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ON A RESPONSE FUNCTION AND TWO-BODY DENSITY MATRIX OF NUCLEAR MATTER*

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Abstract

We present a summary of on-going calculations that address the static and dynamic structure of nuclear matter. Specific projects include (i) evaluation of the density-density response function and corresponding dynamic structure factor, based on the correlated random-phase approximation ($CRPA_I$) and generalizations of this method, and (ii) low-order variational calculation of the reduced two-body density matrix and corresponding generalized momentum distribution. The numerical applications involve the model interaction V2.

1. Introduction

In this paper, we discuss *ab initio* microscopic calculations of properties of nuclear matter that are connected directly or indirectly with its dynamical behavior. We take into consideration the strong short-range dynamical correlations induced by the nucleon-nucleon

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interaction. Our physical picture is the conventional one, namely nonrelativistic point nucleons interacting via two– or three-body potentials. The theoretical approach is correlated basis function theory (CBF) [1,2] or its approximation, the variational theory [3,4]. CBF was originally introduced by the Washington University group in the late 50's or early 60's. It is considered one of the most successful many-body theories for the treatment of strongly correlated systems and it has been applied to diverse systems: nuclear matter, quantum liquids, mixtures, fluid surfaces, quantum solids, *etc.* The repertoire of many body theories of strongly interacting systems involves, besides CBF, ordinary perturbative approaches (parquet formalism, self-consistent Green's function approach), the coupled-cluster theory with conceptual ties both to CBF and ordinary perturbation methods, and computationally intensive stochastic methods (Green's function Monte Carlo and path-integral Monte Carlo).

In the case of Fermi systems, CBF theory is formulated by introducing a nonorthogonal basis of normalized correlated functions $|\psi_m\rangle = F|\phi_m\rangle\langle\phi_m|F^\dagger F|\phi_m\rangle^{-1/2}$. The $|\phi_m\rangle$ are model functions that embody the statistics, symmetries and possible long-range order of the system (plane-wave Slater determinants in the case of infinite systems), while F is a symmetrical, translationally invariant correlation operator which has the task of incorporating the essential dynamical correlations generated by the strong interactions. Having decided upon a correlation operator and model functions, one faces two technical problems [2].

1. Construction of physical quantities or approximations to these in terms of the matrix elements H_{mn}, N_{mn} of the given Hamiltonian and of the unit operator in the correlated basis.

2. Evaluation of the required matrix elements or the physically relevant combinations of them.

Options for (1) include (among many possibilities)

1.1 Variational treatment in terms of the "ground-state" base ket $|\psi_o\rangle$ which is determined by the static variational principle (variational method).

1.2 Correlated-basis perturbation theory.

1.3 Correlated Hartree-Fock theory.

1.4 Correlated random-phase approximation.

Options for (2) consist of

2.1 Cluster expansion in the number of mutually correlated bodies or in a dynamical correlation bond.

2.2 Diagrammatic analysis of cluster expansions and resummation of classes of diagrams via integral equations (notably, the Fermi-hypernetted chain equations).

2.3 Monte-Carlo or stochastic integration.

CBF can be interpreted as a means for transforming the problem of bare particles in strong interaction to a problem of dressed particles interacting weakly through effective potentials. The nonorthogonal CBF approach has been mainly used with correlation factors of Jastrow type, $F_J = \prod_{i<j} f(r_{ij})$ (wherein f(r) is a central, state independent two-body correlation function), and the effect of the other correlations (three-body, spin-isospin dependent, *etc.*) has been taken into account by CBF perturbative corrections to the Jastrow variational result. An orthogonalized version of CBF has also been formulated [5]. Initially, CBF was developed for calculating ground state properties and especially for improving the results obtained from the variational method. Subsequently it has been extended (through developments that continue) to describe excited states and dynamical behavior, as well as behavior at finite temperatures and superfluid phases. CBF has been primarily applied to uniform infinite systems, but there is a growing body of applications to nonuniform systems and to finite systems.

Regarding the nuclear problems that have been studied, variational-CBF calculations have been carried out mainly for the ground state properties of (infinite) nuclear matter including the ground state energy per particle and the equation of state at zero temperature, the radial distribution function $g(r_{12})$ and the static structure factor S(q), the average momentum distribution n(p), *etc.* However, serious efforts are now being devoted to the description of elementary excitations and dynamical behavior. Some results are also available

for the static and dynamic properties of finite nuclei based on a number of restrictive approximations. Rigorous CBF calculations in finite nuclei are still at an early stage [6,7].

