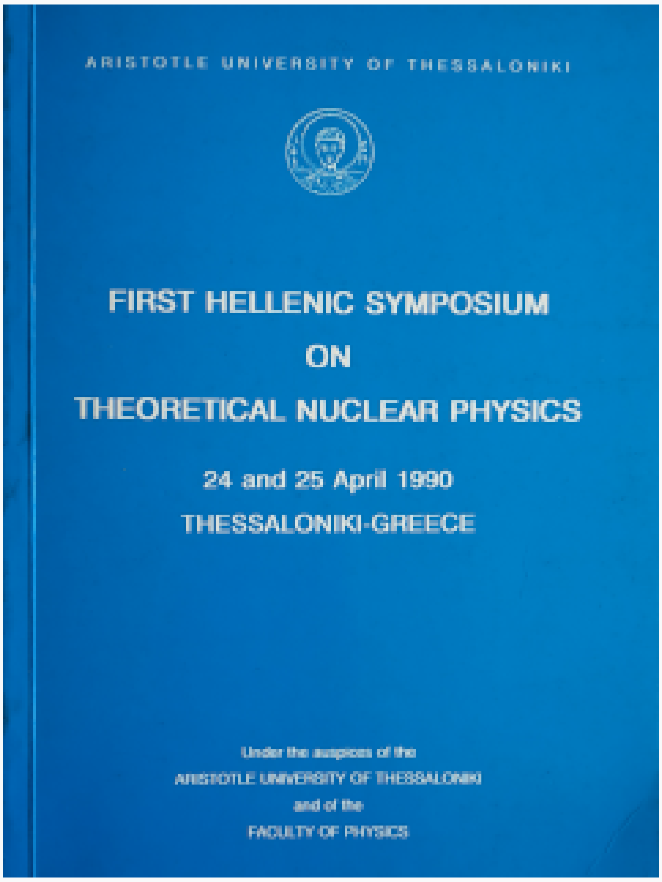
Annual Symposium of the Hellenic Nuclear Physics Society

Τόμ. 1 (1990)

HNPS1990



Approximate Relativistic Mass-Formulae for the Ground State Binding Energy of a Λ in Hypernuclei

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doi: 10.12681/hnps.2837

Approximate Relativistic Mass-Formulae for the Ground State Binding Energy of a Λ in Hypernuclei

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Abstract The Dirac equation with potentials having attractive and repulsive parts is considered in a simplified model and approximate semiempirical mass formulae for the ground state binding energy of a Λ in hypernuclei are derived and discussed.

1. Introduction

In recent years it became possible by means of (π^+,K^+) experiments to determine[1] the binding energies of the Λ particle in its ground and excited states for a number of hypernuclei ranging from $^{9}_{\Lambda}Be$ up to $^{89}_{\Lambda}Y$.
These results can be analysed by using either non-relativistic or relativistic quantum mechanics[2-6]. In this work we shall use the latter kind of analysis. In the following section the formalism used, following ref.7, is described. In the third section the results for the ground state wave functions and the eigenvalue equation are given in a simple model. In section 4 the asymptotic expression for the ground state binding energy for the Λ particle is given and compared with the non-relativistic one. An (approximate) relativistic semi-empirical mass formula for B_Λ is also given which appears to be fairly satisfactory even for rather small values of the mass number ($A \approx 16$). In the final section numerical results are given and comments are made.

2. The formalism

It is assumed that the differential equation governing the motion of the Λ-particle in hypernuclei is the following Dirac equation[4,5]

$$[c\vec{\alpha}\vec{p} + \beta\mu c^2 + \beta U_S(r) + U_V(r)]\Psi = E\Psi \tag{1}$$

where the average Λ-nucleus potential is composed of an attractive component $U_S(r)$ and a repulsive component $U_V(r)$. μ is taken here to be the Λ-core reduced mass.

