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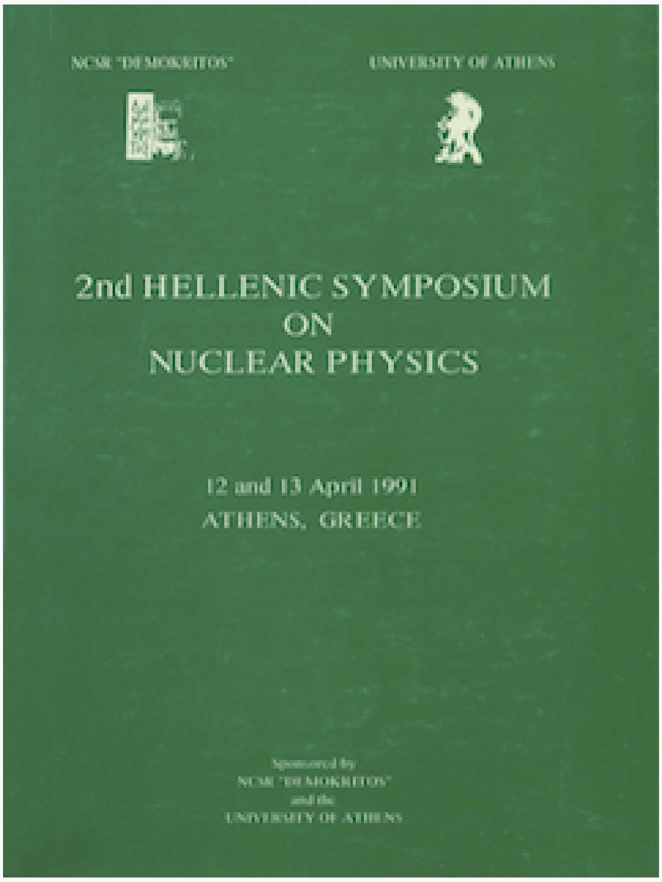

COMPARISON BETWEEN THE RELATIVISTIC AND NON-RELATIVISTIC TREATMENT OF THE Λ-HYPERNUCLEI

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COMPARISON BETWEEN THE RELATIVISTIC AND NON-RELATIVISTIC TREATMENT OF THE Λ-HYPERNUCLEI

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Abstract: Using as an example potentials of the form $U_{\pm}(r)=-D_{\pm}(\cosh^2(r/R))^{-1}$, the binding energies as well as the root mean square radii of the orbits of the Λ particle in hypernuclei in the ground and excited states were calculated in the relativistic and non-relativistic cases and the results are compared.

1. Introduction

During the works of the First Hellenic Symposium on Theoretical Nuclear Physics which took place in Thessaloniki in 1990 we have presented a study concerning Λ-hypernuclei in which the Dirac equation was employed.[1,2] A question which remained hanging in the air at that time was the following: Do the relativistic results derived using the Dirac equation differ essentially from the non-relativistic ones in which the Schrödinger equation is used? This question we try to answer in this contribution using as an example the potential of the form

$$U(r) = -D(\cosh^2(r/R))^{-1} \tag{1}$$

The reason behind the choice of this potential is that it was used by Grypeos, Lalazissis and Massen[3-5] in the non-relativistic study of the Λ-hypernuclei and so the comparison with the relativistic case was much easer.

2. Numerical results

Using the formalism outlined in ref(1) and applying a least-squares fitting procedure we have found in the relativistic case assuming that the Λ-nucleus potential is made up of the components

$$U_{\pm}(r) = -D_{\pm}(\cosh^2(r/R))^{-1} \qquad (2)$$

that the potential parameters are

$D_+=39.69$ MeV, $D_-=201.59$ MeV, $r_o=0.984$ fm.

(where the extra decimals are used for the sake of comparison).We notice that the value of D_+ is very close to the value of the well depth D in the non-relativistic case while the values of r_o in both cases are almost the same. Using the potential parameters given above we have calculated the binding energies of the ground state 1s and of the excited states $1p_{3/2}$ and $1p_{1/2}$ in the relativistic case and also the binding energies of the 1s and 1p states in the non-relativistic case for a number of Λ-hypernuclei and the results obtained in both cases are given and compared in table 1.

Next using the Dirac radial wavefunctions G(r),F(r) we have calculated numerically the root mean square radii of the orbits of the Λ particle in the Λ-hypernuclei in the ground state 1s and in the excited states $1p_{3/2}$ and $1p_{1/2}$ with the help of the formula

$$<r_\Lambda^2>^{1/2}=\left(\frac{\int_0^\infty r^2(G^2(r)+F^2(r))dr}{\int_0^\infty (G^2(r)+F^2(r))dr}\right)^{1/2} \qquad (3)$$

Also using the radial wave functions ψ(r) of the Schrödinger equation we have calculated numerically the root mean square radii of the orbits of the Λ particle in various hypernuclei in the states 1s, 1p using the formula

$$<r_\Lambda^2>^{1/2}=\left(\int_0^\infty \psi^*(r)r^2\psi(r)dr\right)^{1/2} \qquad (4)$$

where the wavefunctions are considered normalized.The results obtained in both cases are given and compared in table 2.