In Sect. II, we discuss CBF calculations of the density-density response function. Section III takes up the variational calculation of the two-body density matrix of nuclear matter. In Sec. IV, we make some general observations on these calculations and indicate further investigations.

2. Density-Density Response Function and Dynamic Structure Factor: Calculations with $CRPA_I$ and Beyond

In this section we briefly survey our calculations of the density-density response function $\Pi(q,\omega)$ and corresponding dynamic structure factor $S(q,\omega)$ [8] of nuclear matter with the correlated random-phase approximation ($CRPA_I$) and generalizations of it (see also Ref. [9]). $CRPA_I$ is the extension of the usual 1p–1h random-phase approximation (RPA_I) within the context of CBF theory [6,10].

Such calculations are interesting mainly for the following two reasons. First, a microscopic evaluation of $\Pi(q,\omega)$ for nuclear matter, together with a consistent evaluation of the self-energy $\Sigma(p,E)$, furnishes important information about the elementary excitations of nuclear systems. The nature of single-particle excitations is revealed by $\Sigma(p,E)$, from which one may derive an energy-dependent effective mass m^*. The properties of collective modes, typified by the zero sound dispersion relation, may be extracted from $\Pi(q,\omega)$. En route, one gets valuable information about the particle-hole interaction. Second, the quantities $\Pi(q,\omega)$ and $S(q,\omega)$ of extended nuclear matter are related to the longitudinal response function of medium and heavy nuclei, $R_L(q,\omega)$, which is measured in inclusive (e,e') scattering experiments in the quasielastic energy regime [11,9]. The data obtained so far show a quenching with respect to values derived by mean-field calculations. It is evident that before one can definitely conclude that effects beyond those implied by the conventional nuclear picture (*e.g.*, meson currents, swelling of nucleons, *etc.*) have come into play, one must perform the conventional calculation as completely as possible by including many-body

effects of increasing complexity. Our calculations supplement the other microscopic calculations of $S(q,\omega)$ [12-14] by providing information on the contribution to the response from RPA-type excitations of the correlated medium.

The $CRPA_I$ equations [6,10] have been derived by applying the Kerman-Koonin least-action principle [15]

$$\delta S = 0 \ , \quad S = \int_{t_1}^{t_2} L[\psi(t),\psi^*(t)]dt \ , \quad L = \langle\psi(t)|i\hbar\frac{\partial}{\partial t} - H|\psi(t)\rangle \tag{1}$$

on the set of correlated basis functions

$$|\psi(t)\rangle = F|\phi(t)\rangle\langle\phi(t)|F^\dagger F|\phi(t)\rangle^{-\frac{1}{2}} \tag{2}$$

In these expressions F is a suitable correlation operator, *e.g.*, of Feenberg or Jastrow type, and $|\phi(t)\rangle$ is a Slater determinant of single particle states ($= e^{\sum c_{ph}(t)a_p^\dagger a_h}|\phi_o\rangle$, where $|\phi_o\rangle$ is the ground-state Slater determinant). The operator F is kept independent of time and equal to that determined for the ground state. By linearizing the equations for $c_{ph}(t)$ and $c_{ph}^*(t)$ resulting from the least-action principle and assuming periodic solutions (*e.g.*, $c_{ph}(t) = x_{ph}\, e^{-i\omega t} + y_{ph}^*\, e^{i\omega t}$), one obtains the $CRPA_I$ equations

$$\begin{bmatrix} A & B \\ B^* & A^* \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \hbar\omega\begin{bmatrix} M & 0 \\ 0 & -M^* \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} \tag{3}$$

Here, X and Y are column matrices and A, B and M are square matrices whose elements carry particle-hole(p-h) labels (*e.g.*, $X = (x_{ph})$ and $A = (A_{ph;p'h'})$). The matrices A and B (respectively Hermitian and symmetric) are constructed in terms of the CBF effective interaction vertex V(12) and CBF single-particle energies e(p) and e(h) assigned to particles and holes, while the metric matrix M (Hermitian), which appears due to the nonorthogonality of the correlated basis, is constructed in terms of the nonorthogonality vertex N(12) [2]. Explicitly,

$$A_{ph;p'h'} = [e(p)-e(h)]\delta_{pp'}\delta_{hh'} + \langle ph'|V(12)|hp'\rangle_a \ ,$$

$$B_{ph;p'h'} = \langle pp'|V(12)|hh'\rangle_a \quad ,$$

$$M_{ph;p'h'} = \delta_{pp'}\delta_{hh'} + \langle ph'|N(12)|hp'\rangle_a \quad , \tag{4}$$

