenvalue equation

$$\frac{R\frac{dj_l(nr)}{dr}/_{r=R} + (k+1)j_l(nR)}{j_l(nR)} =$$

$$\left[1 - D_-(2\mu c^2 - B_\Lambda)^{-1}\right] \frac{R\frac{dh_l^{(1)}(in_0 r)}{dr}/_{r=R} + (k+1)h_l^{(1)}(in_0 R)}{h_l^{(1)}(in_0 R)} \tag{3a}$$

or equivalently

$$\frac{in_0 R h_{l-1}^{(1)}(in_0 R)}{h_l^{(1)}(in_0 R)} - \frac{nRj_{l-1}(nR)}{j_l(nR)} = \frac{D_-}{2\mu c^2 - B_\Lambda}\left[k - l + \frac{in_0 R h_{l-1}^{(1)}(in_0 R)}{h_l^{(1)}(in_0 R)}\right] \tag{3b}$$

which holds for all bound states (i.e. ground and excited). The quantities n , n_0 and k are defined as follows.

$$n = \left[\frac{2\mu}{\hbar^2}(D_+ - B_\Lambda)\left[1 - (B_\Lambda + D_-)(2\mu c^2)^{-1}\right]\right]^{1/2} \tag{3c}$$

$$n_0 = \left[\frac{2\mu}{\hbar^2} B_\Lambda [1 - B_\Lambda (2\mu c^2)^{-1}]\right]^{1/2} \tag{3d}$$

$$k = \pm(j + \frac{1}{2}), j = l \mp \frac{1}{2}, B_\Lambda = -E + \mu c^2$$

Also l is the eigenvalue of the orbital angular momentum $(l = 0, 1, 2, ...)$, while j_l and $h_l^{(1)}$ stand for the spherical Bessel and spherical Hankel functions of the first kind, respectively.

The above eigenvalue equation reduces for the ground state to the form

$$cot(nR) = -\frac{n_0}{n} + \frac{D_-}{2\mu c^2 - B_\Lambda}\left(\frac{n_0}{n} + \frac{1}{nR}\right) \tag{4}$$

which has been derived earlier [1].

In the excited states $p_{3/2}$ and $p_{1/2}$ equation (3) reads respectively:
State $p_{3/2}$:

$$cot(nR) = \left[1 - \frac{D_-}{2\mu c^2 - B_\Lambda}[3(\frac{1}{n_0 R} + \frac{1}{n_0^2 R^2}) + 1]\right]^{-1} \frac{n}{n_0}(1 + \frac{1}{n_0 R}) + \frac{1}{nR} \tag{5}$$

State $p_{1/2}$:

$$cot(nR) = \left[1 - \frac{D_-}{2\mu c^2 - B_\Lambda}\right]^{-1} \frac{n}{n_0}(1 + \frac{1}{n_0 R}) + \frac{1}{nR} \tag{6}$$

It is immediately realized that if the second term in the brackets of the above equations is set equal to zero, each of them reduces to the corresponding non-relativistic eigenvalue equation for the p-state [7].

Similar expressions can be derived for the higher states $d_{5/2}, d_{3/2}, f_{7/2}, f_{5/2}$ etc but these become more complicated. Furthermore, analytic expressions are easily obtained for the large (G) and small (F) component wave functions. These are generalizations to any bound state of expressions (25) and (26) of ref. [1] which refer to the ground state.

3. Approximate expressions for the determination of B_Λ in the various states

The binding energy B_Λ appears in the eigenvalue equation in an implicit form and hence its determination is carried out numerically. Thus, it is desirable to investigate the possibility of deriving approximate expressions which give B_Λ as a function of the mass number of the core nucleus A, which are appropriate for sufficiently large values of A. The "mass formulae" obtained in this way are given in ref [1] for the ground state. Here we attempt to investigate this problem for the excited states as well.

Using the asymptotic expressions of the Bessel and Hankel functions for large $|z|$, namely [8]

$$j_l(z) \simeq \frac{1}{z} cos(z - (l+1)\frac{\pi}{2})$$

$$h_l^{(1)}(z) \simeq \frac{1}{z} e^{i(z - \frac{l\pi}{2} - \frac{\pi}{2})}$$

the general eigenvalue equation (3b) reduces to the following approximate one

$$cot(nR - \frac{l\pi}{2}) = -\frac{n_0}{n}\left[1 - \frac{D_-}{2\mu c^2 - B_\Lambda}\left(1 - \frac{(k-l)}{n_0 R}\right)\right] \tag{7}$$

It is seen that for the ground state ($l = 0$) this equation corresponds in fact to the exact eigenvalue equation. From expression (7) one obtains for sufficiently large R:

$$B_\Lambda = D_+ - \frac{\hbar^2\pi^2(N+\frac{l}{2})^2}{2\mu g(1+(\tilde{f}_l n_0 R)^{-1})^2 R^2} \tag{8}$$

where N is the principal quantum number (N=1,2,...) and

$$g = 1 - \frac{B_\Lambda + D_-}{2\mu c^2} \tag{9}$$

$$\tilde{f}_l = \frac{(1-(B_\Lambda + D_-[1-(k-l)(n_0R)^{-1}])(2\mu c^2)^{-1})}{[1-B_\Lambda(2\mu c^2)^{-1}]} \tag{10}$$

It should be noted that the quantities g and $\tilde{f}_l$ depend upon the unknown binding energy B_Λ. This dependence is, however, usually quite weak and could therefore be taken into account by using an approximate expression for B_Λ : $B_\Lambda^{ap.}$. The same choice for B_Λ in n_0 (eq.(3d)) was made, though in this case the dependence of the corresponding term on B_Λ is not mostly so weak. For the ground strate the simplest choice for $B_\Lambda^{ap.}$ would be D_+.