3. Discussion

Our aim in this contribution was the comparison between the relativistic and non-relativistic results obtained in a phenomenological treatment of Λ hypernuclei.[6] We had chosen for this comparative study the potentials (2) and (1) respectively.The quantities chosen to be compared are the binding energies of the Λ particle in hypernuclei as well as the root mean square radii of its orbits in them.

From tables 1 and 2 we observe that the relativistic results differ from the non-relativistic ones, as far as the binding energies are concerned, very little in the ground state namely (0.2%-0.5%) while in the excited state 1p(which in the rel.case is taken as the average of the binding energies of the states $1p_{3/2}$ and $1p_{1/2}$)the difference becomes greater namely (0.7%-7%).The difference, as far as the root mean square radii are concerned is more apparent even in the ground state and is of the order of (2.1%-2.7%).

Despite the fact that the differences between the relativistic and non-relativistic treatment are not large as to make the non-relativistic calculations unreliable yet the relativistic treatment has some advantages like for instance that it incorporates the spin-orbit coupling the magnitude of which is found to be small for the Λ-hypernuclei an information which we cannot have with the non-relativistic treatment.

Table 1

The binding energies of various Λ-hypernuclei obtained relativistically and non-relativistically are given in columns II-VI and compared in VII-VIII

Relativistic					Non-Relativistic		% Difference	
	U_+=39.7MeV, U_-=201.6MeV r_o=0.984fm				U=38.93MeV r_o=0.986fm			
Hyp.	1s B_Λ	$1p_{3/2}$ B_Λ	$1p_{1/2}$ B_Λ	1p B_Λ	1s B_Λ	1p B_Λ	% in $B_\Lambda(s)$	% in $B_\Lambda(p)$
$^{9}_{\Lambda}Be$	8.66	-	-	-	8.62	-	0.37	-
$^{13}_{\Lambda}C$	11.57	0.67	0.36	0.56	11.59	0.60	0.15	6.95
$^{16}_{\Lambda}O$	13.12	1.96	1.57	1.83	13.15	1.92	0.27	4.68
$^{28}_{\Lambda}Si$	16.91	6.00	5.58	5.86	16.95	6.04	0.25	2.98
$^{32}_{\Lambda}S$	17.74	6.99	6.58	6.86	17.77	7.05	0.18	2.75
$^{40}_{\Lambda}Ca$	19.07	8.64	8.25	8.51	19.09	8.72	0.13	2.39
$^{89}_{\Lambda}Y$	23.27	14.26	13.95	14.15	23.23	14.36	0.16	1.44
$^{138}_{\Lambda}Ba$	25.24	17.05	16.80	16.97	25.15	17.14	0.35	1.00
$^{208}_{\Lambda}Pb$	26.89	19.46	19.26	19.39	26.75	19.52	0.52	0.65

Table 2

The r.m.s. radii of the Λ-particle orbits in various hypernuclei obtained relativistically and non-relativistically are given (II-VI) and compared (VII-VIII)

Relativistic					Non-Relativistic		% Difference	
Hyp	1s fm	$1p_{3/2}$ fm	$1p_{1/2}$ fm	1p fm	1s fm	1p fm	% in 1s	% in 1p
$^{9}_{\Lambda}Be$	2.34	-	-	-	2.29	-	2.10	-
$^{13}_{\Lambda}C$	2.25	-	-	-	2.20	-	2.41	-
$^{16}_{\Lambda}O$	2.24	4.08	4.31	4.19	2.19	4.04	2.52	3.71
$^{28}_{\Lambda}Si$	2.27	3.50	3.56	3.53	2.21	3.43	2.62	2.88
$^{32}_{\Lambda}S$	2.29	3.46	3.51	3.49	2.23	3.39	2.65	2.83
$^{40}_{\Lambda}Ca$	2.32	3.43	3.47	3.45	2.26	3.35	2.65	2.77
$^{89}_{\Lambda}Y$	2.50	3.50	3.52	3.51	2.44	3.42	2.63	2.66
$^{138}_{\Lambda}Ba$	2.63	3.62	3.63	3.62	2.56	3.53	2.66	2.63
$^{208}_{\Lambda}Pb$	2.76	3.76	3.77	3.77	2.69	3.67	2.68	2.62

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