where V(12) is in turn determined by W(12) (the CBF interaction vertex), N(12), and e(k). At nuclear densities these CBF ingredients may be calculated rather accurately by Fermi-hypernetted chain (FHNC) procedures in the case of the Jastrow choice F_J. By solving eq. (3) one obtains the excitation energies $\hbar\omega_n$ and the amplitudes $x_{ph}^{(n)*}$, $y_{ph}^{(n)*}$. One can then construct $\Pi(q,\omega)$ and $S(q,\omega)$ [10]. For example, the latter is given by

$$S(q,\omega) = \frac{1}{\hbar A} \sum_{\omega_n>0} |\sum_h (\bar{x}_{h+q,h}^{(n)*} + \bar{y}_{h-q,h}^{(n)*})|^2 \, \delta(\omega-\omega_n) \quad , \tag{5}$$

where

$$\bar{x}_{h+q,h}^{(n)} = \sum_{h'} M_{h+q,h;h'+q,h'} x_{h'+q,h'}^{(n)} \tag{6}$$

and similarly for $\bar{y}_{h-q,h}^{(n)}$. It is important to note that the particle-hole interaction U that is generated by the formalism is energy dependent.

The $CRPA_I$ eqs. (3)-(4) as formulated in Ref. [10] differ from those of ordinary RPA_I because of the energy dependence that has been introduced by the nonorthogonality of the correlated basis. Krotscheck [16] has derived $CRPA_I$ equations in a form similar to the usual RPA_I equations by rewriting the theory in terms of a p–h irreducible p–h interaction U that shows only a slight energy dependence. Due to the difficulty involved in complete numerical solution of the $CRPA_I$ equations (in either formulation), Krotscheck also proposed a simple approximation (the local correlated random-phase approximation, denoted LCRPA) in which a local particle-hole interaction U(q) (function of the momentum transfer $q = |p-h|$) is constructed from CBF matrix elements and is used to express $\Pi(q,\omega)$ and $S(q,\omega)$ by formulas identical to those of ordinary RPA_I. LCRPA has been used for the approximate calculation of $\Pi(q,\omega)$ and $S(q,\omega)$ of nuclear matter based on hard-sphere and V2 model potentials [17,18]. LCRPA has the advantage of easy application and it correctly reproduces some qualitative trends. However, it has shortcomings, mainly at low q, due to the omission of nonlocalities.

Therefore we have returned to the problem of numerical solution of the $CRPA_I$ equations (3), exploiting the method which has been developed by Kwong [19,20]. First, the method has been developed to solve the HF/RPA_I equations for the infinitely extended nuclear medium using schematic finite-range, density dependent and simplified meson-exchange varieties of nuclear interaction. Due to the existing symmetries in the system, one can define $\mathbf{q} = \mathbf{p}-\mathbf{h} = \mathbf{h}'-\mathbf{p}'$ and $a_h = \cos_{h,q}$ and express x_{ph} *etc.* in terms of q and a_h. For a given q, this reduces the original paired six-dimensional integral equations to two-dimensional ones. The solution then proceeds by partial-waving the various Fourier transforms involved and imposing Legendre-Gaussian quadrature on the double mesh. At a given k_F, for each q considered, diagonalization of a $2N \times 2N$ matrix yields N distinct eigenvalues ω_n, $n = 1,...,N$ and the associate amplitudes $X^{(n)}$ and $Y^{(n)}$. The dynamic structure factor is then calculated by eq. (5). However, due to the discretization imposed in solving the relevant integral equations, the measure of the smooth part of the dynamic form factor is drawn into artificial poles. A smoothing procedure is followed, after removing possible collective modes present, to recover the continuum of $S(q,\omega)$. This strategy seeks the best compromise between reliability and computing time. The fact that $M \neq 1$ in $CRPA_I$ leads to an additional diagonalization. The different matrix elements involved in eqs. (3) and (4) can be evaluated to two levels of accuracy, namely (i) two-body cluster order and (ii) FHNC resummation.

