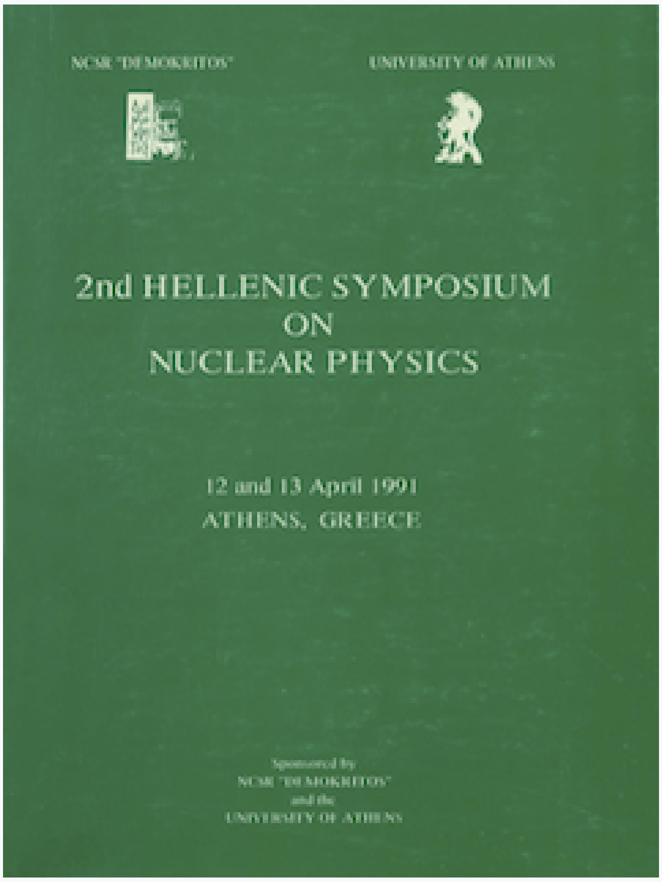
Annual Symposium of the Hellenic Nuclear Physics Society

Τόμ. 2 (1991)

HNPS1991



FRACTAL GEOMETRY AND QUARK-GLUON PLASMA PHYSICS

N. G. Antoniou

doi: 10.12681/hnps.2839

FRACTAL GEOMETRY AND QUARK-GLUON PLASMA PHYSICS

N.G.ANTONIOU

Department of Physics, University of Athens
GR-15771 Athens, Greece

Abstract

Incorporating fractal geometry in the Regge-Mueller approach to strong interaction dynamics one may formulate a model for the one-dimensional critical sector of the hadronic S-matrix in a high energy collision. A non conventional component of the correlation functions in rapidity space is obtained. the phenomenological implications of which are related with the intermittency effects in quark-gluon plasma physics.

1. Introduction

Recent developments in strong interaction dynamics suggest that the S-matrix of the hadronic system in a high-energy collision is likely to have a non conventional component when the energy deposited during the development of the collision has a sufficiently high density [1]. This component describes a critical system in a quark-hadron phase transition process (at $T = T_c$) and gives rise to intermittency effects in multihadron configurations in rapidity space [2]. Physically one may consider the totality of the hadronic events as a superposition of two components : (a) the normal hadronic fluid with short range correlations in rapidity, originated by conventional excitations of the initial hadrons through the exchange of standard Regge singularities and (b) the critical hadronic fluid (critical Feynman-Wilson (FW) fluid) which is generated by hadronization in a quark-hadron phase transition process and is characterized by long range correlations (infinite range at $T = T_c$) in rapidity space.

Phenomenologically, this picture resembles the two-component model for superfluidity which was advocated during the early studies of this phenomenon and was based on the assumption that the density of a fluid ρ is the superposition of a normal component ρ_n and a superfluid one ρ_s ($\rho = \rho_n + \rho_s$) which is related with new microscopic processes [3]. Similarly, the critical component of the hadronic fluid is expected to have a direct connection with non perturbative QCD phenomena at the quark level (microscopic level) leading to specific signatures for the quark-hadron phase transition at the level of the hadronic S-matrix (FW fluid).