It should be also noted that in the ground state expression (8) goes over to the expression (45) of ref.[1]. Various improved approximate expressions for B_Λ may be obtained which, however, become more complicated. Some of these have been used or may be used in the nonrelativistic case.

The above discussion refers to the case of rather deeply bound states in sufficiently heavy hypernuclei. Another limiting case deserves particular attention. It is the case of states in which the Λ is very loosely bound. In this case analytic expressions for B_Λ can also be obtained.

4. Numerical results and comments

The eigenvalue equation (3b) was solved numerically by using the folowing values of the potential parameters D_+=30.55 MeV D_-=300.0 MeV and r_0=1.01 fm. These were obtained by choosing D_-=300 Mev and determining D_+ and r_0 by least squares fitting[1] of the ground state energies of the Λ to the corresponding experimental values. If the choice D_-=430 Mev is made the results for B_Λ are similar. The values of the binding energies of the Λ in the ground and excited states $p_{3/2}, p_{1/2}, d_{5/2}, d_{3/2}$ etc, obtained for a number of hypernuclei are given in table 1. In the same table the binding energies of the unsplitted

Table1: Relativistic binding energies of the Λ particle in hypernuclei using the exact eigenvalue equation. The following potential parameters $D_+=30.55$MeV, $D_-=300$MeV, $r_o=1.01$fm were used.

Hypernuclei	$s_{1/2}$ MeV	$p_{3/2}$ MeV	$p_{1/2}$ MeV	p MeV	$d_{5/2}$ MeV	$d_{3/2}$ MeV	d MeV	$f_{7/2}$ MeV	$f_{5/2}$ MeV	f MeV
$^{9}_{\Lambda}Be$	7.5	-	-	-	-	-	-	-	-	-
$^{11}_{\Lambda}B$	9.6	-	-	-	-	-	-	-	-	-
$^{12}_{\Lambda}C$	10.5	-	-	-	-	-	-	-	-	-
$^{13}_{\Lambda}C$	11.2	-	-	-	-	-	-	-	-	-
$^{16}_{\Lambda}O$	13.1		-	-	-	-	-	-	-	-
$^{28}_{\Lambda}Si$	17.5	5.6	4.5	5.2	-	-	-	-	-	-
$^{32}_{\Lambda}S$	18.4	7.1	6.1	6.8	-	-	-	-	-	-
$^{40}_{\Lambda}Ca$	19.8	9.5	8.7	9.2		-	-	-	-	-
$^{51}_{\Lambda}V$	21.1	11.9	11.3	11.7	1.6	0.2	1.0	-	-	-
$^{89}_{\Lambda}Y$	23.6	16.7	16.3	16.6	8.4	7.5	8.0	-	-	-
$^{138}_{\Lambda}Ba$	25.1	19.7	19.4	19.6	13.0	12.4	12.8	5.4	4.3	4.9
$^{208}_{\Lambda}Pb$	26.3	21.9	21.8	21.9	16.5	16.2	16.4	10.3	9.6	10.0