This method has been previously employed in a calculation of $S(q,\omega)$ of nuclear matter at low values of the momentum transfer q, based on the OMY-6 test potential [20]. With a view towards more realistic NN interactions, our present calculations have begun with the model potential V2 [21], which consists of the central part of the $^3S_1-^3D_1$ component of the Reid soft-core interaction, assumed to act in all partial waves. The momentum transfer q ranges from 0 to about 4 fm^{-1} and the density considered is near saturation ($\rho = 0.182$ fm^{-3}). For the correlation function in the Jastrow pair product assumed for F, we use the parametrized form

$$f(r) = \exp[-C_1 e^{-C_2 r}(1-e^{-r/C_3})/r] \qquad (7)$$

with parameters determined by Ceperley *et al.* [22]. Our aim is to explore and improve the method of solution before applying it to more realistic interactions, and also to compare with the results of the corresponding LCRPA calculation [18].

In addition to the refinement of methods for the numerical solution of $CRPA_I$, we are investigating a number of possibilities for generalizations of $CRPA_I$ that incorporate correlated multipair effects in the description of elementary excitations and dynamical response. In $CRPA_I$ the correlation factor is kept time independent, and is taken the same as that determined (*e.g.*, variationally) for the ground state. This approximation is expected to be valid when the wavelength of the excitations is long compared to the range of the correlations. In our "CFRPA" treatment we generalize $CRPA_I$ by considering a time-dependent correlation factor F(t). Pursuing this generalization, it will be interesting to see how the short-range correlations in the excited states differ from those of the ground state, and whether the high-momentum excitations involved in F are "orthogonal" to the low momentum excitations introduced by $CRPA_I$. Time-dependent correlation factors have been considered by Saarela and Suominen [23] in a study of elementary excitations of liquid ^{4}He. Also Mädler [24] has considered time and angular momentum-dependent two-body, correlation functions in a low cluster order application in finite nuclei. It would be worthwhile to implement the latter approach for the simpler case of nuclear matter.

The derivation of the CFRPA equations is similar to the derivation of the $CRPA_I$ equations, again being based on the least-action principle (1). The variation of the Slater-determinant part $\phi(t)$ is treated exactly in the same way. If, in addition, one assumes that the correlation factor F(t) is Hermitian and that $[F(t),\dot{F}(t)] = 0$ (as is the case for example of F_J) one ends up with a set of coupled nonlinear equations (the CFRPA equations). The equations resulting from variation of the model function $\phi(t)$ have the same form as $CRPA_I$ (eq. (3)) but F is time dependent. The remaining equations amount to Euler-Lagrange equations for the time-dependent correlations in F(t). We are currently examining different options for explication of the latter equations and for solving the full set of CFRPA equations.

An alternative approach to a CFRPA treatment specializes to time-dependent Jastrow-type excitations at the outset, expressed in terms of density-fluctuation operators.

3. Variational Calculation of the Reduced Two-Body Density Matrix and Corresponding Generalized Momentum Distribution

In this section we describe our calculation of the reduced two-body density matrix (and the corresponding generalized momentum distribution) of the ground state of nuclear matter. The complete two-body density matrix of a system of A particles is defined by [1]

$$\rho_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1,\mathbf{r}'_2) = A(A-1)\int\psi^*(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3...)\psi(\mathbf{r}'_1,\mathbf{r}'_2,\mathbf{r}_3...)d\mathbf{r}_3 \cdots d\mathbf{r}_A \quad , \tag{8}$$

while the half-diagonal version is given by

$$\tilde{\rho}_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1) = \rho_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1,\mathbf{r}_2) \quad . \tag{9}$$

Here ψ is the normalized wave function of the ground state. The corresponding generalized momentum distributions $n(\mathbf{p},\mathbf{q},\mathbf{Q})$ and $n(\mathbf{p},\mathbf{Q})$ are defined as Fourier transforms of these quantities:

$$n(\mathbf{p},\mathbf{q},\mathbf{Q}) = \frac{\rho^2}{A^2}\int\rho_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1,\mathbf{r}'_2)e^{-i\mathbf{p}\cdot(\mathbf{r}_1-\mathbf{r}'_1)}\, e^{-i\mathbf{q}\cdot(\mathbf{r}_2-\mathbf{r}'_2)}\, e^{-i\mathbf{Q}\cdot(\mathbf{r}'_1-\mathbf{r}_2)}\, d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}'_1 d\mathbf{r}'_2 \tag{10}$$

and

$$n(\mathbf{p},\mathbf{Q}) = n(\mathbf{p},\mathbf{q}=0,\mathbf{Q}) \quad . \tag{11}$$