[+] Presented by C.G.Koutroulos

The Dirac equation can be reduced to the following pair or radial differential equations.

$$\frac{dG(r)}{dr} = D(r)F(r) - \frac{k}{r} G(r) \qquad (2)$$

$$\frac{dF(r)}{dr} = H(r)G(r) + \frac{k}{r} F(r) \qquad (3)$$

where $k=\pm(j+1/2)$, $j=l\mp1/2$ and

$$D(r) = \frac{1}{\hbar c}(\mu c^2+E+U_-(r)) = \frac{1}{\hbar c}(2\mu c^2-B_\Lambda+U_-(r)) \qquad (4)$$

$$H(r) = \frac{1}{\hbar c}(\mu c^2-E+U_+(r)) = \frac{1}{\hbar c}(B_\Lambda+U_+(r)) \qquad (5)$$

The potentials $U_+(r)$ and $U_-(r)$ are defined as follows:

$$U_+(r) = U_s(r) + U_v(r) \qquad (6)$$

$$U_-(r) = U_s(r) - U_v(r) \qquad (7)$$

The pair of radial differential equations (2)-(3) is reduced to a second order differential equation of the Schrodinger type i.e.[4)]

$$g''(r) - \{ \frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2} (V_{centr} + V_{S.O.} + B_\Lambda) \}g(r) = 0 \qquad (8)$$

where the central part of the potential is

$$V_{centr} = U_+(r) + \frac{\hbar^2}{2\mu} \{ \frac{1}{\hbar^2 c^2} (U_+(r) + B_\Lambda)(U_-(r) - B_\Lambda) -$$
$$-D^{-1}(r)D'(r)r^{-1} -(2D(r))^{-1} D''(r) + \frac{3}{4} D^{-2}(r)(D'(r))^2\} \qquad (9)$$

while the spin orbit part is given by the expression

$$V_{S.O.} = - \frac{\hbar}{2\mu} \frac{1}{(2\mu c^2 -B_\Lambda+U_-(r))} \frac{1}{r} \frac{dU_-(r)}{dr} \vec{l}.\vec{\sigma} \qquad (10)$$

It should be noted that the potentials V_{centr} and $V_{S.O.}$ depend on r as well as on B_Λ i.e. they are energy dependent.

The large G(r) and small F(r) components may be expressed in terms of g(r) as follows:

$$G(r) = g(r)\sqrt{D(r)} \tag{11a}$$

$$F(r) = [G'(r) + \frac{k}{r} G(r)]D^{-1}(r) \tag{11b}$$

In the following we shall apply the above formalism to the rectangular potential model.

3. Rectangular Potential shapes

We assume that the potentials $U_+(r)$ and $U_-(r)$ are of rectangular shape with the same radius R and depths D_+ and D_- respectively i.e.

$$U_\pm(r) = -D_\pm(1-\theta(r-R)] \tag{12}$$

where θ(r-R) is the unit step function.

The ground state (l=0 , k=-1) wave functions may be obtained analytically in a way similar to that for the usual Dirac equation. The result is:

$$G(r) = No\{[1-\theta(r-R)]\sin\eta r+\theta(r-R)\sin\eta R e^{-\eta_o(r-R)}\}, \quad 0\leq r<\infty \tag{13}$$

$$F(r) = N_o c\hbar\{[1-\theta(r-R)\ ] \frac{1}{(2\mu c^2-B_\Lambda-D_-)} [\eta\cos\eta r- \frac{1}{r}\sin\eta r]+\theta(r-R) \frac{-\sin\eta R}{(2\mu c^2-B_\Lambda)} [\eta_o+ \frac{1}{r}]e^{-\eta_o(r-R)}\} , \quad 0\leq r<\infty \tag{14}$$

The constant N_o is determined by using the normalization contition

$$\int_0^\infty (G^2(r)+F^2(r))dr = 1 \tag{15}$$

The corresponding energy eigenvalue equation is the following

$$\cot\eta R = -\frac{\eta_o}{\eta}\left[\frac{1-\{B_\Lambda+D_-[1+(\eta_o R)^{-1}]\}(2\mu c^2)^{-1}}{1-B_\Lambda(2\mu c^2)^{-1}}\right] \qquad (16)$$

where η_o and η are given by the following expressions:

$$\eta_o = \{\frac{2\mu}{\hbar^2} B_\Lambda[1-B_\Lambda(2\mu c^2)^{-1}]\}^{1/2} \qquad (17)$$

$$\eta = \{\frac{2\mu}{\hbar^2}(D_+-B_\Lambda)[1-(B_\Lambda+D_-)(2\mu c^2)^{-1}]\}^{1/2} \qquad (18)$$

The eigenvalue equation (16) will be used in the following section to obtain approximate explicit expressions for B_Λ.