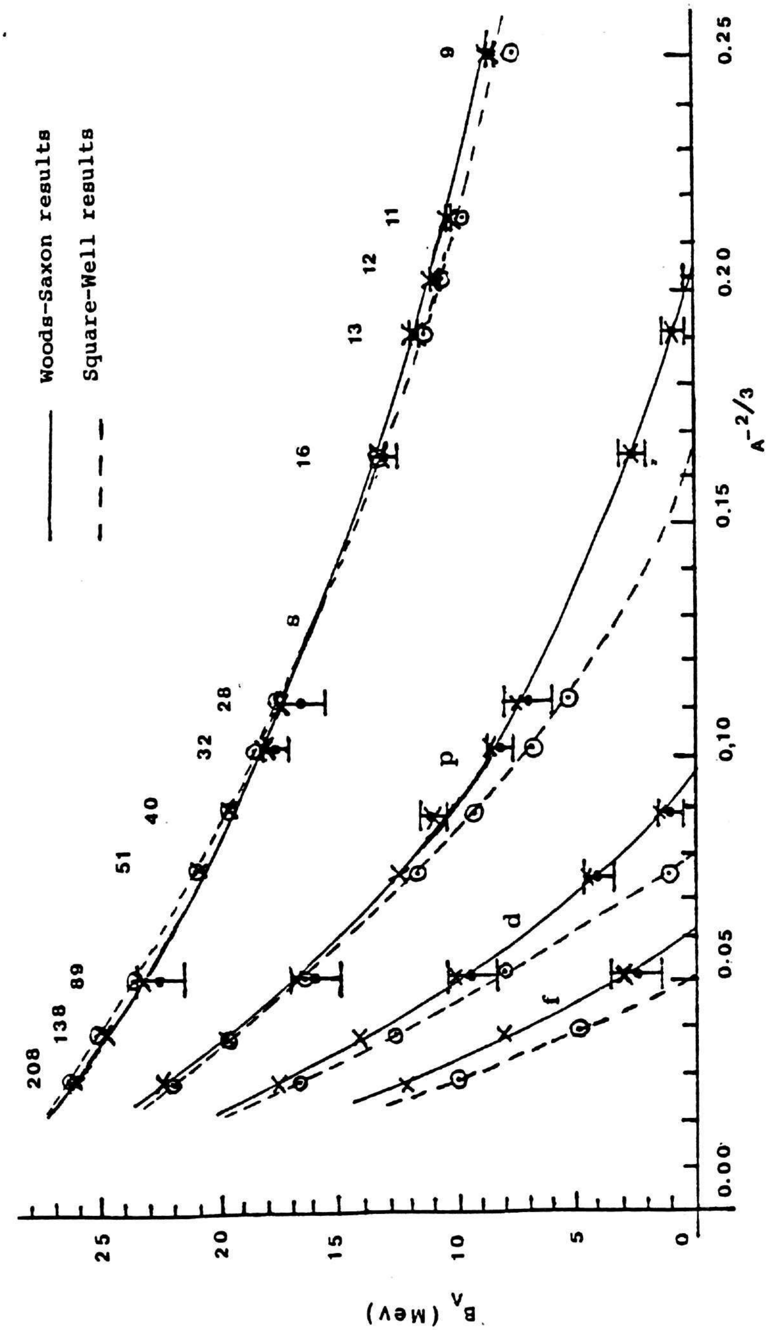
Woods-Saxon results
Square-Well results
B_Λ (Mev)
$A^{-2/3}$
208
138
89
51
40
32
28
16
13
12
11
9
s
p
d
f
0.00
0.05
0,10
0.15
0.20
0.25
0
5
10
15
20
25

fig. 1

<u>Table 2</u> Binding energies of the Λ particle using the approximate expression (8) with potential parameters $D_+=30.55$ MeV, $D_-=300$ MeV, $r_o=1.01$ fm

Hypernuclei	$s_{1/2}$ MeV	$p_{3/2}$ MeV	$p_{1/2}$ MeV	$d_{5/2}$ MeV	$d_{3/2}$ MeV	$f_{7/2}$ MeV	$f_{5/2}$ MeV
${}^{9}_{\Lambda}Be$	5.2	-	-	-	-	-	-
${}^{11}_{\Lambda}B$	7.9	-	-	-	-	-	-
${}^{12}_{\Lambda}C$	9.0	-	-	-	-	-	-
${}^{13}_{\Lambda}C$	10.0	-	-	-	-	-	-
${}^{16}_{\Lambda}O$	12.2			-	-	-	-
${}^{28}_{\Lambda}Si$	17.1	6.9	4.1	-	-	-	-
${}^{32}_{\Lambda}S$	18.1	7.8	5.5	-	-	-	-
${}^{40}_{\Lambda}Ca$	19.5	9.6	7.9	-	-	-	-
${}^{51}_{\Lambda}V$	20.9	11.5	10.4	5.9	0.2	-	-
${}^{89}_{\Lambda}Y$	23.5	15.9	15.4	8.1	5.7	-	-
${}^{138}_{\Lambda}Ba$	25.1	18.9	18.6	11.5	10.3	6.4	2.3
${}^{208}_{\Lambda}Pb$	26.3	21.2	21.0	14.8	14.2	8.6	6.6

states p,d,f are given. These were obtained by means of the usual weighted average of the splitted states. In fig. 1 the binding energies of the ground state 1s and of the unsplitted excited states p and d are plotted versus $A^{-2/3}$ and compared whith the experimental binding energies and also with the results obtained in ref. [9] using the Woods-Saxon potential. It is seen that the W-S results are closer to the experimental values as one should expect in view of its more realistic shape in the surface region. In certain cases, however, the results obtained with the Square-Well potential are fairly satisfactory.

Finally in table 2 the values of B_Λ are given for a number of states using the approximate formula (8). For the $B_\Lambda^{ap.}$ which is needed in the expression of $g, \tilde{f}_\ell$ and n_0, the choice $B_\Lambda^{ap.} = D_+$ was used for the ground state. For the excited states a value close to the estimated value of the lower (unsplitted) state was used for $B_\Lambda^{ap.}$. It was realized that such a choice leads to better results compared to those obtained if a value of $B_\Lambda^{ap.}$ closer to the actual energy is used. This seems to be due to the approximations involved in deriving expression (8). From the results of this table, it is seen that this expression gives better results for the lower states. In addition in order to obtain satisfactory values of B_Λ it is necessary the hypernucleus to be sufficiently heavy. It would be interesting to investigate whether it would be possible to obtain better approximate expressions for B_Λ. Work in this direction is in progress.

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