A microscopic calculation of the properties (8)-(11) in nuclear matter will provide us with far more complete information on the correlation structure of the nuclear medium than that already extracted from the calculation of the diagonal part of the two-body density matrix (which gives the radial distribution function $g(r_{12})$). In addition, one can obtain information on multiparticle-multihole excitations through the ω^3-sum rule [1,25]. Moreover, $\tilde{\rho}_2$ is an essential input of theories of final-state interactions in quasi-elastic electron nucleus scattering [26]. Final-state interactions are known to persist even to high momentum transfer q, and the impulse approximation [27] is only approximately valid. In order to extract

reliable information on the momentum distribution of the nucleons n(p) from data in the high q quasielastic regime, one has to calculate corrections to the impulse approximation. It is in these corrections that $\tilde{\rho}_2$ appears in one way or another. This can be shown schematically if we write n(**p**,**Q**) as

$$n(\mathbf{p},\mathbf{Q}) = \sum_{\hat{q}} \langle\psi| a^{\dagger}_{\hat{q}+\hat{Q}}\, a^{\dagger}_{\hat{p}-\hat{Q}}\, a_{\hat{p}} a_{\hat{q}} |\psi\rangle \qquad (12)$$

(where $\hat{p}-\hat{Q} = (\mathbf{p}-\mathbf{Q},\sigma)$ and $\hat{q}+\hat{Q} = (\mathbf{q}+\mathbf{Q},\sigma')$ and the sum is over all momenta $\hbar\mathbf{q}$ and spin projections σ' of orbital $\hat{q}$) and insert the density fluctuation operator $\rho_{\hat{Q}} = \sum_{\hat{q}} a^{\dagger}_{\hat{q}+\hat{Q}} a_{\hat{q}}$ $(\hat{Q} \neq 0)$

$$n(\mathbf{p},\mathbf{Q}) = \langle\psi|\rho_{\hat{Q}} a^{\dagger}_{\hat{p}-\hat{Q}} a_{\hat{p}}|\psi\rangle - n(\mathbf{p}) \quad . \qquad (13)$$

The expectation value in (13) can be interpreted as a transition matrix element for scattering of a particle out of orbital $\hat{p}$ to another orbital $\hat{p}-\hat{Q}$ (without spin-flip), the process being mediated by a (spin-independent) density fluctuation (phonon). This transition corresponds to the final-state scattering process that occurs at the right-most vertex of Fig. 1. The latter is one of the many scattering processes that are expected to contribute to final-state effects in quasi-elastic electron-nucleus scattering.

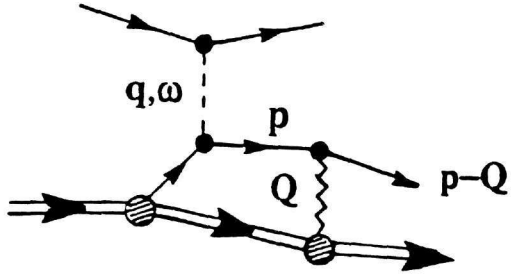

Fig. 1. Inelastic scattering process involving final-state interactions mediated by a density fluctuation (a phonon of wave vector **Q**).

The one-body density matrix $\rho_1(\mathbf{r}_1,\mathbf{r}'_1)$ and its Fourier transform, the momentum distribution n(p), have been studied extensively [28-30]. Recently, microscopic analyses of the two-body density matrices of Bose [31] and Fermi [32] systems (aimed respectively at liquid ^{4}He and liquid ^{3}He) have been carried out within the variational method. We use the

same formalism for our calculation of $\bar{\rho}_2$ and $n(\mathbf{p},\mathbf{Q})$ for the ground state of nuclear matter [33]. Assuming a Jastrow wavefunction ψ_J ($=F_J\phi_0$), implementing a suitable cluster expansion of $n(\mathbf{p},\mathbf{Q})$ and going to the thermodynamic limit ($A \rightarrow \infty$, ρ finite), one obtains an infinite series of cluster terms

$$n(\mathbf{p},\mathbf{Q}) = n_0(\mathbf{p},\mathbf{Q}) + (1-\delta_{Q0})[n_{(2)}(\mathbf{p},\mathbf{Q}) + n_{(3)}(\mathbf{p},\mathbf{Q}) + ...] \quad . \tag{14}$$