4. Approximate semi-empirical mass formulae for the ground state binding energy of the Λ

Following a method analogous to the one used for the non-relativistic case by D.D.Ivanenco and N.N.Kolesnikov[8] and also by J.D.Walecka[9] the eigenvalue equation (16) can be solved approximately for the ground state binding energy $B^{g.s.}=B_\Lambda$ if the radius R is sufficiently large.Thus, various approximate expressions for B_Λ as functions of the mass number of the core nucleus A_{core} may be derived. These play the role of (approximate) relativistic "mass formulae". We mention two of these "semi-empirical mass formulae". More complicated expressions may be also derived[10].

i) The asymptotic expression for B_Λ

$$B_\Lambda^{as} = D_+ - \frac{\hbar^2\pi^2}{2m^*R^2} \qquad (19)$$

where $R=r_0A_{core}^{1/3}$. The above formula which is valid for very large values of R looks-like the non-relativistic one except that the effective mass m^* of the Λ-particle:

$$m^*=m[1-\frac{D_++D_-}{2mc^2}] \qquad (20)$$

appears instead of its usual mass m.

ii) An improved expression for B_Λ

An expression which gives fairly satisfactory values for B_Λ even for rather small values of A_{core} is the following:

$$B_\Lambda^{approx.} = D_+ - \frac{\hbar^2 \pi^2}{2\mu g[1+(\tilde{f}\eta_0 R)^{-1}]^2 R^2} \quad (21)$$

where

$$g = 1-(D_+ + D_-)(2\mu c^2)^{-1} \quad (22)$$

and

$$\tilde{f} = \frac{(1-(D_+ + D_-(1+(\eta_0 R)^{-1}))(2\mu c^2)^{-1}}{1-D_+(2\mu c^2)^{-1}} \quad (23)$$

The quantity η_0 is estimated from (17) with $B_\Lambda \approx D_+$

5. Numerical results and comments:

The potential parameters have been determined by an (unweighted) least-square fitting of B_Λ to the experimental values for a number of hypernuclei (see table 1) by considering the D_+, D_- and r_0 as adjustable parameters. The "exact" values of B_Λ were obtained by solving the eigenvalue equation (16) numerically. The best fit values are:

$D_+ = 489.20$ MeV, $D_- = 30.57$ MeV, $r_0 = 1.05$ fm

Some of our results are displayed in table 2. It is seen that expression (21) gives values fairly close to those obtained by solving the eigenvalue equation for values of the mass number larger than A=16. The values of parameters in the r.h.s. of table 2 are those corres-

TABLE 1

Experimental B_Λ values used for the fitting.

Hyper-nucleus	B_Λ^{exp} (MeV)	A_{core}	B_Λ^{exp} (MeV)
${}^{10}_{\Lambda}B$	8.89±0.12	63	21.90±0.20
${}^{13}_{\Lambda}C$	11.69±0.12	72	23.00±0.20
${}^{15}_{\Lambda}N$	13.59±0.15	80	22.50±0.20
${}^{16}_{\Lambda}O$	13.00±2.0	93	22.80±0.20
${}^{32}_{\Lambda}S$	17.50±0.5	103	23.70±0.20

TABLE 2

Comparison between the values of B_Λ("exact") and $B_\Lambda^{approx.}$.

A	D_-=489.20 MeV D_+=30.57 MeV r_o=1.054 fm m^*=0.767m	D_-=590.15 MeV D_+=29.50 MeV r_o=1.123 fm m^*=0.722m
	B_Λ("exact") MeV	$B_\Lambda^{approx.}$ MeV
12	11.59	9.92
16	13.97	12.71
20	15.66	14.61
32	18.78	17.95
40	20.05	19.27
90	23.75	22.98
140	25.23	24.42
208	26.30	25.46

ponding to the fitting of $B_\Lambda^{approx.}$. These values have been used in obtaining the results for $B_\Lambda^{approx.}$displayed in this table. More numerical results can be found in ref.7. The values of the effective mass of the Λ-particle estimated using the above values of the parameters is found to be $m^* \approx 0.8m$ which is in agreement with other estimates.

