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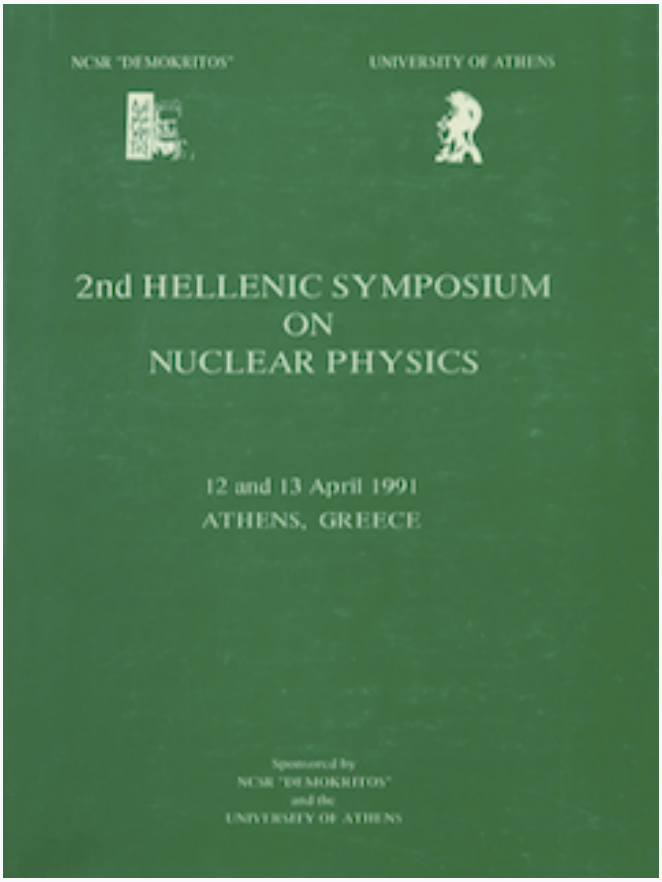

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THE "COSH" OR SYMMETRIZED WOODS-SAXON NUCLEAR POTENTIAL*

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Abstract

The so-called "cosh" or symmetrized Woods-Saxon potential which has been used in nuclear physics is discussed and certain comments are made regarding its form-factor, depth and radius. A rough estimate of its parameters is also attempted by using various possibilities. This is done by considering either a number of individual nuclei or by least squares fitting. The potential depth parameter V_o apart from being state dependent varies also with the mass number of the nucleus. Comments on relevant limitations and inaccuracies are also made.

1. Introduction

The very-well known Woods-Saxon (WS) potential

$$V_{WS}(r) = -V_o f_{WS}(r) = \frac{-V_o}{1+e^{(r-R)/a}} \qquad (1)$$

has been extensively used in various calculations in nuclear

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*Presented by M.E. Grypeos

physics and is discussed in standard texts [1]. Such a potential does not satisfy the physical requirement which is usually imposed [2] that the force experienced by a particle at the centre of a spherically symmetric potential be zero, though the value of $\left.\frac{dV_{WS}}{dr}\right|_{r=0}$ is very small for R/a >>1, that is apart from the light nuclei.

An alternative potential has been considered, however, by Buck and Pilt[3], the so-called "cosh" potential, which is a symmetrized form of the WS one, namely

$$V_c(r) = -V_o f_c(r) = -V_o \frac{1+\cosh(R/a)}{\cosh(r/a)+\cosh(R/a)} \qquad (2)$$

and has the desirable feature: $\left.\frac{dV}{dr}\right|_{r=0}=0$. Note that we have mostly dropped the corresponding subscripts in the potential parameters in order to simplify the notation. The form factor of this potential is very similar to that which had been used [4] to describe very successfully the densities of nuclei for a wide range of mass number. Buck and Pilt have used potential (2) as the central part of a cluster-core potential and have realized that by suitably choosing R and a, potential shapes can be obtained which are remarkably similar to the folding potentials used in ref 5 and hence they also exhibit rotational spectra.