The leading term $n_0(\mathbf{p},\mathbf{Q})$, given by

$$n_0(\mathbf{p},\mathbf{Q}) = \delta_{Q0}(A-1)n(p) \quad , \tag{15}$$

refers to uncorrelated bosons (except that the momentum distribution $n(p)$ must be calculated for the correlated Fermi system). In our calculation, which is carried out in lowest cluster order, we approximate $n(\mathbf{p},\mathbf{Q})$ by

$$n(\mathbf{p},\mathbf{Q}) = n_0(\mathbf{p},\mathbf{Q}) + (1-\delta_{Q0})n_{(2)}(\mathbf{p},\mathbf{Q}) \quad , \tag{16}$$

where $n_{(2)}(\mathbf{p},\mathbf{Q})$ is expressed as a sum of seven two-body cluster terms of the form

$$n_{(2)}(\mathbf{p},\mathbf{Q}) = \frac{\rho^3}{v}\,\frac{1}{A}\sum_{i=1}^{7}\int \bar{\rho}^{(i)}_{2(2)}\,(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1)e^{-i\mathbf{p}\cdot(\mathbf{r}_1-\mathbf{r}'_1)-i\mathbf{Q}\cdot(\mathbf{r}'_1-\mathbf{r}_2)}d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}'_1 \quad . \tag{17}$$

(here v is the spin degeneracy of each single particle state, equal to 4 in the case of nuclear matter). The factor in the integrand, $\bar{\rho}^{(i)}_{2(2)}$, is given by one of the following expressions:

(17a) $l(k_F r_{11'})\zeta(r_{12})$; (17b) $l(k_F r_{11'})\zeta(r_{1'2})$; (17c) $l(k_F r_{11'})\zeta(r_{12})\zeta(r_{1'2})$;

(17d) $-l(k_F r_{12})l(k_F r_{1'2})$; (17e) $-l(k_F r_{12})\zeta(r_{1'2})l(k_F r_{1'2})$; (17f) $-l(k_F r_{12})l(k_F r_{1'2})\zeta(r_{12})$;

(17g) $-l(k_F r_{12})\zeta(r_{12})l(k_F r_{1'2})\zeta(r_{1'2})$;

where $l(x)$ is the Slater function $3j_1(x)/x$ and $\zeta(r_{ij}) = f(r_{ij})-1$. The seven two-body cluster contributions are represented graphically in Fig. 2 by Ursell-Mayer diagrams. (For the precise diagrammatic rules, see in the Appendix of Ref. [32].)

One can attribute a physical significance to these terms by observing that they coincide with the lowest-order terms of some of the quantities that enter the FHNC expression of $n(\mathbf{p},\mathbf{Q})$ [32]. The latter expression achieves a separation of contributions from various

scattering processes that contribute to the generalized momentum distribution n(**p**,**Q**). In particular, the term $n_o(\mathbf{p},\mathbf{Q})$ comes from the scattering of dynamically and kinematically uncorrelated nucleons. The terms (17a) and (17b) describe, in lowest order, the scattering of a nucleon from orbital $\hat{\mathbf{p}}$ to another orbital $\hat{\mathbf{p}}-\hat{\mathbf{Q}}$ with the intervention of a phonon to conserve momentum, and the corresponding time-reversed process. The terms ((17d)-(17g)) describe the scattering of kinematically correlated nucleons, which, due to their dynamical correlations, can populate states even above the Fermi sea. Finally, the term (17c) is the leading example of "higher order" terms that are supposed to correct the various processes just considered (*cf.* Ref. [32]).

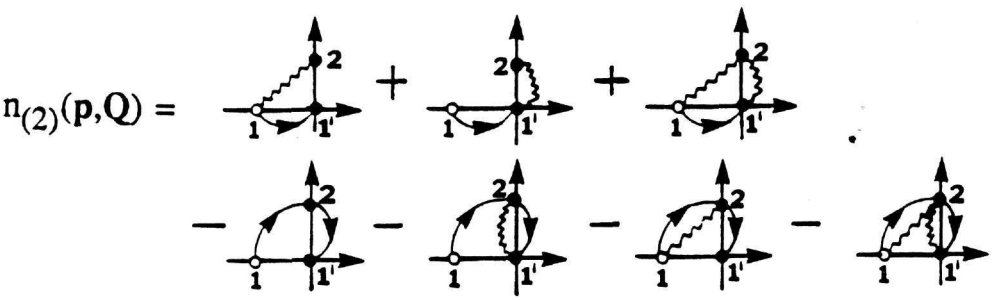

Fig. 2. Diagrammatic representation of the two-body cluster contributions to the generalized momentum distribution function n(**p**,**Q**).

In our calculation, which is in progress, we are again using (as in Sec. 2) the model interaction V2 [21] and the correlation function (7) for $\rho = 0.182$ fm^{-3}.