More results related to the present approach can be found in ref. 7 . It should be finally noted that the relativistically estimated values of B_Λ are very close to the non-relativistic ones.

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Phenomenological relativistic study of the Λ-hypernuclei with the Woods-Saxon and other potentials

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Abstract A phenomenological relativistic analysis of the ground and excited states binding energies of a Λ in hypernuclei, as these have been determined by recent (π^+,K^+) experiments, is made by using various Λ-nucleus potentials. The r.m.s. radii of the orbits of the Λ-particle are also determined, in the case of the Woods-Saxon potential.

1.Introduction

The relativistic treatment of Λ hypernuclei discussed recently[1,2] has the advantage that approximate analytic expressions for B_Λ can be obtained but it has also the disadvantage that the assumed rectangular shapes of the potentials are quite unrealistic, in particular for lighter hypernuclei. Thus, we have extended our phenomenological relativistic treatment to other potentials like the Gaussian[3,4] or the Woods-Saxon[3,5] e.t.c. The former is expected to be suitable for the lighter hypernuclei while the latter should be quite a realistic representation of the average Λ-nucleus potential for a wide range of mass numbers. Our results concerning the parameters of these potentials, the binding energies of the Λ particle in various hypernuclei in the ground and excited states as well as the mean square radii[6] of the orbits of the Λ particle are given in the following sections. More results can be found in references 3-6.

2. Potential shapes and parameters.

In this section we give the values of the potential parameters of the various potentials, some or all of which were determined by a least square fitting procedure, using the experimental values of the binding energies for the ground state of the Λ in hypernuclei displayed in table 1 of the preceding contribution.

i) Woods-Saxon potential

$$U_\pm(r)=\frac{-D_\pm}{1+e^{(r-c)/a}} \tag{1}$$

where c can be either of the simple form[7]

$$c=r_0A^{1/3} \tag{2}$$

or of the more complicated one[8]

$$c=(1/2)^{1/3}\, r_0A^{1/3}\, [\, [1+(1+2^2\, 3^{-3}(\frac{\pi\, a}{r_0A^{1/3}})^6\,)^{1/2}\,]^{1/3}+ +[1-(1+2^2\, 3^{-3}(\frac{\pi\, a}{r_0A^{1/3}})^6)^{1/2}\,]^{1/3}\,] \tag{3}$$

We have considered the following cases with $c=r_0A^{1/3}$:

a) The parameters a and D_ are kept fixed to the values a=0.6fm and D_=443MeV.The values of the remaining two parameters are then the following: D_+ =30.0MeV and r_0=1.211fm

b) a=0.6 fm (fixed) and D_=300 MeV (fixed). In this case we obtain D_+=29.8 MeV and r_0=1.198 fm.

c) All the parameters are taken as adjustable . Their values are found to be D_+=29.5 MeV , D_=416.9 MeV , r_0=1.153 fm and a=0.32 fm

Also we have considered the following cases when c is given by the more complicated expression (3):

a) The parameters a and D_ are kept fixed to the values a=0.6 fm and D_=443 MeV. The values of the remaining two parameters are then the following: D_+ =28.3 MeV and r_0=1.45fm.

b) All the parametrs are taken as adjustable. Their values are found to be D_+=28.2 MeV, D_=427.1 MeV, r_0=1.475 fm and a=0.63 fm. The advantage of using the more complicated expression for c is that with this the fitting is a little better i.e. the x^2 is smaller.

ii) Gaussian potential

$$U_\pm(r)=-D_\pm e^{-r^2/R^2} \tag{4}$$

where $R = r_0A^{1/3}$

The potential parameters obtained are: D_+=33.6 MeV , D_=427 MeV , r_0=1.27 fm.