In order to specify this complementarity and quantify the connection of the critical S-matrix in rapidity space with the non perturbative sector of QCD near the critical temperature, we propose to explore the consequences of the fractal structure found recently in a study of a simplified gauge theory on a lattice, in the deconfinement region and near the transition temperature. In fact it was shown in a Monte Carlo calculation of the $SU(2)$ gauge model [4] that the dominant configurations in the three dimensional space of the deconfinement region, near $T = T_c$, form a fractal structure with $D_F \simeq 2$ (fractal dimension). Although a confirmation of this result for a realistic QCD on a lattice is still missing, we consider this structure as a fundamental property of the non perturbative QCD sector relying upon the fact that such a geometry is expected to be develoved in the interface of any critical system as a universal property [5]. Quantitatively, one expects a strong dependence of the fractal dimension D_F on the detailed structure of the underlying gauge theory and for the realistic case (QCD with $SU(3)$ gauge group combined with correctly flavoured quarks) we make the crucial assumption of maximal fractality, $0 < D_F < 1$, which may have severe phenomenological implications. Geometrically this assumption implies that the projection of the fractal structure under consideration onto any one-dimensional Euclidean space is a Cantor like set with the same fractal dimension D_F [6]. In what follows we argue that this remarkable behaviour at the quark level of the strongly interacting system, near the transition temperature, may have important consquences for the critical component of the S-matrix in the hadronic phase ($T = T_c$) leading to measurable effects. For this purpose we consider a high-energy density collision along the z axis in which a quark-gluon plasma state is generated through the excitation of the vacuum. This system expands along the space-time hyperbolas

$$\begin{aligned} z &= \tau c \sinh y \\ t &= \tau \cosh y \end{aligned} \tag{1.1}$$

and for a typical hadronization time-scale $\tau_h \simeq 10\ fms$, a rapidity domain, $\delta y = 1$, covers the range $\delta z \simeq 10\ fms$ of the ordinary one-dimensional z-space. For this rapidity interval the mapping $y \to z$ is approximately linear, $z = c\tau_h y$ ($|y| \leq 0.5$), and therefore the assumed fractal structure in the three-dimensional space, near the transition temperature, inducing a Cantor like set in the one-dimensional z-space, leads to a fractal geometry in the rapidity space with fractal dimension D_F ($0 < D_F < 1$). This geometrical structure may be considered as a characteristic property of the critical S-matrix, imposing, together with analyticity and unitarity, the constraint of self-similarity in rapidity space which coincides with the longitudinal phase space of the collision. This last property distinguishes the critical sector of the S-matrix from the conventional component with standard singularities in the angular momentum plane. In the following sections we study a simple one-dimensional model for the critical S-matrix introducing a fractal measure in the rapidity space. In Section 2 we review the theory of the bare pomeron within the Regge-Mueller approach. In Section 3 the analogue model of the bare pomeron is constructed in the fractal rapidity space preserving the factorizability and the uniformity of the Mueller propagator (fractal pomeron). The properties of this model are investigated and the intermittent behaviour of the produced hadronic system is especially discussed. In Section 4 the two-component model is reviewed and its phenomenological implications are presented. Finally, our conclusions and remarks are given in Section 5.

2. The bare pomeron

We consider multiparticle production in rapidity space generated by a factorizable simple pole in the angular momentum plane exchanged in the Regge-Mueller diagrams (inclusive pomeron). For the inclusive rapidity distributions we have :

$$d\sigma_{in}^{(\kappa)} = g_a g_b g_p^\kappa P_{in}(y_1 - y_a) P_{in}(y_2 - y_1) ... P_{in}(y_b - y_\kappa)\{dy\} \quad (2.1)$$

$$(y_a < y_1 < y_2 < ... < y_\kappa < y_b)$$

where g_a, g_b are the couplings of the inclusive pomeron with the initial hadrons (a, b), g_p is the internal coupling in the Regge-Mueller diagrams and $\{dy\}$ the integration measure in the ordinary rapidity space. $\{dy\} \equiv dy_1 dy_2 ... dy_\kappa$. The propagator $P_{in}(y_\lambda - y_{\lambda-1})$ of the inclusive pomeron is

$$P_{in}(y_\lambda - y_{\lambda-1}) = \exp[\xi_p^{in}(y_\lambda - y_{\lambda-1})] \quad (2.2)$$