4. Discussion-Outlook

In the above, we have reported on calculations being undertaken for the density-density response function $\Pi(\mathbf{q},\omega)$ and the corresponding dynamic structure factor $S(\mathbf{q},\omega)$, using $CRPA_I$ and CFRPA approaches. We have also discussed a calculation of the reduced two-body density matrix $\tilde{\rho}_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1)$ and corresponding generalized momentum distribution function n(**p**,**Q**), based on a lowest-order variational method. Both investigations are carried out for the limiting case of infinite nuclear matter. The numerical calculations are performed

with the model interaction V2 as input.

$CRPA_I$ and CFRPA techniques for the evaluation of the response functions have the attractive feature of incorporating long-range as well as short-range correlations. Accordingly they provide information that complements results from other methods [12-14]. The lowest-order variational calculation of $\tilde{\rho}_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1)$ and $n(\mathbf{p},\mathbf{Q})$ is of special interest in that no quantitative treatment of these quantities exists for the nuclear case, to the best of our knowledge. There are only approximate expressions in the literature, which have not been evaluated for nuclear matter [26,32].

We plan to extend our calculations in the directions of (i) improved techniques, (ii) improved interactions and correlation functions, (iii) finite nuclei and (iv) other quantities characterizing nuclear response.

Regarding the improvement of the techniques, the calculation of $\Pi(q,\omega)$ and $S(q,\omega)$ using CFRPA-type equations can evidently be approached by several different paths, depending on how the equation for the time-dependent correlation function f(r,t) (or radial distribution function g(r,t)) is formulated and solved. One has to investigate these paths and find the most efficient one. In evaluating $\tilde{\rho}_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1)$ and $n(\mathbf{p},\mathbf{Q})$, the obvious next step is a numerical FHNC calculation [32]. If it proves necessary, one can also proceed to a CBF perturbative treatment of these quantities, as has been done in the case of $\rho_1(\mathbf{r}_1,\mathbf{r}'_1)$ and n(p) [29,34,35].

Regarding the introduction of more realistic interactions and correlations, we make the following comments. The V2 potential is a simple model interaction that has been used by numerous authors, mainly for the purpose of comparing different many-body methods. However, a realistic description of nuclear matter requires the use of interactions of at least V6 type with spin-isospin and tensor parts, in conjunction with suitable state-dependent correlation functions [4]. We plan first to use the $CRPA_I$ code with V4 interactions (as was done for few cases in Ref. [20]), and then proceed to the V6 case. A similar generalization of the calculation of $\tilde{\rho}_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1)$ and $n(\mathbf{p},\mathbf{Q})$ is planned.

The calculations discussed so far refer to nuclear matter. A crude estimation of the corresponding properties of finite nuclei can be obtained by simply identifying the nuclear matter quantities with those of medium-to-heavy nuclei. However, one can proceed and calculate relevant quantities of finite nuclei (for example the particle-hole interaction and the generalized momentum distribution $n(\mathbf{p},\mathbf{Q})$) by inserting the nuclear matter results in a kind of a local density approximation. Such an approach has been followed, for example, in calculations of the odd parity states of ^{16}O and ^{40}Ca (Ref. [36]) and in estimating the momentum distribution n(p) of finite nuclei (Ref. [37]). It may even be possible, *albeit* with enormous efforts, to extend the formalism to allow rigorous (*ab initio*) calculations of the aforementioned quantities for finite nuclei.

Finally, we should mention that the procedures we follow in the calculation of $\Pi(q,\omega)$, $S(\mathbf{q},\omega)$ and of $\tilde{\rho}_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1)$, $n(\mathbf{p},\mathbf{Q})$ may be generalized for the calculation of other response functions and dynamical structure functions (longitudinal, spin-isospin) and of the full two-body density matrix $\rho_2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}'_1,\mathbf{r}'_2)$ and corresponding generalized momentum distribution $n(\mathbf{p},\mathbf{q},\mathbf{Q})$.

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References

[1] E. Feenberg, *Theory of Quantum Fluids* (Academic Press, New York, 1969).

[2] J. W. Clark and E. Krotscheck, in *Recent Progress in Many Body Theories*, eds. H. Kummel and M. L. Ristig (Springer-Verlag, Berlin, 1984), p. 127.

[3] J. W. Clark, in *Progress in Particle and Nuclear Physics*, ed. D. H. Wilkinson (Pergamon, Oxford, 1979), Vol. 2, p. 89.

[4] V. R. Pandharipande and R. B. Wiringa, Rev. Mod. Phys. **51** (1979) 821.