iii) Symmetrized Woods-Saxon potential

$$U_\pm(r)=-D_\pm[(1+e^{(r-c)/a})^{-1}+(1+e^{(-r-c)/a})^{-1}-1] \tag{5}$$

The potential parameters obtained (with $c=r_0A^{1/3}$) are: $D_+=29.9$ MeV, $D_-=697.5$ MeV, $r_0=1.178$ fm, a=0.31 fm.

iv) Potential of the form[9,10]

$$U_\pm(r)=\frac{-D_\pm}{\cosh^2(r/R)} \tag{6}$$

where $R = r_0A^{1/3}$

This potential has been used in ref.(9,10) for the non relativistic treatment of hypernuclei, where its region of validity is discussed.The potential parameters in this case are: $D_+=39.7$ MeV, $D_-=201.6$ Mev, $r_0=0.98$ fm.

Using the parameters of these potentials we have calculated in each case the binding energies of the ground and excited states of various hypernuclei and the results are shown in fig. 1.

3. Root mean square radii of the orbits of the Λ-particle in hypernuclei.

Using the wave functions G(r) and F(r),we have calculated in the case of the Woods-Saxon potential, the root mean square radii of the orbits of the Λ-particle in hypernuclei in the ground state $1s_{1/2}$ and also in the excited states $1p_{3/2}$ and $1p_{1/2}$ by means of the formula

$$<r^2>^{1/2} = \left(\frac{\int_0^\infty r^2[G^2(r)+F^2(r)]dr}{\int_0^\infty [G^2(r)+F^2(r)]dr}\right)^{1/2} \tag{7}$$

For the 1s state we have the approximate formula

$$\hbar\omega_\Lambda \approx \frac{3\hbar^2}{2\mu}\frac{1}{<r^2>} \approx \Delta_{sp} \tag{8}$$

for the determination of the r.m.s. radii. The Δ_{sp} is given by the formula

$$\Delta_{sp} = B_\Lambda(1s)-B_\Lambda(1p) \tag{9}$$

The results obtained with the "exact" and approximate formulae, (7) and (8), respectively, are given in table 1.

4. Comments

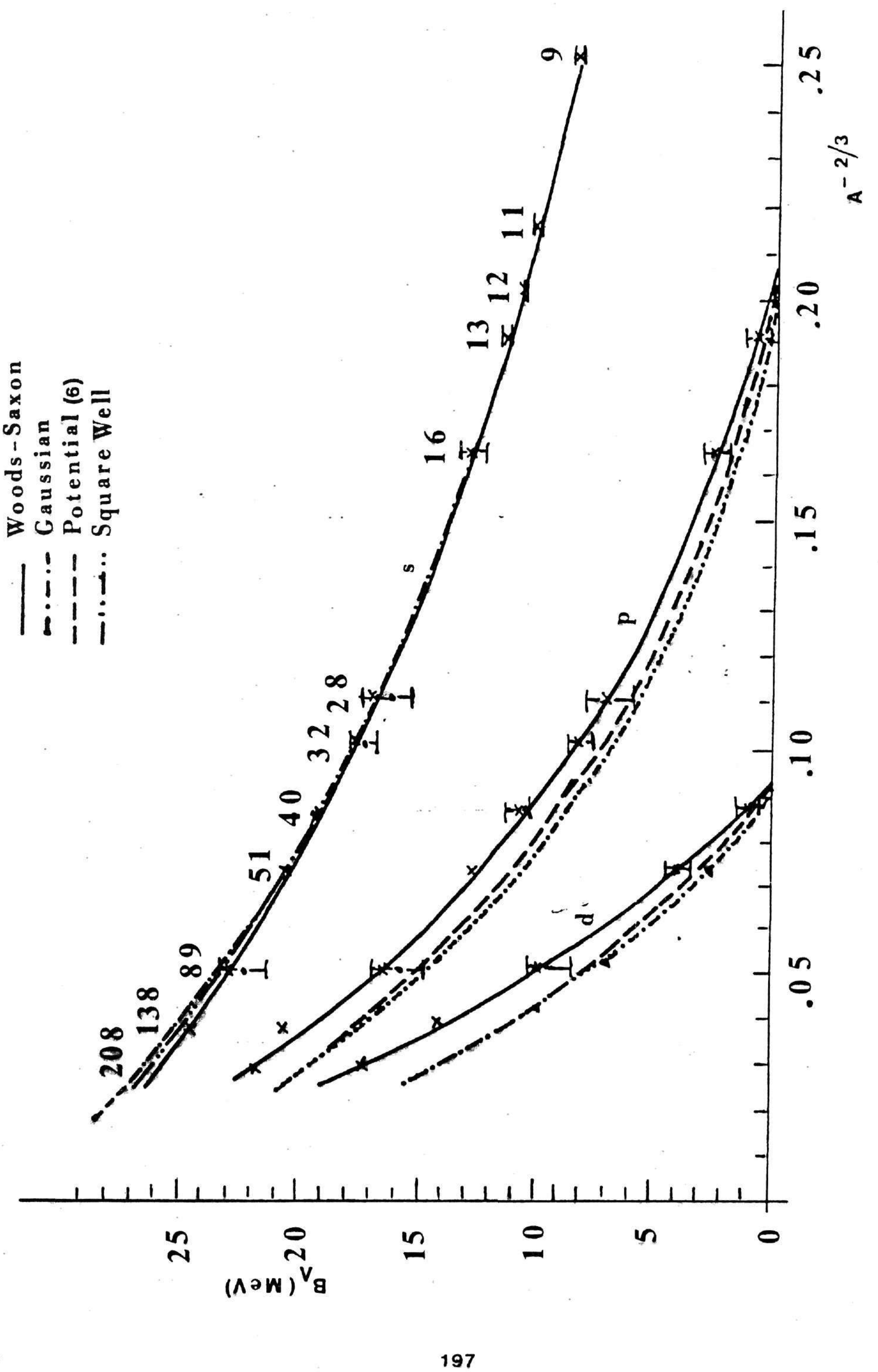
Our results concerning the binding energiers B_Λ of the ground and excited states of the Woods-Saxon potential are in a very good agreement with the experimental results of Chrien[11]. This is in agreement with the findings of the non-relativistic treatment of ref. 12. It should be noted also that the symmetrized Woods-Saxon potential gives very close results with those of the Woods-Saxon potential. The results are not so good with the other potentials for the excited states.