where $\xi_p^{in} = \alpha_p^{in} - 1$ (α_p^{in} is the position of the singularity in the angular momentum plane). From eqs.(2.1) and (2.2) we finally obtain :

$$\begin{aligned} d\sigma_{in}^{(\kappa)} &= g_a g_b g_p^\kappa \exp[\xi_p^{in}(y_b - y_a)]\{dy\} \\ \sigma_{ab} &= g_a g_b \exp[\xi_p^{in}(y_b - y_a)] \end{aligned} \quad (2.3)$$

where σ_{ab} is the total cross section of the collision $a + b$. The hadronic fluid generated by the bare pomeron has the behaviour of an ideal system without correlations and, in particular, it has the following properties which are easily extracted from eqs.(2.3) :

(1) The densities $\rho(y_1, y_2, ..., y_\kappa)$ are constant and their level is fixed by the internal coupling g_p as follows :

$$\rho(y_1, y_2, ..., y_\kappa) = \frac{1}{\sigma_{ab}} \frac{d\sigma_{in}^{(\kappa)}}{\{dy\}} = g_p^\kappa \quad (2.4)$$

(2) The average multiplicity $< N >$ is proportional to the size Δ of the system ($\Delta = y_b - y_a$) :

$$< N > = \int_\Delta \rho(y) dy = g_p \Delta \quad (2.5)$$

(3) The multiplicity moments $C_\kappa = < N^\kappa > / < N >^\kappa$ are equal to unity, $C_\kappa = 1$, corresponding to a totaly uncorrelated system with a Poisson multiplicity distribution :

$$P(N, \Delta) = \frac{(g_p \Delta)^N}{N!} e^{-g_p \Delta} \quad (2.6)$$

(4) The generating function $Q(\zeta,\Delta)$, which defines also the grand partition function of the FW fluid, has the following form :

$$Q(\zeta,\Delta)=\sum_{N}P(N,\Delta)\zeta^{N}=\exp[g_{p}\Delta(\zeta-1)] \qquad (2.7)$$

Equation (2.7) corresponds to a perfect thermodynamic system with a normal analogue pressure $p(\zeta)=g_p(\zeta-1)$.

(5) The exclusive pomeron propagator $\tilde{P}_{ex}(\xi)$ in the ξ-plane ($\xi=J-1$) is given by the equation,

$$\tilde{P}_{ex}(\xi)=g_p(\xi-\xi_p^{in}+g_p)^{-1} \qquad (2.8)$$

which is a particular form of the unitarity constraint. Equation (2.8) gives the position $J=\alpha_p^{ex}$ of the singularity (pole) of $\tilde{P}_{ex}(\xi)$ in the J-plane (angular momentum plane) in terms of the position α_p^{in} of the inclusive singularity in the same plane and the coupling g_p as follows :

$$\alpha_p^{in}-\alpha_p^{ex}=g_p \qquad (2.9)$$

Equation (2.9) raises the question of self-consistency for the pomeron singularity which would require $\alpha_p^{in}=\alpha_p^{ex}$. This defect of the one-dimensional bare pomeron model may be eliminated by taking into account higher order corrections, within the framework of the standard Reggeon Field Theory, leading to more complicated solutions for the pomeron singularity. Nevertheless, the simplicity of the above model [7] combined with the phenomenological evidence that the conventional (non critical) hadronic system behaves like a normal fluid with short range correlations, qualify the bare pomeron as the archetype of the non critical FW fluid. In the next section the construction of the fractal analogue of the bare pomeron is presented considering again a simple, factorizable singularity as the leading Regge-Mueller propagator and imposing self-similarity by introducing the appropriate fractal measure in the rapidity space. Our conjecture is that this new system is the archetype of a critical hadronic system generated in a quark-hadron phase transition process.