[5] S. Fantoni and V. R. Pandharipande, Phys. Rev. C **37** (1988) 1697.

[6] J. W. Clark, in *The Many Body Problem, Jastrow Correlations versus Brueckner Theory*, eds. R. Guardiola and J. Ros (Springer-Verlag, Berlin, 1981), p. 184.

[7] R. B. Wiringa, S. C. Pieper, and V. R. Pandharipande, in *Recent Progress in Many Body Theories*, eds. T. L. Ainsworth and B. E. Clements (Plenum Press, New York) to be published.

[8] A. Fetter and J. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[9] E. Mavrommatis and J. W. Clark, in *Proceedings of the First Hellenic Symposium on Theoretical Nuclear Physics 1990*, eds. M. E. Grypeos *et al.* (University of Thessaloniki, 1991), p. 11.

[10] J. M. Chen, J. W. Clark and D. G. Sandler, Zeits. Physik **A305** (1982) 223.

[11] I Sick, in *Progress in Electron-Nucleus Scattering*, ed. A. Fabrocini, *et al.* (World Scientific, Singapore, 1989), p. 552.

[12] M. Butler and S. Koonin, Phys. Letters **B205** (1988) 123.

[13] S. Fantoni and V. R. Pandharipande, Nucl. Phys. **A473** (1987) 234; A. Fabrocini and S. Fantoni, Nucl. Phys. **A503** (1989) 375.

[14] A. S. Rinat and W. H. Dickhoff, Phys. Rev. B **42** (1990) 10004.

[15] A. K. Kerman and S. E. Koonin, Ann. Phys. **100** (1976) 332.

[16] E. Krotscheck, Phys. Rev. A **26** (1982) 3536.

[17] E. Mavrommatis, R. Davé and J. W. Clark, in *Condensed Matter Theories*, eds. P. Vashishta, R. K. Kalia and R. F. Bishop (Plenum Press, New York, 1987), Vol. 2, p. 249.

[18] E. Mavrommatis and J. W. Clark, in *Condensed Matter Theories*, ed. V. C. Aguilera-Navarro (Plenum Press, New York, 1990), Vol. 5, p. 97.

[19] N. H. Kwong, Ph.D. Thesis, California Institute of Technology (1983) (unpublished).

[20] D. Sandler and N. H. Kwong, Phys. Letters **B136** (1984) 1.

[21] V. R. Pandharipande, R. B. Wiringa, and B. D. Day, Phys. Letters **57B** (1975) 205.

[22] D. Ceperley, G. V. Chester, and M. H. Kalos, Phys. Rev. B **16** (1977) 3081.

[23] M. Saarela and J. Suominen, in *Condensed Matter Theories*, ed. J. Keller (Plenum Press, New York, 1989), Vol. 4, p. 377.

[24] P. Mädler, Nucl. Phys. **A404** (1983) 58.

[25] F. Dalfovo and S. Stringari, Phys. Rev. Lett. **63** (1989) 532.

[26] J. W. Clark and R. N. Silver, in *Proceedings of the Third International Conference on Nuclear Reaction Mechanisms*, ed. E. Gadioli (Universite degli Studi di Milano, 1988), p. 531 and references therein.

[27] P. C. Hohenberg and P. M. Platzman, Phys. Rev. **152** (1966) 198.

[28] J. W. Clark and M. L. Ristig, in *Momentum Distributions*, eds. R. N. Silver and P. E. Sokol (Plenum Press, New York, 1989), p. 39 and references therein.

[29] O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. **A505** (1989) 267; Phys. Rev. C **41** (1990) 24.

[30] B. E. Vonderfecht, W. H. Dickhoff, A. Polls, and A. Ramos, Phys. Rev. C **44** (1991) R1265.

[31] J. W. Clark and M. L. Ristig, Phys. Rev. B **40** (1989) 4355.

[32] J. W. Clark and M. L. Ristig, Phys. Rev. B **41** (1990) 8811.

[33] We use here **q** and **Q** instead of **k** and **q** of references [31], [32].

[34] S. Fantoni and V. R. Pandharipande, Nucl. Phys. **A427** (1984) 473.

[35] M. Flynn, Ph.D. Thesis, Washington University (1986) (unpublished).

[36] J. W. Clark, E. Krotscheck and B. Schwesinger, Phys. Lett. **143B** (1984) 287.

[37] S. Stringari, M. Traini, and O. Bohigas, Nucl. Phys. **A516** (1990) 33.