More recently, Carbonell et al [6] have used potential (2) in connection with a semiclassical method of quantization for a particle in a nuclear potential and for the comparison of their results with those obtained with the Schrödinger equation. In their case the WS potential is inadequate (according to ref 6b) to calculate the classical trajectories since the numerical methods used for the integration of the equations of motion are inapplicable in the vicinity of r=0 for this potential.

In this paper we would like to discuss the "cosh" potential and comment on the possibility of using instead of expression (2) an alternative one, which is closely related to it. The relevant remarks are made in the next section where the dependence of the radius R and of the various moments of the potential on the mass number is also discussed. In section 3 a rough estimate of the

parameters of the potential is attempted by considering for this purpose various possibilities. The last section is devoted to the presentation of numerical results and comments.

2. The potential form factor, depth and radius

Instead of using potential (2) an alternative possibility is to use a symmetrized Woods-Saxon potential in the form which has been used in the case of nuclear densities (and very recently in the analysis of hypernuclei [7]), namely to use instead of (2):

$$V_{SF}(r) = -V_o f_{SF}(r) = -V_o \frac{\sinh(R/a)}{\cosh(r/a)+\cosh(R/a)} =$$

$$= -Vo\{[1+\exp(\frac{r-R}{a})]^{-1}+[1+\exp(\frac{-r-R}{a})]^{-1}-1\} \qquad (3)$$

Both potentials (2) and (3) are of symmetrized Fermi form and have zero slope at the origin, but their form factors differ, their ratio being:

$$\frac{f_{SF}}{f_c} = \frac{\sinh(R/a)}{1+\cosh(R/a)} \qquad (4)$$

if the values of R and a in the two potentials do not differ appreciably.

This ratio is almost unity for $R/a>>1$, that is for the heavier elements for which both potentials are very close to the Woods-Saxon one.

The choice of the numerator in potential (2) seems to originate from the requirement the form factor to become unity at the origin, where the potential is minimum. Thus, V_o is the actual depth of the "cosh" potential: $D_c = V_o$. This is not the case for the other two potentials (see table 1). Therefore, for each of these potentials V_o is a "depth-parameter", which determines its depth to a large extent, but not entirely, since D depends also upon R and a.

Analogous remarks can be made regarding the meaning of R as it is clear from table 1. For all three potentials, R is a parameter which characterizes the range of the potential, but it is somehow smaller than the "half-depth radius".

Table 1

Potential	Depth of the potential	Value of the potential at R
Woods-Saxon $V_{WS}(r) = \frac{-V_0}{1+\exp[(r-R)/a]}$	$D_{WS} = V_0[1+\exp(-R/a)]^{-1}$	$\lvert V_{WS}(R)\rvert = \frac{1}{2} V_0 =$ $= \frac{1}{2} D_{WS}[1+\exp(-R/a)] > \frac{1}{2} D_{WS}$
"cosh" potential $V_c(r) = - V_0 \frac{1+\cosh(R/a)}{\cosh(r/a)+\cosh(R/a)}$	$D_c = V_0$	$\lvert V_c(R)\rvert = \frac{1}{2} D_c\left[1+ \frac{2\exp(-R/a)}{1+\exp(-2R/a)}\right] > \frac{D_c}{2}$
Symmetrized Woods-Saxon $V_{SF}(r) = - V_0 \frac{\sinh(R/a)}{\cosh(r/a)+\cosh(R/a)}$	$D_{SF} = V_0 \frac{1-\exp(-R/a)}{1+\exp(-R/a)}$	$\lvert V_{SF}(R)\rvert = \frac{1}{2} D_{SF}\left[1+ \frac{2\exp(-R/a)}{1+\exp(-2R/a)}\right] > \frac{D_{SF}}{2}$

The above remarks, as well as, the following one are pertinent to the cases in which the condition R/a>>1 is not quite satisfied that is to the lighter nuclei.