In this contribution we gave also an estimate of the root mean square radii of the orbits of the Λ-particle in hypernuclei. The calculations were performed using the Woods-Saxon potential and they refer to the ground state as well as the excited states $1p_{3/2}$ and $1p_{1/2}$. In the ground state the r.m.s. radii were also calculated using the approximate formula (8) (coll. II of table (1)). Comparing these values with the "exact" ones, given in col.(I) of the same table we see that this formula underestimates the r.m.s. radii, the difference being of the order of 4% - 6% . The r.m.s. radii obtained here do not differ very much from the non-relativistic results obtained by Rayet[13], with a different approach, the values of Rayet being slightly smaller than our values. Also our values for the ground state are smaller than the non-relativistic values found by Daskaloyannis et.al.[14], who used a square well potential.

TABLE 1

Root mean square radii of the orbits of the Λ-particle in hypernuclei using the Woods-Saxon potential

Hyper-nuclei	$1s_{1/2}$ "exact"	$1s_{1/2}$ approx.	$1p_{3/2}$	$1p_{1/2}$
${}^{13}_{\Lambda}C$	2.39	2.30		
${}^{16}_{\Lambda}O$	2.40	2.27		
${}^{28}_{\Lambda}Si$	2.52	2.36		
${}^{32}_{\Lambda}S$	2:54	2.39	3.32	3.32
${}^{40}_{\Lambda}Ca$	2.67	2.49	3.37	3.35
${}^{51}_{\Lambda}V$	2.79	2.61	3.47	3.45
${}^{89}_{\Lambda}Y$	3.16	2.93	3.82	3.79
${}^{138}_{\Lambda}B$	3.54	3.32	4.22	4.18
${}^{208}_{\Lambda}Pb$	3.97	3.72	4.69	4.65



Woods-Saxon
Gaussian
Potential (6)
Square Well
B_Λ (MeV)
$A^{-2/3}$
0
5
10
15
20
25
.05
.10
.15
.20
.25
208
138
89
51
40
32
28
16
13
12
11
9
s
p
d

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