3. The fractal pomeron

In this section we attempt to study the properties of the S-matrix generated by a self-consistent singularity in the ξ-plane, introducing a fractal measure, $\delta_F y \sim y^{D_F-1}dy$, in the rapidity space. The integral transform which relates the fractal y-space with the ξ-plane is written as follows :

$$\tilde{K}(\xi) = \int_0^\infty e^{-y\xi} K(y) \delta_F y \qquad (3.1)$$

where $K(y)$ is a typical propagator in the y-space (rapidity space). Equation (3.1) shows that the fractal analogue $K^{(F)}(y)$ of the propagator $K(y)$ has the form, $K^{(F)}(y) = y^{D_F-1}K(y)$ and therefore in the analogue bare pomeron model one obtains the corresponding inclusive propagators as follows :

$$P_{in}(y_\lambda - y_{\lambda-1}) = \exp[\xi_0(y_\lambda - y_{\lambda-1})] \qquad (3.2)$$

$$P_{in}^{(F)}(y_\lambda - y_{\lambda-1}) = G(D_F, \delta_0)|y_\lambda - y_{\lambda-1}|^{D_F-1} \exp[\xi_0(y_\lambda - y_{\lambda-1})] \qquad (3.3)$$

$$(|y_\lambda - y_{\lambda-1}| \gg \delta_0)$$

where G is a constant and δ_0 a characteristic scale in the rapidity space. One notices that the fractal dimension D_F $(0 < D_F < 1)$ introduces a power law in the propagator $P_{in}^{(F)}(y_\lambda - y_{\lambda-1})$ if the singularity lies at the point $\xi_0 = 0$ (pomeron singularity at $J = 1$). Combined with the factorization property, the above singularity at $J = 1$ leads to the following inclusive distributions :

$$d\sigma_{in,c}^{(\kappa)} = g_a^{(c)} g_b^{(c)} g_F^{(\kappa)} (y_1 + \frac{\Delta_c}{2})^{D_F-1} (\frac{\Delta_c}{2} - y_\kappa)^{D_F-1} \prod_{\lambda=2}^{\kappa} (y_\lambda - y_{\lambda-1})^{D_F-1}$$

$$\sigma_{ab}^{(c)} = g_a^{(c)} g_b^{(c)} \Delta_c^{D_F-1} \qquad (3.4)$$

$$(\Delta_c \gg \delta_0 \ , \ y_i - y_{i-1} \gg \delta_0 \ , \ i = 2, 3, \ldots \kappa)$$

The corresponding correlation functions of the FW fluid satisfy the power laws :

$$< \rho_c(y_1)\rho_c(y_2)\ldots\rho_c(y_\kappa) >= g_F^\kappa \Delta_c^{1-D_F} (y_1 + \frac{\Delta_c}{2})^{D_F-1} (\frac{\Delta_c}{2} - y_\kappa)^{D_F-1} \prod_{\lambda=2}^{\kappa} (y_\lambda - y_{\lambda-1})^{D_F-1} \qquad (3.5)$$

where Δ_c denotes the size of the hadronic system in rapidity space. In particular the two particle correlation function satisfies the self-similarity condition :

$$< \rho_c(y_1)\rho_c(y_2) > \sim |y_1 - y_2|^{D_F - 1} \qquad (|y_1 - y_2| \gg \delta_0) \tag{3.6}$$

in the limit $\Delta_c \to \infty$ where the end effects of the system are negligible. Equation (3.6) specifies the fractal nature of the hadronic distribution (random fractal) for $\delta_0 \ll \delta y \ll \Delta_c$ and it is of interest to note that the hadronic fluid defined by eqs.(3.5) is also a critical system with the following distinct properties :

(1) The average multiplicity grows according to a power law $< N > \sim \Delta_c^{D_F}$

(2) The scaled factorial moments [8]

$$F_p(\delta, \Delta_c) = \frac{< N(N-1)...(N-p+1) >_\delta}{< N >_\delta^p} \tag{3.7}$$

have an intermittent behaviour in the region $\delta_0 \ll \delta \ll \Delta_c$ as follows :

$$F_p(\delta, \Delta_c) = \frac{p![\Gamma(D_F)]^{p-1}}{\Gamma[2 + D_F(p-1)]} \left(\frac{4\delta}{\Delta_c}\right)^{(D_F-1)(p-1)} \tag{3.8}$$

(3) The multiplicity distribution $P(N, \Delta_c)$ satisfies KNO scaling,

$$\begin{aligned} &< N > P(N, \Delta_c) = \Psi(N/ < N >) \\ &\Psi(x) \sim x^{1/2\eta} \exp(-ax^{1/\eta}) \qquad (\eta = D_F - 1) \end{aligned} \tag{3.9}$$

where $a = (1 - D_F) D_F^{D_F/1-D_F} \left[\frac{\Gamma(D_F)}{\Gamma(2D_F)}\right]^{1/1-D_F}$.