The choice of the numerator in the "cosh" potential discussed earlier seems, however, to have, another implication. The volume integral of the potential becomes a transcendental function of the radius R:

$$|\bar{V}_c| = |4\pi \int_0^\infty V_c(r) r^2 dr| = \frac{4\pi V_o}{3} R^3 [1+\frac{(\pi a)^2}{R^2}] [\frac{1+\cosh(R/a)}{\sinh(R/a)}] \qquad (5)$$

and it is not easy to use it in order to express R as a function of the mass number of the nucleus. Transcendental terms appear also in the volume integral of the Woods-Saxon potential[8]. On the contrary for the $V_{sf}(r)$ these terms are absent as the results given in ref 4 for the symmetrized Fermi density show and our detailed calculations have verified. One can therefore write on the basis of a folding model and the well-known properties of the convolution (in analogy, for example, with the approximate treatment of ref 9 for the WS hypernuclear potential):

$$\frac{4\pi}{3} V_o R^3 [1+ \frac{(\pi a)^2}{R^2}] = A_c |\bar{V}_{NN}| \qquad (6)$$

where, in the case of a bound nucleon, A_c=A-1, A being the mass number of the nucleus. $|\bar{V}_{NN}|$ is the volume integral of the spin-average, (central), nucleon-nucleon potential $|\bar{V}_{NN}| = |\int \bar{V}_{NN}(\vec{r})d\vec{r}|$ The above equation is a third-order equation with respect to R and can be solved exactly. In fact it has the same structure with the equation arising from the normalization integral of a trapezoidal distribution [10]. The corresponding equation for the Fermi distribution [11] is approximate.

Thus, in the case of the V_{sf} potential, one may obtain the following "exact" expression of the radius R in terms of the mass number

$$R=\frac{1}{2^{1/3}} r_o A_c^{1/3} \left[\{1+[1+\frac{2^2}{3^3}(\frac{\pi a}{r_o A_c^{1/3}})^6]^{1/2}\}^{1/3}+\{1-[1+\frac{2^2}{3^3}(\frac{\pi a}{r_o A_c^{1/3}})^6]^{1/2}\}^{1/3}\right]$$

$$= r_o A_c^{1/3}\left[1-\frac{1}{3}\left(\frac{\pi a}{r_o A_c^{1/3}}\right)^2+\frac{1}{81}\left(\frac{\pi a}{r_o A_c^{1/3}}\right)^6+\frac{1}{243}\left(\frac{\pi a}{r_o A_c^{1/3}}\right)^8+\ldots\right] \qquad (7)$$

where $r_o = (3|\bar{V}_{NN}|/4\pi V_o)^{1/3}$. The leading term of this expansion (see section 4 for the expression of V_o): $R \approx r_o(A-1)^{1/3}$ is of the form which is often used for the radius of the Woods-Saxon potential. In ref 12, however, the r_o in such an expression was dependent on A.

It should be noted that the higher moments of potential $V_{SF}(r)$ may also be given analytically and contain no exponential terms. The same holds for the "cosh" potential but not for the WS one. The expression of the n-th moment is (see also ref. 8):

$$\langle r^n\rangle_{SF} = \langle r^n\rangle_c = \frac{\int_0^\infty V_c(r) r^{n+2} dr}{\int_0^\infty V_c(r) r^2 dr} =$$

$$= \frac{3R^n}{n+3}\left[1+\frac{(\pi a)^2}{R^2}\right]^{-1}\left[1+(n+3)!\sum_{k=0}^{n+2}\frac{(1-(-1)^k)(1-2^{-k})}{(n+2-k)!}\,\zeta(k+1)\left(\frac{a}{R}\right)^{k+1}\right]$$

$$n = 1,2,3,\ldots \qquad (8)$$

where ζ is Riemman's ζ-function.

The expressions of the first moments are quite simple: We give the expression for the second and fourth moment which we shall use in the next section

$$\langle r^2\rangle_{SF} = \frac{3}{5}R^2\left[1+\frac{7}{3}\left(\frac{\pi a}{R}\right)^2\right] \qquad (9)$$

$$\langle r^4\rangle_{SF} = \frac{3}{7}R^4\left[1+\left(\frac{\pi a}{R}\right)^2\right]^{-1}\left[1+7\left(\frac{\pi a}{R}\right)^2+\frac{49}{3}\left(\frac{\pi a}{R}\right)^4+\frac{31}{3}\left(\frac{\pi a}{R}\right)^6\right] \qquad (10)$$

Use of expressions (7) for R in the above formulae leads to the dependence of the various moments on the mass number. It is obvious that the leading term of $\langle r^n\rangle$ is proportional to $A_c^{n/3}$.

3. The estimate of the potential parameters

In this section we discuss the problem of the estimate of the potential parameters V_o, R and a. We have paid attention to three possible ways in determining these parameters. In the two of them the R and a will be determined from the known corresponding quantities of the charge density distribution.

The first one is related to a rather recent proposal by Salcedo et al. [13] who estimated, in their study of inclusive pion nuclear reactions the radius R and difuseness parameter a for a point-proton density distribution

$$\rho(r) = \frac{\rho_o}{1+e^{(r-R)/a}} \tag{11}$$

from the corresponding quantities of the same type charge density distribution

$$\rho_e(r) = \frac{\rho_{oe}}{1+e^{(r-R_e)/a_e}} \tag{12}$$

Here we shall use the relations

$$\rho_o \approx \rho_{oe}(1-\frac{r_p^2}{6Ra}) \tag{13}$$

$$\langle r^2\rangle + r_p^2 = \langle r^2\rangle_e \tag{14}$$

$$\int\rho(\vec{r})d\vec{r} = \int\rho_e(\vec{r})d\vec{r} \tag{15}$$

which result from the properties of the convolution [14] and also the formulae for the m.s. radius and for the volume integral of the symmetrized Fermi density distribution. The use of (13) instead of $\rho_{oe} \approx \rho_o$ of Salcedo et al (13) should be more appropriate and is derived from expressions (11) and (12) of ref. 15b (see also 15a) , which in the present case take the form

$$\rho_e(r) = \int\rho(|\vec{r}-\vec{r}'|)\rho_p(|\vec{r}'|)d\vec{r}' = \left[1+\sum_{n=1}^{\infty}\frac{\langle r'^{2n}\rangle_p}{(2n+1)!}\Delta^n\right]\rho(r) \tag{16}$$

where $\Delta^n\rho(r) = [(d/dr)^{2n}+(2n/r)(d/dr)^{2n-1}]\rho(r)$ and $\langle r^{2n}\rangle_p$ is the 2n-th moment of the charge density distribution of the proton ($\langle r^2\rangle_p = r_p^2$). The above expansion is expected to be valid for va-

lues of r sufficiently larger than the proton size and was used here for r=R.

A linearization procedure leads after some algebra to the following expressions for R and a:

$$R = R_\circ + \varepsilon$$

$$= \left[1 + \frac{(\pi r_0^2/6)^2 R_\circ^2 [1+(\pi a_\circ/R_\circ)^2]^2 - (\pi a_\circ)^4 B + 2(\pi a_\circ)^2 B^2 - B^3}{-\frac{6}{7}(\pi a_\circ)^4 R_\circ^2 + 2(\pi a_\circ)^2 [(\pi a_\circ)^2 - \frac{9}{7} R_\circ^2] B + 2[-3(\pi a_\circ)^2 + \frac{12}{7} R_\circ^2] B^2 + 4B^3}\right] R_\circ \tag{17}$$

$$a = \left[\frac{-6R_\circ \varepsilon + 7(\pi a_\circ)^2 - 5r_p^2}{7\pi^2}\right]^{1/2} \tag{18}$$

where $B=(\pi a_\circ)^2 - \frac{5}{7} r_p^2$ and the expression of ε in (18) follows from (17).