(4) The constraint of self-consistency for the fractal pomeron, specifies the internal coupling g_F in the Regge-Mueller diagrams (eqs.3.5) as follows [2] :

$$g_F = \delta_0^{-D_F} \frac{D_F}{\Gamma(D_F)} \left(\frac{2 - D_F}{2 - 2D_F}\right)^{D_F - 1} \tag{3.10}$$

(5) The fractal Pomeron propagators (exclusive and inclusive) have the following structure in the ξ-plane :

$$\tilde{P}_{ex}^{(F)}(\xi) = \exp(-b\xi^{D_F}) \qquad \tilde{P}_{in}^{(F)}(\xi) = [\exp(b\xi^{D_F}) - 1]^{-1} \tag{3.11}$$

where $b = \delta_0^{D_F} D_F^{-1}(2-D_F)^{1-D_F}(2-2D_F)^{D_F-1}$. We notice that both propagators have a leading singularity at $\xi = 0$ indicating a stability property for the fractal pomeron at $J = 1$ in contrast to the intercept gap problem of the ordinary bare pomeron (eq.2.9).

(6) The contribution of the fractal pomeron to the total cross section has the following structure :

$$\sigma_{ab}^{(c)} = g_a^{(c)} g_b^{(c)} f(\Delta_c/\delta_0) \qquad (\Delta_c \equiv \ln(s/s_0))$$
$$f(\tau) = \frac{(1+\eta)^{1/2}}{2\eta\sqrt{\pi}} \tau^{-\frac{1+\eta}{2\eta}} \sum_{\nu=1}^{+\infty} \nu^{\frac{1}{2\eta}} \exp\left[-\frac{1+\eta}{2(1-\eta)} \nu^{\frac{1}{\eta}} \tau^{\frac{\eta-1}{\eta}}\right] \tag{3.12}$$

where $\sqrt{s_0}$ is the energy threshold for producing a critical system in the collision $a+b$. The scaling function $f(\tau)$ has a non conventional threshold behaviour at $\tau = 0$ (essential singularity) with a pronounced maximum at $\tau \approx 1$ and a power law behaviour $f(\tau) \sim \tau^{-\eta}$ for $\tau \gg 1$, reflecting the fractal geometry in rapidity space.

(7) The effective two-particle potential in the critical hadronic system has the following form in rapidity space :

$$V(y_i - y_j) = \frac{2-D_F}{1-D_F} \ln\left|\frac{y_i - y_j}{\delta_0}\right| + (1-D_F) D_F^{\frac{D_F}{1-D_F}} \left|\frac{y_i - y_j}{\delta_0}\right|^{-\frac{D_F}{1-D_F}} \tag{3.13}$$

The long range effect of this potential is a necessary mechanism for the criticality of the FW fluid since short range forces cannot generate a phase transition in a one-dimensional system.

(8) The grand partition function $Q_c(\zeta, \Delta_c)$ of the critical system under consideration has the following structure in the ξ-plane

$$Q_c(\zeta, \Delta_c) = \left[\exp(b\xi^{D_F}) - \zeta\right]^{-1} \tag{3.14}$$

Taking the contribution of the leading singularity for $\zeta > 1$ we obtain in the limit $\Delta_c \to \infty$ (thermodynamic limit)

$$\ln Q_c(\zeta, \Delta_c) \sim \frac{\Delta_c}{\delta_0} (\ln\zeta)^{1/D_F} \quad , \qquad p(\zeta) \sim (\ln\zeta)^{1/D_F} \tag{3.15}$$

Equation (3.15) gives the analogue pressure of the FW fluid, specifies its critical behaviour for $\zeta \to 1$ and shows that, in the same limit, Kadanoff scaling is fulfilled, as expected, due to the constraint of self-similarity in rapidity space.