Having determined the parameters of the point proton density which is taken to be the same with the point nucleon one, we may now proceed in obtaining the parameters of the (symmetrized) Woods-Saxon potential which is assumed to be given in the folding model by the expression

$$V(r) = A_c \int \rho(|\vec{r} - \vec{r}'|)\, V_{NN}(\vec{r}')\, d\vec{r}' =$$

$$= \left[A_c \int V_{NN} d\vec{r}'\right] \left[1 + \sum_{n=1}^{\infty} \frac{\langle r'^{2n} \rangle_{NN}}{(2n+1)!} \Delta^n\right] \rho(r) \tag{19}$$

Again the above expansion is valid for large r compared to the range of the nucleon-nucleon potential V_{NN} and for sufficiently short range V_{NN} only the first term in the sum may be taken into account.

The relation between the depth parameter $V_\circ$ of the (symmetrized) Woods-Saxon potential and the parameters of the (symmetrized) Fermi distribution for point nucleons is found in a way analogous to that of $\rho_{c\varepsilon}$ and $\rho_\circ$. The result is

$$-V_c \approx (A_c \rho_\circ \int V_{NN}(r) d\vec{r}') \left[1 - \frac{\langle r^2 \rangle_{NN}}{6Ra}\right] \tag{20}$$

The potential radius R_v and difuseness parameter a_v are related with the corresponding parameters of the point-nucleon distribution by the relation (which is obtained now without linearization):

$$R_v = (-\frac{a_3}{2})^{1/3}\{[1+\sqrt{1+\frac{2^2}{3^3}\frac{a_2^3}{a_3^2}}]^{1/3}-[-1+\sqrt{1+\frac{2^2}{3^3}\frac{a_2^3}{a_3^2}}]^{1/3}\} \quad (21)$$

$$a_v = [\frac{3(R^2-R_v^2)+7(\pi a)^2+5\langle r^2\rangle_{NN}}{7\pi^2}]^{1/2} \quad (22)$$

where

$$a_2 = \frac{3}{4}R^2+\frac{7}{4}B' \qquad a_3 = -\frac{7}{4}[R^2+(\pi a)^2]R/[1-\frac{\langle r^2\rangle_{NN}}{6Ra}] \quad (23)$$

and $$B' = (\pi a)^2 - \frac{5}{7}\langle r^2\rangle_{NN} \quad (24)$$

Another possible way of determining the parameters R_v and a_v is the one which is outlined in Ref. 17 (see also ref. 16) in the case of the Λ-nucleus potential. According to this approach one uses relation (14) between the second moments of the (symmetrized) Fermi distribution and the corresponding one between the fourth moments (The factor 7/3 in expression A3 of ref 17 should be written 10/3 [18]):

$$\langle r^4\rangle_e = \langle r^4\rangle + r_p^4 + \frac{10}{3}\langle r^2\rangle r_p^2 \quad (25)$$

Analogous relations are used between the moments of the potential and point-nucleon distribution. The analytic expressions (9) and (10) for the corresponding moments in the case of the (symmetrized) Fermi form factors are also used.

Again in this approach the parameters R and a, R_v and a_v are given analytically [23].

It should be clear that in both approaches, described above, the approximation is made, as in ref.17 that the convolution of a (symmetrized) Fermi distribution is a (symmetrized) Fermi distribution. This should be reasonable as long as the "folding" distribution (either proton charge density or nucleon-nucleon potential) are of sufficiently short range, compared to the folded distribution. Having determined R_v and a_v in one way or the other the potential depth parameter V_0 is adjusted so that the values of the experimental single particle energies, are reproduced by solving the Schrodinger equation.

Finally, all parameters could be determined by a suitable least-squares fitting to the experimental single particle energies. It appears, however, that in most cases such a procedure would not be satisfactory, mainly because of the large experimental uncertainties. For this reason one of the parameters, namely a_v, (on which the single particle energies, at least for the ground state [10], are not expected to depend strongly, unless A_c is small) is fixed from our previous experience and the remaining parameters are determined by least-squares fitting (see also next section).