We have listed the characteristic properties of a hadronic system generated by a factorizable Regge-Mueller singularity at the position $J = 1$ and corresponding to a uniform fractal in rapidity space (fractal pomeron). Our model describes a critical system [2] in one dimension and the corresponding critical index $\eta = 1 - D_F$ $(0 < \eta < 1)$ has its geometrical origin in the fractal dimension D_F of the rapidity space. The other fundamental parameter which, together with the critical index η, completely specifies the system, is the minimal scale δ_0 which introduces a lower limit to the fractal structure leading to a violation of self-similarity in the domains $\delta y \approx O(\delta_0)$ of the rapidity space.

From the point of view of the S-matrix approach, our one-dimensional solution in rapidity space incorporates unitarity, maximal analyticity (leading singularity at $J = 1$), self-similarity (fractal measure in y-space) and maximal simplicity (factorizability and uniformity of the propagator in the fractal y-space) as the basic ingredients leading to a unique, self-consistent and non trivial component of the S-matrix associated with a number of new, non conventional and measurable effects (intermittency, new threshold in high-energy diffraction, intermittency breaking).

The physical origin of the hadronic system corresponding to our solution (critical FW fluid) may be consistently identified with the hadronization process, at the transition temperature $T = T_c$, of a quark-gluon system generated during the development of the collision. The strong density fluctuations in the critical FW fluid, found in our model as a result of the fractal geometry in the rapidity space, reflect at the level of the S- matrix, the non perturbative aspects of QCD and especially the fractal structure of the ordinary space in the deconfinement region [4]. We therefore propose to consider our solution as a model for the critical component of the S-matrix (describing a newly hadronized system) which may be combined with an ordinary component for normal events in a two-fluid treatment of multiparticle production in a high-energy collision.

4. The two-component model

In this model we consider the fluid of hadrons produced in a high-energy collision as the mixture of a normal component corresponding to the ordinary hadronic system with short range correlations in rapidity and a critical component generated in a second order phase transition process during the development of the hadronization. The inclusive rapidity densities of hadrons in this model are written as follows :

$$\rho(y_1, y_2, .., y_\kappa) = \rho_c(y_1, y_2, .., y_\kappa) + \rho_n(y_1, y_2, .., y_\kappa) \tag{4.1}$$

where,

$$\begin{aligned} \rho_c(y_1, y_2, ..., y_\kappa) &= \lambda_c < \rho_c(y_1)\rho_c(y_2)...\rho_c(y_\kappa) > \\ \rho_n(y_1, y_2, ..., y_\kappa) &= (1-\lambda_c) < \rho_n(y_1)\rho_n(y_2)...\rho_n(y_\kappa) > \end{aligned} \tag{4.2}$$

The normal component $< \rho_n(y_1)...\rho_n(y_\kappa) >$ lies in the physical sector of the S-matrix generated by the bare pomeron dynamics, enlarged with secondary Regge trajectories and properly modified by higher order corrections. The component $< \rho_c(y_1)...\rho_c(y_\kappa) >$ on the other hand. belongs to the critical sector of the S-matrix (critical FW fluid) and is identified with the solution (3.5) generated by the fractal pomeron dynamics [2]. The factor λ_c in eqs.(4.2) is an energy dependent mixing parameter which gives the probability for producing a critical system in the collision $a+b$, $\lambda_c = \sigma_{ab}^{(c)}/\sigma_{ab}$ $(0 \leq \lambda_c \leq 1)$. Using eqs.(3.12) one may express the parameter λ_c in terms of the external couplings g_a, g_b of the fractal pomeron and the fundamental parameters of the critical system as follows :

$$\lambda_c^{-1} = 1 + \frac{\sigma_{ab}^{(0)}}{g_a g_b} \delta_0 f^{-1}\left(\frac{\Delta_c}{\delta_0}\right) \tag{4.3}$$

In eq.(4.3) $\sigma_{ab}^{(0)}$ is the normal component of the total cross section σ_{ab} corresponding to conventional multihadron intermediate states in the unitarity equation. Integrating now eqs.(4.1) in a rapidity interval δ, in the central region, $|y| \leq \delta/2$, we find the corresponding two-component model for the moments $F_p(\delta, \Delta_c)$ as follows :

$$F_p(\delta, \Delta_c) = \lambda_c \left[\frac{< \rho_c(0) >}{< \rho(0) >}\right]^p F_p^{(c)}(\delta, \Delta_c) + (1-\lambda_c)\left[\frac{< \rho_n(0) >}{< \rho(0) >}\right]^p F_p^{(n)}(\delta, \Delta_c) \tag{4.4}$$

For the critical component $F_p^{(c)}(\delta, \Delta_c)$ one may use the power law (3.8) which is valid in the fractality region, $\delta_0 \ll \delta \ll \Delta_c$, whereas for the normal term $F_p^{(n)}(\delta, \Delta_c)$ one may adopt in practice a phenomenological model for the conventional system assuming an exponential two-particle correlation function for the lowest moment ($p = 2$) and a negative binomial distribution prescription for the higher moments ($p > 2$). For completeness we write the corresponding expressions as follows :

$$\begin{aligned} F_2^{(n)}(\delta, \Delta) &= 1 + \frac{\gamma \delta_c}{\delta}(1 - e^{-\delta/\delta_c}) \\ F_p^{(n)} &= F_{p-1}^{(n)}(\delta, \Delta)[1 + (p-1)(F_2^{(n)} - 1)] \end{aligned} \tag{4.5}$$

where δ_c is the correlation length in rapidity and γ the strength of the two-particle correlation [9]. With these specifications, the two component model (eq.4.4) may confront the measurements of the scaled factorial moments in several experiments leading to a number of qualitative results for the corresponding pattern and the underlying mechanism as follows [2].

(a) The scaled factorial moments of lowest order ($p = 2, 3$) are dominated by the conventional component whereas a clear intermittency pattern appears at the level of higher moments. The intermittency indices form a linear spectrum indicating a second order phase transition.

(b) The intermittency effect due to a quark-hadron phase transition may be significant even if the production cross section for the critical system is very small ($\lambda_c \simeq 10^{-5}$). The magnitude of the mixing parameter λ_c is a measure of the population of exceptional events in the sample characterized by strong fluctuations in rapidity density. These events belong to a critical hadronic system which may be interpreted as a newly hadronized quark-gluon plasma.

(c) The intermittency pattern breaks down for rapidity intervals close to a minimal scale, $\delta_m \sim \delta_0$, as a result of self-similarity violation in these domains of the rapidity space.

Furthermore, the presence of the critical component in the S-matrix leads to measurable effects in the diffractive sector of the collision introducing a new energy threshold, $\sqrt{s_0}$, associated with the production of a critical hadronic system. In particular, the elastic amplitude $A_{ab}(s,t)$ in the forward direction ($t = 0$) admits a contribution from the critical component as follows [2]

$$A_{ab}(s,0) = A_{ab}^{(0)}(s,0) + \frac{s g_a^{(c)} g_b^{(c)}}{\pi} \int_0^{+\infty} \frac{f(\tau')d\tau'}{\sinh[\delta_0(\tau' - \tau)]} \tag{4.6}$$

where $f(\tau)$ is given by eq.(3.12) and $A_{ab}^{(0)}(s,0)$ is the conventional elastic amplitude corresponding to a standard, rising total cross section at high energies and a slowly varying real part [10]. Using the convention $\sigma_{ab} = s^{-1} Im A_{ab}$, we obtain from eq.(4.6) the following measurable quantities :

$$Re A_{ab}(s,0) = Re A_{ab}^{(0)}(s,0) + \frac{s g_a^{(c)} g_b^{(c)}}{\pi} \mathbf{P} \int_0^{+\infty} \frac{f(\tau')d\tau'}{\sinh[\delta_0(\tau' - \tau)]} \tag{4.7}$$

$$\sigma_{ab}(s) = \sigma_{ab}^{(0)} + g_a^{(c)} g_b^{(c)} \delta_0^{-1} \theta(s - s_0) f(\delta_0^{-1} \ln \frac{s}{s_0}) \tag{4.8}$$

For $p\bar{p}$ collisions, the onset of the critical component at CERN collider energies may explain the unexpectedly large value of the real part found by the UA4 experiment [11]. The precise remeasurement of this effect in a forthcoming experiment at CERN [12] together with more accurate measurements of the factorial moments, may constraint the parameters of the model providing us with the basic ingredients for a proposal towards a phenomenological theory of the critical sector of the S-matrix.