4. Numerical results and discussion

In this section we determine the parameters for the symmetrized Woods-Saxon potential (3) for a number of nuclei for which the parameters of the corresponding charge density distribution have been determined [4] (together with those of some other nuclei) from the analysis of the elastic electron scattering experiments. The corresponding values given in ref. 4 are displayed also in table 1. In this table the values of the point proton density parameters R and a and potential parameters R_v and a_v are also given by using the improved and extended version of the approach of Salcedo et al.[13] for the densities, described in the previous section, namely our expressions (17),(18),(21) and (22).

In table 2 the same values are given using the method of second and fourth moments (ref.17) that is expressions (36) and (37) of ref. 23 and the corresponding ones for the potential parameters. The second and fourth moments of the nucleon-nucleon potential were estimated by using the interaction 4 of Volkov (see table 1 of ref 19). The reason for such a choice is that this interaction is of rather short range, which is essential for the validity of the approaches we are using.

It is seen from the results of tables 1 and 2 that the values of R_v and a_v obtained with these two approaches do not differ very appreciably. It is observed, however, that the first approach leads to larger values of R_v and to smaller values of a_v

Table 2 Parameters of the Symmetrized Woods-Saxon potential (3) determined with the first method described in the text. The values in parentheses were determined by using for the experimental single particle energies those obtained from the corresponding curves in fig.117 of ref 22b.

Nucl	R_s(fm)	a_s(fm)	R(fm)	a(fm)	R_v(fm)	a_v(fm)	V_0(MeV) 1s-state	V_0(MeV) 1p-state	V_0(MeV) 1d-state	V_0(MeV) 2s-state	V_0(MeV) 1f-state
$^{12}_{6}C$	2.214	0.488	2.122	0.454	3.287	0.463	57.0	49.5			
$^{16}_{8}O$	2.562	0.497	2.468	0.467	3.342	0.528	66.3	53.1			
$^{24}_{12}Mg$	2.934	0.569	2.859	0.541	3.427	0.648	(74.1)	64.3			
$^{28}_{14}Si$	3.085	0.563	3.006	0.537	3.563	0.641	75.5	71.2	71.7	70.6	
$^{32}_{16}S$	3.255	0.601	3.184	0.576	3.660	0.687	(77.8)	(70.7)	(71.5)	70.7	
$^{40}_{20}Ca$	3.556	0.578	3.474	0.556	3.959	0.659	82.4	71.4	63.8	64.2	
$^{56}_{26}Fe$	4.054	0.600	3.970	0.583	4.401	0.682	(81.7)	(73.4)	(68.3)	(57.5)	
$^{58}_{28}Ni$	4.153	0.566	4.055	0.553	4.525	0.642	84.1	77.9	64.2	59.4	65.6

compared to those of the second approach. The differences are larger for the lighter elements though they are smaller for the medium ones ($30 \lesssim A \lesssim 70$). It is indicated that the values with the

Table 3 Parameters of the symmetrized Woods-Saxon potential (3) determined with the second method described in the text. The values in parentheses were determined by using for the experimental single particle energies those obtained from the corresponding curves in fig.117 of ref 22b.

Nucl	R(fm)	a(fm)	R_v(fm)	a_v(fm)	V_0(MeV) 1s-state	V_0(MeV) 1p-state	V_0(MeV) 1d-state	V_0(MeV) 2s-state	V_0(MeV) 1f-state
$^{12}_{6}C$	2.092	0.459	2.802	0.585	65.7	62.2			
$^{16}_{8}O$	2.475	0.465	3.037	0.603	71.7	60.5			
$^{24}_{12}Mg$	2.858	0.541	3.341	0.667	(75.5)	66.3			
$^{28}_{14}Si$	3.020	0.533	3.454	0.666	77.2	73.8	75.2	73.5	
$^{32}_{16}S$	3.192	0.574	3.604	0.699	(78.7)	(71.9)	(73.2)	72.1	
$^{40}_{20}Ca$	3.512	0.546	3.837	0.690	84.1	73.8	66.9	66.9	
$^{54}_{26}Fe$	4.023	0.566	4.268	0.717	(83.2)	(75.6)	(71.2)	(59.8)	
$^{58}_{28}Ni$	4.131	0.528	4.340	0.695	86.1	81.0	68.0	62.6	70.6