5. Concluding remarks

We have formulated a model for the one-dimensional critical sector of the hadronic S-matrix introducing a fractal measure in the rapidity space. Hence, together with the conventional principles of maximal analyticity, unitarity and self-consistency we have imposed the property of self-similarity in rapidity space as a new constraint of the S-matrix, associated with the criticality of the system and the related non conventional production mechanism (quark-hadron phase transition). Within the framework of the fractal geometry introduced in the rapidity space we have found a self-consistent solution corresponding to a factorizable singularity at the point $J = 1$ of the angular momentum plane (fractal pomeron) which leads to a hadronic system (FW fluid) undergoing a second order phase transition. The fundamental parameters of the model are (a) the critical exponent η related with the fractal dimension, $\eta = 1 - D_F$ $(0 < \eta < 1)$ and (b) the minimal scale δ_0, physically connected with the maximal time scale in the hadronization process [1]. At large intervals in rapidity $(\delta \approx \Delta)$ the system obeys KNO scaling whereas at small scales $(\delta \ll \Delta)$ the system becomes intermittent. The intermittency effect breaks down if the rapidity interval δ is comparable with the minimal scale δ_0. In order to study phenomenologically these remarkable effects we have constructed a simple model for the complete S-matrix in rapidity space taking into account the totality of the events in a high-energy collision, including the conventional, non critical states of the hadronic fluid. We have adopted a two-component mechanism for the inclusive densities, introducing a mixing parameter in terms of the couplings of the fractal pomeron with the initial hadrons. In this model the non conventional properties of the hadronic fluid compete with the standard behaviour of the hadronic system at the level of higher order factorial moments. Moreover, the production of a critical system may generate a new threshold $\sqrt{s_0}$ in high energy collisions with measurable effects in the real part of the forward elastic amplitude ($\sqrt{s_0} \simeq 542\ GeV$ for $p\bar{p}$ collisions). It is therefore suggestive that in order to test these theoretical ideas which attempt to connect the non perturbative sector of QCD with a critical (fractal) sector of the S-matrix, measurements of high statistics are required both of the scaled factorial moments in the limit $\delta \rightarrow 0$ and the elastic-diffractive scattering in the limit $s \rightarrow s_0$ [2]. Finally, the forthcoming extensive experimental programme with relativistic heavy ions is expected to play a decisive role in establishing the critical properties of the strongly interacting system and the nature of the quark-hadron phase transition.

References

[1] N.G.Antoniou et *al.*. CERN preprint TH-5815/90 ; Phys.Lett.**B245** (1990) 619.

[2] N.G.Antoniou et *al.*, Phys.Lett.**B260** (1991) 199 ; Phys.Lett.**B245** (1990) 624 ; N.G.Antoniou, F.Diakonos and C.G.Papadopoulos "The phenomenology of the fractal pomeron" Phys.Lett. to be published.

[3] Landau and Lifshitz, Statistical Physics (Part 2) Pergamon Press (1980)

[4] M.I.Polikarpov, Phys.Lett.**B236** (1990) 61.

[5] K.Binder, Ann.Phys.**98** (1976) 390.

[6] K.Falconer, Fractal Geometry, John Wiley (1990).

[7] R.L.Crawford and R.Jennings (Editors), "Phenomenology of particles at high energies" Academic Press (1974).

[8] A.Bialas and R.Peschanski, Nucl.Phys.**B273** (1986) 703 ; **B308** (1988) 803.

[9] P.Carruthers and I.Sarcevic, Phys.Rev.Lett.**63** (1989) 1562.

[10] A.Martin, Proc.of the 2nd International Conference on Elastic and Diffractive Scattering, ed.K.Goulianos (Editions Frontiéres) p.145 (1987).

[11] UA4 Collaboration. D.Bernard et *al.*, Phys.Lett.**B198** (1987) 583.

[12] Proposal CERN/SPSC 90-9, SPSC/P250 (1990).