first approach are less reliable in particular for the lighter nuclei. In fact this approach does not work at all if it is applied to ^{4}He while for ^{6}Li gives unrealistic results. The situation becomes worse, as it is expected, if a nucleon-nucleon force of a longer range like the Volkov 1 force is used. In this case, even for ^{12}C this approach is not applicable.

In tables 2 and 3 the values of the potential depth parameter V_0 determined by using the values found for R_v and a_v and the nuclear contribution to the proton energies for various states are also displayed. These energies were estimated from the experimental proton energies given in ref.22 (or by the corresponding curves, of fig.117 of ref.22b), subtracting ap-

proximate values for the Coulomb energies E_c. The same expression for E_c as in ref. 7 has been used for the present rough estimates using the values of table 2, for R_v and a_v. The values of E_c obtained in this way are quite close to those of ref 7 in which the R_v and a_v of the corresponding (or neighbouring) nuclei had been taken from ref. 20 and 21. The values of V_o obtained for each set of values of R_v and a_v given in tables 2 and 3 do not differ very much for A>24. There is, however, a marked state dependence of V_o. V_o is smaller for the 1p-states and (usually) even smaller for the 1d or 2s states. In addition V_o varies rather smoothly with A for the 1s and 1p states, though for the 1d and 2s states it behaves rather irregularly. This should be mainly attributed to the existing uncertainties (see below). We may also note that the values of V_o we have found are in some cases rather similar to the corresponding ones for the Woods-Saxon potential (with spin-orbit term) obtained by Elton and Swift [12], though they are usually different.

We have also made a least squares fitting to the single particle energies by assuming for a_v the value 0.65 fm. In view of the variation of V_o with the mass number which is also indicated from the results of tables 2 and 3, we have attempted a least-squares fitting by considering symmetric nuclei with $12 \leq A \leq 40$ and assuming the following variation of V_o with A :

$$V_o = V_1 + \frac{V_2}{A} \qquad (26)$$

taking as adjustable parameters the V_1, V_2 and $|\bar{V}_{NN}|$. The best fit values by fitting to the single-particle energies of the 1s states are V_1 = 80.4MeV, V_2= -166MeV and $|\bar{V}_{NN}|$=845MeV.fm^3, while for the 1p states, we find V_1= 66.2MeV, V_2= -190MeV and $|\bar{V}_{NN}|$=852MeV.fm^3.

It should be pointed out that the folding model approach used in section 3 with an average spin and isospin independent nucleon-nucleon potential is essentially limited to the case of light symmetric nuclei. Although the described approach could perhaps be also attempted for some assymetric nuclei, its validity in these cases should be in general quite doubtful.

We note finally that in ref. [23] where some more details of this investigation are given the state dependence of V_0 is discussed in relation with the concept of an effective nucleon mass (as it was done for the Woods-Saxon potential [24]) as well as the addition of a spin-orbit term in the potential.

In conclusion we would like to make clear that we have attempted in this paper only a rough estimate of the potential parameters. This is mainly due to the approximations involved, to the rough estimate of the Coulomb energies and to the rough experimental values used for the single particle energies. In order to become free of the second ambiguity and diminish the third one we have also used the more recent neutron separation energies reported in ref. 25. Unfortunately only the ^{12}C and ^{16}O nuclei (and some lighter ones) were studied in this reference. The values of the potential depth parameters obtained by using the experimental values for the neutron energies in the 1s and 1p state are now smaller than those of tables 1 and 2. The differences are however of the order of ≈10% or less.

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